وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع الاجتماع والاقتصاد

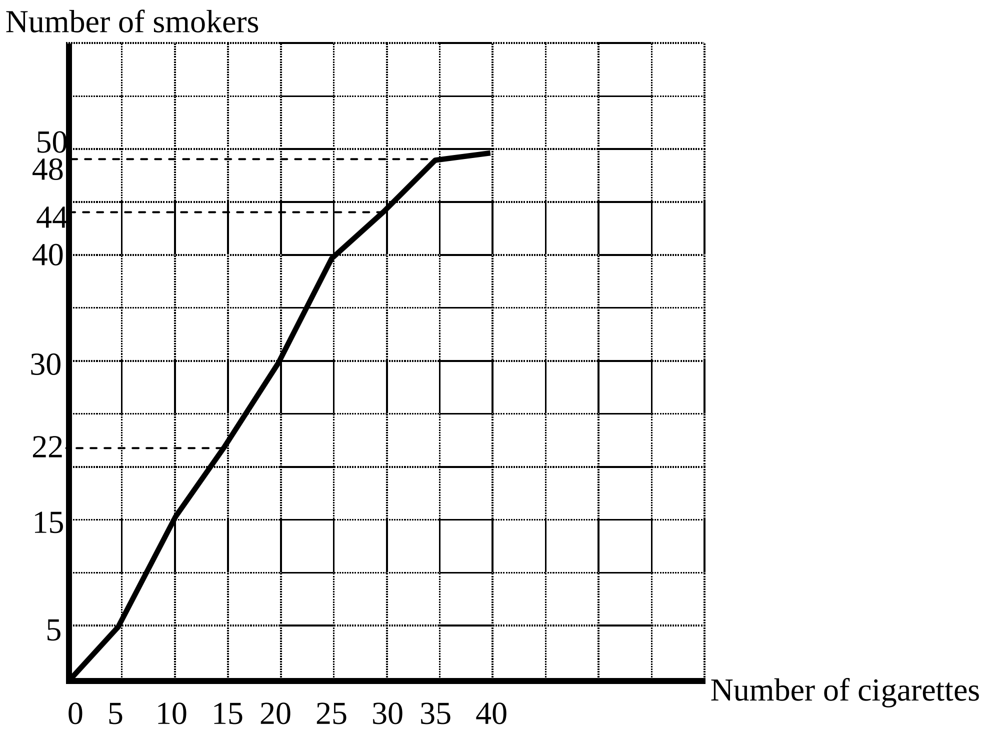
دورة سنة ٢٠٠٤ الاستثنائية

عدد المسائل :اربع	مسابقة في الرياضيات المدة :ساعتان	الاسم : الرقم :

ملاحـظة يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المُرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (2 points)

A survey was conducted on 50 smokers to study their daily consumption of cigarettes.
The graph below represents the increasing cumulative frequency polygon of these smokers.



1) Copy and complete the following frequency table of this distribution:

Number of cigarettes	[0 ;5[				[20 ;25[			[35 ;40]
Number of smokers	5		7		10			2

2) Determine the median of this distribution, to the nearest unit, and give a significance of the value thus obtained.

II- (4 points)

An employee receives the amount of 2 000 000 LL .
On the first day, he spends 20% of this amount.
On the second day he spends 20% of the amount remaining with him from the previous day, and so on for every new day.
Designate by U_n ($n \geq 1$) the amount, in LL, left with this employee at the end of the nth day.

1) Verify that $U_1 = 1\ 600\ 000$.
2) Prove that (U_n) is a geometric sequence whose ratio is to be determined.
3) Calculate U_n in terms of n.
4) At the end of which day, the amount left with this employee would become for the first time less than 500 000 LL ?

III- (4points)

A bag contains **seven** balls :

one red ball carrying the number n

two yellow balls each carrying the number -5

four green balls each carrying the number 4 .

Two balls are drawn, simultaneously and at random, from this bag.

1) Prove that the probability of drawing **one** red ball and **one** green ball is equal to $\frac{4}{21}$.

2) Calculate the probability of drawing **two** green balls.

3) Calculate the probability of drawing **two** balls having the same colour.

4) Let X designate the random variable that is equal to the product of the two numbers carried by the **two** drawn balls.
   a- Justify that the possible values of X are : $-5n$; $4n$; -20 ; 16 ; 25.
   b- Determine the probability distribution of X.
   c- Determine the value of n for which the mathematical expectation $E(X)$ is equal to -1 .

IV- (10 points)

A- Let f be the function that is defined on $[-1\,;+\infty[$ by $f(x) = x - 2 - 2xe^{-x}$, and let (C) be its representative curve in an orthonormal system $(O;\ \vec{i}\ ,\ \vec{j}\ )$.

1 a- Calculate $\lim\limits_{x \to +\infty} f(x)$, and prove that the line (d) of equation $y = x - 2$ is an asymptote of (C) .
   b- Study, according to the values of x, the relative positions of (C) and (d) .
   c- Calculate $f(0)$ and $f(-1)$.

2) Given below the table of signs of $f'(x)$.

x	-1		0.3		$+\infty$
$f'(x)$		$-$	0	$+$	

Set up the table of variations of f .

3) a- Draw (d) and (C) .
   b- Show, graphically, that the equation $f(x) = 0$ has a unique positive solution α .
   Verify that $2.4 < \alpha < 2.5$.

B- **In all what follows, suppose that** $\alpha = 2.45$.

A factory produces a certain chemical liquid.

The function M_C ,defined on [0;10] by $M_C(x) = 1 + 2(1 - x)e^{-x}$, expresses the daily marginal cost of this production .

x is expressed in thousands of liters, and $M_C(x)$ in millions LL.

The fixed cost of this production amounts to 2 million LL.

1) Prove that the total cost function C is expressed by $C(x) = x + 2 + 2xe^{-x}$.

2) The whole production is completely sold at the price of 2000 LL per liter.
   a- Prove that the profit function is expressed by $P(x) = x - 2 - 2xe^{-x}$.
   b- Determine the quantity that should be produced daily by this factory in order that the profit is zero.
   Does the factory achieve a profit if the daily production of this liquid is 2000 liters? Justify your answer.

<table>
<tr><th colspan="2">Q</th><th></th><th>M</th></tr>
<tr><td rowspan="2">I</td><td>1</td><td>
<table>
<tr><td>Number of cigarettes</td><td>[0 ;5[</td><td>[5 ;10[</td><td>[10 ;15[</td><td>[15 ;20[</td><td>[20 ;25[</td><td>[25 ;30[</td><td>[30 ;35[</td><td>[35 ;40]</td></tr>
<tr><td>Number of smokers</td><td>5</td><td>10</td><td>7</td><td>8</td><td>10</td><td>4</td><td>4</td><td>2</td></tr>
</table>
</td><td>1 ½</td></tr>
<tr><td>2</td><td>Graphically, the line of equation y = 25 cuts the given polygon at a point of abscissa x such that $16.8 \le x \le 16.9$ hence x = 17.

♦OR : Let A(15 ; 22) and B(20 ;30), (AB) : $\frac{y-30}{x-20}=\frac{8}{5}$.

For y = 25 we get x = 16.875 then x = 17.

♦OR : By using the formula.

Interpretation: 25 persons of these smokers smoke a number of cigarettes ≥ 17.</td><td>2</td></tr>
<tr><td rowspan="4">II</td><td>1</td><td>$U_1 = 2\,000\,000 - 2\,000\,000 \times \frac{20}{100} = 1\,600\,000$.</td><td>1 ½</td></tr>
<tr><td>2</td><td>$U_{n+1} = U_n - U_n\left(\frac{20}{100}\right) = 0.8\,U_n$.

It is a geometric seqeuence of common ratio 0.8.</td><td>2</td></tr>
<tr><td>3</td><td>$U_n = U_1 q^{n-1} = 1\,600\,000\,(0.8)^{n-1}$.</td><td>1 ½</td></tr>
<tr><td>4</td><td>$U_n < 500\,000$

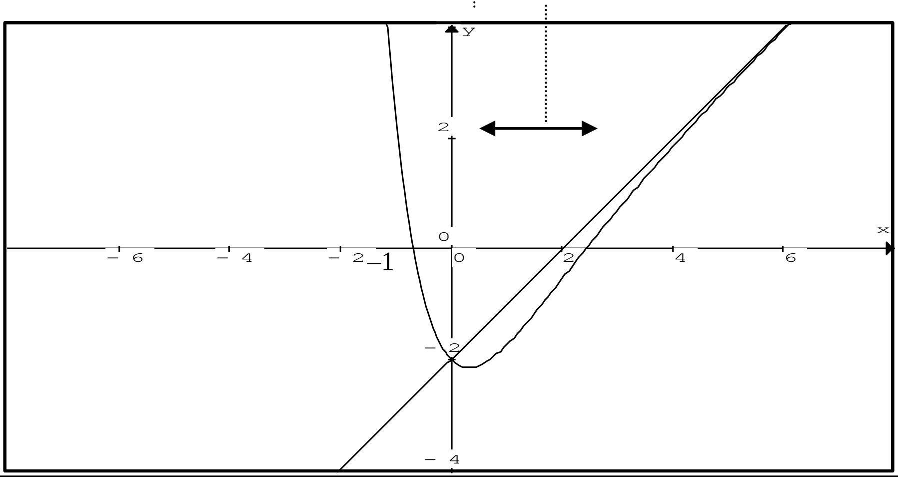
$(0.8)^{n-1} < \frac{5}{16}$; $(n-1)\ln(0.8) < \ln\left(\frac{5}{16}\right)$; $n-1 > \frac{\ln\left(\frac{5}{16}\right)}{\ln(0.8)}$; $n > 6.21$.

At the end of the 7th day the amount will be less than 500 000 LL.</td><td>2</td></tr>
<tr><td rowspan="5">III</td><td>1</td><td>Number of possible outcomes $C_7^2 = 21$.

Probability to draw a red ball and a green one is $\frac{C_1^1 C_4^1}{21} = \frac{4}{21}$.</td><td>1</td></tr>
<tr><td>2</td><td>Probability of drawing two green balls is $\frac{C_4^2}{21} = \frac{6}{21} = \frac{2}{7}$.</td><td>1</td></tr>
<tr><td>3</td><td>Drawing two balls of the same colour is drawing two yellow, or two green :

$p = \frac{C_4^2 + C_2^2}{21} = \frac{7}{21} = \frac{1}{3}$.</td><td>1</td></tr>
<tr><td>4-a-</td><td>The possible outcomes : (RY), (RY), (YY), (YG), (GG).

$X(\Omega) = \{-5n,\ 4n,\ 25,\ -20,\ 16\}$.</td><td>1</td></tr>
<tr><td>4-b-</td><td>
<table>
<tr><td>X_i</td><td>-20</td><td>$-5n$</td><td>$4n$</td><td>16</td><td>25</td></tr>
<tr><td>p_i</td><td>$\frac{C_2^1 \times C_4^1}{21} = \frac{8}{21}$</td><td>$\frac{C_1^1 \cdot C_2^1}{21} = \frac{2}{21}$</td><td>$\frac{4}{21}$</td><td>$\frac{6}{21}$</td><td>$\frac{C_2^2}{21} = \frac{1}{21}$</td></tr>
</table>
</td><td>2</td></tr>
</table>

	4-c-	$E(X) = \frac{1}{21}(-160 - 10n + 16n + 96 + 25) = \frac{1}{21}(6n - 39)$. $E(X) = -1$; $6n - 39 = -21$; $n = 3$.	1
	A. 1-a-	$f(x) = x - 2 - 2xe^{-x}$; $D_f = [-1\ ;\ +\infty[$ $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(x - 2 - \frac{2x}{e^x}\right) = +\infty$ $\lim\limits_{x\to+\infty}[f(x)-(x-2)] = \lim\limits_{x\to+\infty}\frac{-2x}{e^x} = 0$. (d): $y = x-2$ is an asymptote of (C).	2
	A. 1-b	$f(x) - (x - 2) = -2x\,e^{-x}$ If $x = 0$, then (C) cuts (d) at point $(0\ ;\ -2)$ If $x < 0$, then (C) is above (d) If $x > 0$, then (C) is below (d).	1 ½
	A. 1-c	$f(0) = -2$ and $f(-1) = 2.436$.	1
	A. 2	$\begin{array}{c\|ccccc} x & -1 & & 0.3 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 2.436 & \searrow & -2.144 & \nearrow & +\infty \end{array}$	1 ½
IV	A. 3-a-		2 ½
	A. 3-b-	Over $[0\ ;\ +\infty[$, (C) cuts the axis of abscissas at a unique point Hence $f(x) = 0$ has a unique solution α. $f(2.4) = -0.0354$; $f(2.5) = 0.0895$,$f(2.4)<0$ and $f(2.5)>0$ then $2.4<\alpha<2.5$	2
	B.1	$\alpha = 2{,}45$ $C'(x) = 1 + 2e^{-x} - 2x\,e^{-x} = 1 + 2(1 - x)e^{-x}$ and $C(0) = 2$	2 ½

		Therefore C (x) is the total cost. ♦**OR** : C(x) is an anti-derivative of M_C (x) that takes the value 2 at x = 0.	
	B. 2-a-	Selling price of a unit is 2000 × 1000 = 2 millions. Selling price of x units is 2x. $P(x) = 2x - (x + 2 + 2x\,e^{-x}) = x - 2 - 2x\,e^{-x}$.	2
	B 2-b-	P(x) = 0 for f(x) = 0 and $x \geq 0$, hence x = 2.45. The profit vanishes for a production of 2450 liters. For a production of 2000 liters x = 2 and f(x) = – 0.541. The factory loses 541 000 LL	2 ½