عدد المسائل ستة	مسابقة في مادة الرياضيات	الاسم:
	المدة: ساعتان	الرقم:

إرشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (3 points)

Consider the polynomial $p(x) = (x-2)^2 - (2-x)(x+4)$

1) Factorize p(x).

2) Expand and reduce p(x).

3) **a.** Expand and reduce $2(x-3)(x+2)$.

b. Calculate p(3).

c. Solve the equation $p(x) = 8$.

II- (2 points)

Given the two numbers A and B :

$A = (\sqrt{3}+\sqrt{2})^2 + (\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$; $B = \sqrt{50}+\sqrt{150}+\sqrt{96}+\sqrt{54}-5\sqrt{2}$.

1) Calculate A and write it in the form of $a + b\sqrt{6}$ where a and b are integers.

2) Calculate B and write it in the form of $x\sqrt{6}$ where x is an integer.

3) By using the answers in questions 1 and 2, rationalize the denominator of the expression $\frac{A}{B}$ and simplify the answer.

III- (2 points)

In what follows,we have the survey of the scores of 30 students of a class :

12 ; 18 ; 15 ; 11 ; 14 ; 7
14 ; 12 ; 11 ; 8 ; 18 ; 15
7 ; 18 ; 12 ; 14 ; 17 ; 10
14 ; 11 ; 10 ; 18 ; 17 ; 12
7 ; 12 ; 15 ; 8 ; 14 ; 17.

1) Use the above survey to construct a table of scores containing the frequencies and the increasing cumulative frequencies.

2) What is the percentage of the students who got a score less than 13?

3) Calculate the mean of the class scores.

IV- (2 points)

Consider an isosceles triangle ABC, such that AB = AC = 3 cm, and BC = 2 cm
M is the midpoint of [BC].

1) **a.** Calculate AM.

b. Calculate $\sin \widehat{ABC}$.

2) **a.** Calculate the area S of the triangle ABC.

b. Show that $2S = BA \times BC \times \sin \widehat{ABC}$.

V- (6 points)

In an orthonormal system of axes $(x'ox, y'oy)$, Consider the points $E(3;3)$, $F(2;-2)$ and $G(-2;4)$.

1) Locate the points E, F and G.

2) **a.** Given that $EG = \sqrt{26}$ calculate EF and FG.

b. Deduce that the triangle EFG is isosceles and right angled at E.

3) Let (C) be the circle circumscribed about triangle EFG .

a. Find the radius R of (C).

b. Calculate the coordinates of the point I , the center of (C), and deduce that I is on $(y'y)$.

c. Show that $P(-3,3)$ belongs to the circle (C).

4) **a.** Calculate the coordinates of point L, the translate (image) of E under the translation of vector $\overrightarrow{OP}$.

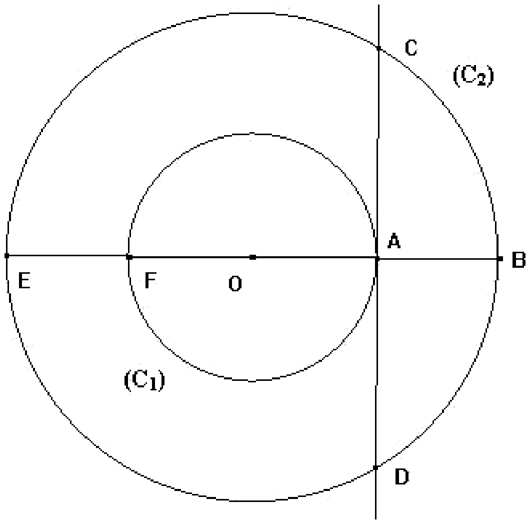
b. Determine the equation of the straight line (OE).

c. Determine the equation of the straight line (d') , the translate (image) of (OE) under the translation of vector $\overrightarrow{OP}$.

d. Show that P, G and L are collinear.

VI- (5 points)

In the figure below we have the 2 circles (C_1) and (C_2), of the same center O and their radii are $R_1 = 2$cm and $R_2 = 4$cm respectively. A straight line passing through O cuts (C_1) at F and A, and cuts (C_2) at B and E. The tangent to circle (C_1) at A cuts the circle (C_2) at C and D.



1) Show that (CD) is the perpendicular bisector of [OB].

2) Determine the nature of the triangle OBC.

3) Show that the quadrilateral OCBD is a rhombus.

4) Let P be the midpoint of [CE].

a. Calculate OP and deduce that P belongs to the circle (C_1).

b. Show that D, O and P are collinear.

c. Show that (OP) is perpendicular to (CE) and deduce that (CE) is tangent to (C_1).

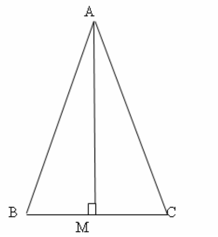
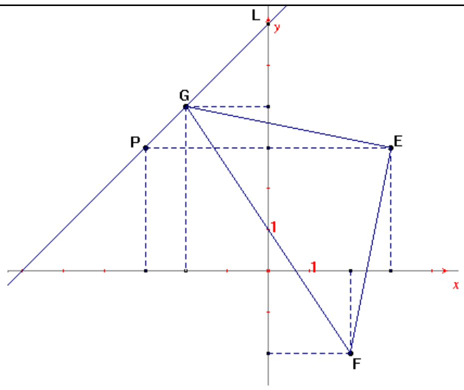
d. Show that (CO) is perpendicular to (DE).

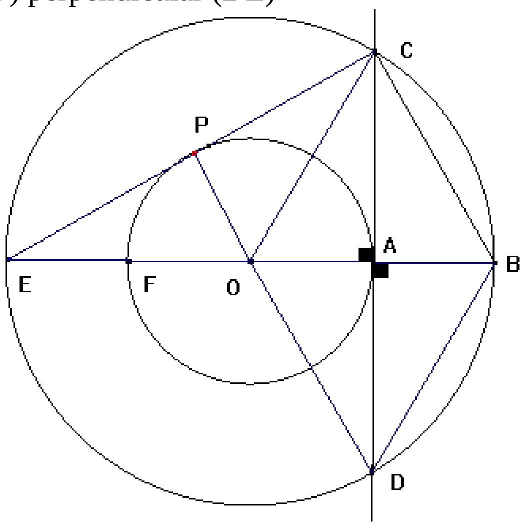
5) Show that the triangle EBC is an enlargement of the triangle EOP and precise the center and ratio of this enlargement.

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

الشهادة المتوسطة
مسابقة الرياضيات

دورة سنة ٢٠٠٨ العادية
اسس التصحيح

<table>
<tr><th>Part of the Q</th><th>Answer</th><th>Mark</th></tr>
<tr><td>I.1</td><td>$p(x) = (x – 2)^2 – (2 – x)(x + 4)$
$p(x) = (x – 2)(x – 2 + x + 4) = (x – 2)(2x + 2)$
$p(x) = 2(x – 2)(x + 1)$.</td><td>0.50</td></tr>
<tr><td>I.2</td><td>$p(x) = 2x^2 + 2x – 4x – 4$
$p(x) = 2x^2 – 2x –4$.</td><td>0.50</td></tr>
<tr><td>I.3.a</td><td>$2(x – 3)(x + 2) =2(x^2 + 2x–3x– 6) = 2x^2 – 2x – 12$.</td><td>0.50</td></tr>
<tr><td>I.3.b</td><td>$p(3) = 2(3 – 2)(3 + 1) = 8$.</td><td>0.50</td></tr>
<tr><td>I.3.c</td><td>If $p(x) = 8$, then $2x^2-2x-4=8$ so $2x^2 – 2x – 12=0$ $x=-2$ $x=3$</td><td>1</td></tr>
<tr><td>II.1</td><td>$A = 6 + 2\sqrt{6}$.</td><td>0.75</td></tr>
<tr><td>II.2</td><td>$B = 12\sqrt{6}$.</td><td>0.75</td></tr>
<tr><td>II.3</td><td>$\frac{A}{B}=\frac{6+2\sqrt{6}}{12\sqrt{6}}=\frac{6+2\sqrt{6}}{12\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{6\sqrt{6}+12}{12\times 6}=\frac{\sqrt{6}+2}{12}$</td><td>0.50</td></tr>
<tr><td>III.1</td><td><table>
<tr><td>score</td><td>7</td><td>8</td><td>10</td><td>11</td><td>12</td><td>14</td><td>15</td><td>17</td><td>18</td><td>Total</td></tr>
<tr><td>Frequency</td><td>3</td><td>2</td><td>2</td><td>3</td><td>5</td><td>5</td><td>3</td><td>3</td><td>4</td><td>30</td></tr>
<tr><td>Cumulative frequency</td><td>3</td><td>5</td><td>7</td><td>10</td><td>15</td><td>20</td><td>23</td><td>26</td><td>30</td><td></td></tr>
</table></td><td>0.75</td></tr>
<tr><td>III.2</td><td>15 students have a score less than 13 the percentage is 50%</td><td>0.25</td></tr>
<tr><td>III.3</td><td>$\overline{X}=\frac{7\times3+8\times2+10\times2+11\times3+12\times5+14\times5+15\times3+17\times3+18\times4}{30}$ $\frac{388}{30}=12.9$</td><td>0.50</td></tr>
<tr><td>IV.1.a</td><td>The triangle AMB is right at M and
BM = 1 cm.
$AB^2 = AM^2 + BM^2$
$9 = AM^2 + 1$;
So , $AM^2 = 8$ and
$AM = 2\sqrt{2}$
</td><td>0.50</td></tr>
<tr><td>IV.1.b</td><td>$\sin\widehat{ABC} = \frac{AM}{AB}=\frac{2\sqrt{2}}{3}$.</td><td>0.50</td></tr>
<tr><td>IV.2.a</td><td>S = Area of ABC = $\frac{1}{2}BC \times AM = \frac{1}{2} \times 2 \times 2\sqrt{2} = 2\sqrt{2}\ cm^2$.</td><td>0.50</td></tr>
<tr><td>IV.2.b</td><td>$2S = 4\sqrt{2}$
$BA \times BC \times \sin\widehat{ABC} = 3 \times 2 \times \frac{2\sqrt{2}}{3} = 4\sqrt{2}$.
Hence, $2S = BA \times BC \times \sin\widehat{ABC}$.</td><td>0.50</td></tr>
<tr><td>V.1</td><td>
Figure</td><td>0.50</td></tr>
<tr><td>V.2.a</td><td>$EF=\sqrt{26}=EG$
$FG=\sqrt{52}$</td><td>1</td></tr>
</table>

V.2.b	$FG^2 = EF^2 + EG^2$ Therefore EFG is a right isosceles triangle	**0.50**
V.3. a	Radius$=\frac{FG}{2}=\frac{\sqrt{52}}{2}=\sqrt{13}$	**0.50**
V.3. b	I(0 ;1) belongs to (y' y)	**0.75**
V.3. c	$IP=\sqrt{13}=$ Radius, therefore P belongs to the circle (C).	**0.25**
V.4. a	$\overrightarrow{OP}=\overrightarrow{EL}$; L(0;6)	**0.75**
V.4. b	Equation of (OE) : y = x	**0.25**
V.4. c	(d') has a translate (PL) : y = x + 6	**1**
V.4. d	G, P and L are points on (d')	**0.50**
VI.1	A is the midpoint of [OB] and (CD) is perpendicular to (OA) then (CD) is the perpendicular bisector of [OB].	**0.75**
VI.2	OC = BC and OB = OC then OBC is an equilateral triangle.	**0.50**
VI.3	OD = BD and OB = OD then OBD is an equilateral triangle therefore OC = BC = OD = BD then OCBD is a rhombus.	**0.50**
VI.4.a	Triangle ECB we have : O midpoint of [EB] and P midpoint of [CE] then (OP) // (BC) (midpoint theorem) and $OP=\frac{BC}{2}=\frac{4}{2}=2=R_2$ then P belongs to circle (C_1).	**0.75**
VI.4.b	(OD) // (BC) since ODBC is a rhombus and (OP) // (BC) (midpoint theorem); then at O we can have only 1 parallel to (BC) then P, O and D are collinear.	**0.50**
VI.4.c	[OP] is the median and height of the isosceles triangle COE then (OP) is the perpendicular to (CE) and (CE) perpendicular at P to OP, then (CE) is tangent to (C_2).	**0.75**
VI.4.d	In the triangle CED ; we have [EA] height, [DP] height then O orthocenter therefore [CO] is the 3rd height then (CO) perpendicular (DE) 	**0.50**
VI.5	Center E and ratio 2.	**0.75**