وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	دورة سنة ٢٠٠٤ الاستثنائية
عدد المسائل : اربع	مسابقة في الرياضيات المدة : ساعتان	الاسم : الرقم :

ملاحظــة: يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (3.5 points).

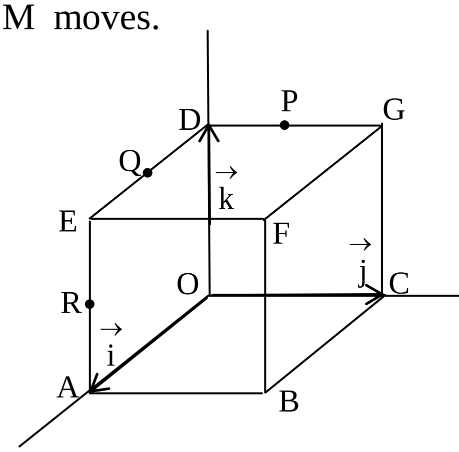
In the plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points

A , B and M of affixes -1 , 4 and z respectively, and let M' be the point of affix z'

such that $z' = \dfrac{z-4}{z+1}$ $(z \neq -1)$.

1) In the case where $z = 1+i$, write z' in its algebraic form, and give its exponential form.
2) Determine the values of z for which $z' = z$.
3) a- Give a geometric interpretation of $|z+1|$, and of $|z-4|$.
   b- Find, when $|z'| = 1$, the line on which the point M moves.

II- (3.5 points).



In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the cube OABCDEFG such that : A(1 ; 0 ; 0) , B(1 ; 1 ; 0) and F(1 ; 1 ; 1) . Designate by P, Q and R the midpoints of the segments [DG] , [DE] and [AE] respectively .

1) a-Show that $2x + 2y + 2z - 3 = 0$ is an equation of the plane (PQR).
   b- Prove that the plane (PQR) passes through the midpoint of [AB] .
   c- Prove that the planes (PQR) and (BEG) are parallel.

2) a- What is the nature of quadrilateral EGCA ?
   b- Let M be a variable point on the line (AC) .
   Show that $\overrightarrow{AM} \times \overrightarrow{EF} = \overrightarrow{AM} \times \overrightarrow{GF}$.

III-(4 points).

A multiple choice test is made up of **three** independent questions. The candidate is required to answer all the questions .Each question has two suggested answers out of which only one is correct.

A candidate answers randomly each of these three questions.

1) a- Show that the probability that he answers the three questions correctly is equal to $\frac{1}{8}$.

b- Consider the event E : « Among the three answers of the candidate, exactly two are correct » .

Calculate the probability of E.

2) The test is marked as follows : **+5** points for each correct answer, and **-3** points for each wrong answer.

Designate by X the random variable that is equal to the total mark obtained by the candidate upon answering the questions of this test.

a- Determine the 4 possible values of X .

b- Determine the probability distribution of X , and calculate the mean (expected value) E (X).

IV-(9 points).

Consider the differential equation (E) : $y'' - 2y' + y = x + 1$.

1) Let $y = z + x + 3$.

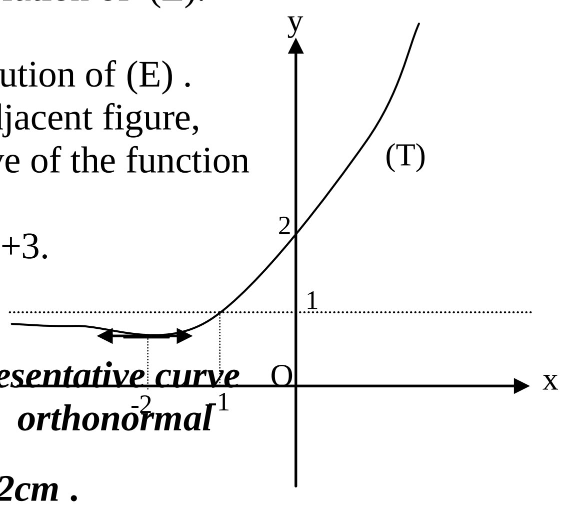
a- Write a differential equation (E') satisfied by z, and solve (E').

b- Deduce the general solution of (E).

2) Let f be a particular solution of (E) .

The curve (T) , in the adjacent figure, is the representative curve of the function f ' **the derivative** of f .

Show that $f(x) = xe^x + x + 3$.



Designate by (C) ***the representative curve of the function*** f ***in an orthonormal system*** $(O;\vec{i},\vec{j})$ ***; unit 2cm .***

3) a- Calculate f(1) and $\lim_{x \to +\infty} f(x)$.

b- Calculate $\lim_{x \to -\infty} f(x)$, and show that the line (d) of equation y = x + 3

is an asymptote of (C) .

c- Determine, according to the values of x, the relative positions of (C) and (d).

d- Verify that I (-2 ; $1 - \frac{2}{e^2}$) is a point of inflection of the curve (C) .

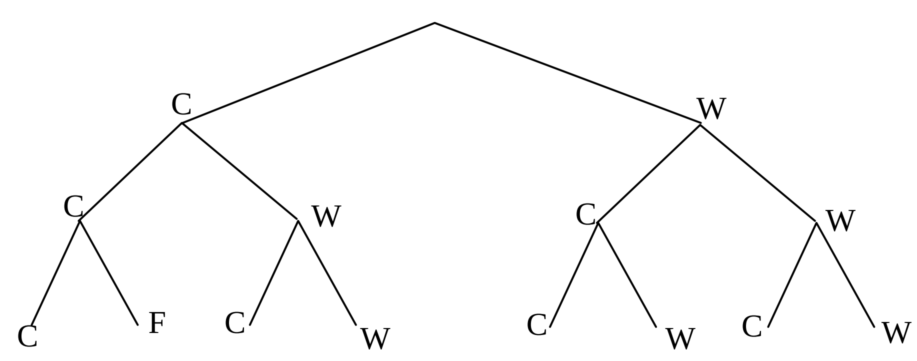
4) a- Verify that f is strictly increasing on IR , and set up its table of variations.

b- Draw (d) and (C).

c- Calculate ,in cm^2, the area of the region bounded by the curve (C), the

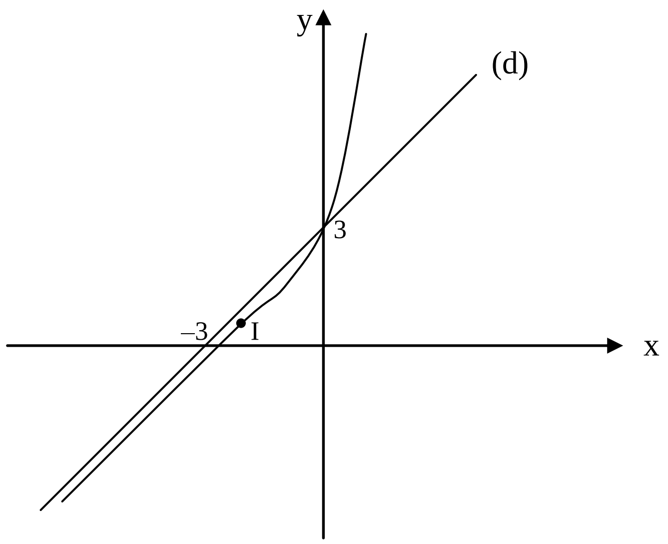
line (d) and the lines of equations x = 0 and x = 1 .

Answer Key

Life Sciences		MATH	2nd Session 2004
Questions		Answers	G
I	1	$z' = \dfrac{1+i-4}{1+i+1} = \dfrac{-3+i}{2+i} = -1 + i = \sqrt{2}e^{i\frac{3\pi}{4}}$	1 ½
	2	$z' = z$; $z = \dfrac{z-4}{z+1}$; $z^2 = -4$; $z = -2i$ or $z = 2i$.	½
	3-a-	$\lvert z+1\rvert = \lvert z_M - z_A\rvert = MA$; $\lvert z-4\rvert = \lvert z_M - z_B\rvert = MB$	½
	3-b-	$\lvert z'\rvert = \dfrac{MB}{MA}$; since $\lvert z'\rvert = 1$ then MB = MA. M moves on the perpendicular bisector of [AB].	1
II	1-a-	P(0 ; ½ ; 1) , Q(½ ; 0 ; 1) , R(1 ; 0 ; ½) the coordinates of P , Q and R verify the equation 2x + 2y + 2z – 3 = 0 ▶ or : M(x ; y ; z) is a point of the plane (PQR) iff $\vec{PM}.(\vec{PQ} \wedge \vec{PR}) = 0$.	½
	1-b-	I(1 ; ½ ;0) : mid point of [AB], the coordinates of I satisfy the equation of the plane (PQR).	½
	1-c-	(PQ) is parallel to (EG) ; (QR) is parallel to (DA) which is parallel to (BG) (PQR) contains two intersecting lines parallel to two intersecting lines in de (EBG) ; then (PQR) and (EBG) are parallel. ▶or: x + y + z – 2 = 0 is an equation of the plane (BEG) . The two distinct planes(PQR) and (BEG) are parallel having two collinear normal vectors.	1
	2-a-	$\vec{EA} = \vec{GC}$ (EA) is perpendicular to the plane (OABC) , then (EA)$\perp$(AC) EAGC is a rectangle.	½
	2-b-	$\vec{AM} \wedge \vec{EF} = \vec{AM} \wedge (\vec{EG} + \vec{GF}) = \vec{AM} \wedge \vec{EG} + \vec{AM} \wedge \vec{GF}$ $\vec{AM}$ and $\vec{EG}$ are collinear, $\vec{AM} \wedge \vec{EG} = \vec{0}$,then $\vec{AM} \wedge \vec{EF} = \vec{AM} \wedge \vec{GF}$ ▶or (AC) : x = – α + 1 ; y = α , z = 0 $\vec{AM}$(-α ; α ; 0) , $\vec{EF}$ (0 ;–1 ;0) , $\vec{GF}$ (1 ;0 ;0) ; $\vec{AM} \wedge \vec{EF} = \vec{AM} \wedge \vec{GF} = \alpha\ \vec{k}$	1
III			
	1-a-	P(CCC) = ½ × ½ ×½ = 1/8	1
	1-b-	P(E) = P(CCW) + p(CWC) + p(WCC) = 1/8 +1/8 + 1/8 = 3/8	1
	2-a-	The four possible values of X are – 9 ; – 1 ; 7 ; 15 .	½
	2-b-	see table below	1 ½

$x = x_i$	–9	– 1	7	15
P_i	1/8	3/8	3/8	1/8

		E(X) = – 9/8 – 3/8 + 21/8 + 15/8 = 3	

IV	1-a-	$y''-2y'+y = x+1$ with $y' = z'+1$ and $y'' = z''$ then $z''-2z'+z = 0$. Characteristic equation $r^2 - 2r + 1 = 0$; $r_1 = r_2 = 1$ and $z = (c_1x + c_2)e^x$.	1 ½
	1-b-	The general solution of (E) is $y = (c_1x + c_2)e^x + x + 3$.	½
	2	According to the graph: $f'(-1) = 1$ and $f'(0) = 2$ $f'(x) = c_1e^x + (c_1x + c_2)e^x + 1$ $f'(-1) = 1$ gives $\frac{c_2}{e} + 1 = 1$ so $c_2 = 0$, $f'(0) = 2$ gives $c_1 + c_2 + 1 = 2$ so $c_1 = 1$, $f(x) = xe^x + x + 3$.	1
	3-a-	$f(1) = e + 4 \approx 6.738$, $\lim\limits_{x\to+\infty} f(x) = +\infty$.	½
	3-b-	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} xe^x + \lim\limits_{x\to-\infty} (x+3) = 0 - \infty = -\infty$. $\lim\limits_{x\to-\infty} [f(x) - (x+3)] = \lim\limits_{x\to-\infty} xe^x = 0$, then the line (d) of equation y = x + 3 is an asymptote of (C) .	1
	3-c-	$f(x) - (x+3) = xe^x$ For x = 0 , (C) cuts (d) at point (0 ; 3) For x > 0 , (C) is above (d) For x < 0 , (C) is below (d) .	1
	3-d-	According to the graph $f''(-2) = 0$, Over $]-\infty ;-2[$: f ’ is decreasing then f ‘’ (x) < 0 Over $]-2 ; +\infty[$: f ’ is increasing then f ‘’ (x) > 0 The point I(–2 ;f(–2)= $1 - \frac{2}{e^2}$) is a point of inflection of (C).	½
	4-a-	(T) is above the axis of abscissas , then f ’(x) > 0 for every x , hence f is strictly increasing over R x: $-\infty$ … $+\infty$ f ’(x): + f (x): $-\infty \nearrow +\infty$	½
	4-b-		1 ½

	4-c-	$A = \int_0^1 xe^{x}dx = (x-1)e^{x} \Big|_0^1 = 1\ u^2$ then $A = 4\ cm^2$.	1