وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٤ العادية

عدد المسائل :اربع

مسابقة في الرياضيات
المدة : ساعتان

الاسم :
الرقم :

ملاحظة : يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (3 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B such that : $z_A = 1$ and $z_B = \frac{3}{2} + i\frac{\sqrt{3}}{2}$.

Let (C) be the circle with center A and radius 1.

1) a - Write $z_B - z_A$ in the exponential form.

b – Determine a measure of the angle $(\vec{u};\vec{AB})$.

c – Show that the point B belongs to the circle (C).

2) To every point M, of non-zero affix Z, associate the point M' of affix z' such that $z' = \frac{\bar{z} + 2}{\bar{z}}$.

a – Prove that $\bar{z}\,(z' - 1) = 2$.

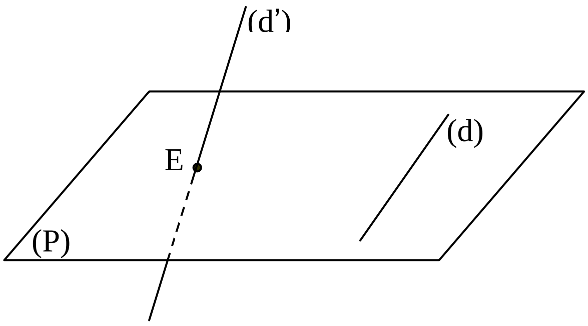
b – Deduce, when M' moves on the circle (C), that M moves on a circle (T) to be determined.

II - (4 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the lines (d) and (d') defined by :

$$(d): \begin{cases} x = t+1 \\ y = 2t \\ z = t-1 \end{cases} \quad \text{and} \quad (d'): \begin{cases} x = 2m \\ y = -m+1 \\ z = m+1 \end{cases}$$

(t and m are two real parameters).



1) Prove that (d) and (d') are skew (not coplanar).

2) a – Show that $x - y + z = 0$ is an equation of the plane (P) determined by O and (d).

b – Determine the coordinates of E, the point of intersection of (P) and (d').

c – Prove that the straight line (OE) cuts (d).

3) a – Calculate the distance from point O to the line (d).

b – Deduce that the circle in plane (P), with center O and passing through E, is tangent to line (d).

III- (9 points)

Let f be the function defined, on $]0\,;+\infty[$ by $f(x) = x + 2\dfrac{\ln x}{x}$. (C) is the representative curve of f in an orthonormal system $(O;\vec{i}\,,\vec{j})$; unit 2 cm .

1) a – Calculate $\lim\limits_{x \to 0} f(x)$ and give its graphical interpretation.

   b – Determine $\lim\limits_{x \to +\infty} f(x)$ and verify that the line (d) of equation y = x is an asymptote of (C).

.

   c – Study according to the values of x , the relative position of (C) and (d) .

2) The table below is the table of variations of the function f ', the derivative of f.

x	0		e		$e\sqrt{e}$		$+\infty$
f "(x)	‖	–	⋮	–	0	+	
f ′ (x)	‖ $+\infty$	↘	1	↘	$1-e^{-3}$	↗	1

   a – Show that f is strictly increasing on its domain of definition, and set up its table of variations.

   b – Write an equation of the line (D) that is tangent to (C) at the point G of abscissa e .

   c – Prove that the curve(C) has a point of inflection L.

   d – Show that the equation f(x) = 0 has a unique root α and verify that $0{,}75 < \alpha < 0{,}76$.

3) Draw (D) , (d) and (C) .

4) Calculate, in cm^2 , the area of the region bounded by the curve (C), the line (d) and the two lines of equations x = 1 et x = e .

IV- (4points)

Consider two urns U and V :

U contains **three** balls numbered 0 and **two** balls numbered 1 .

V contains **five** balls numbered 1 to 5 .

A - One ball is drawn randomly from each urn.

Designate by X the random variable that is equal to the product of the two numbers that are marked on the two drawn balls.

1) Prove that P(X = 0) is equal to $\dfrac{3}{5}$.

2) Determine the probability distribution of X .

B – In this part, the 10 balls that were in urns U and V are all placed in one urn W .

Two balls are drawn, simultaneously and at random, from this urn W.

1) What is the number of possible draws of these 2 balls?

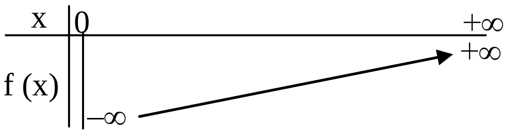
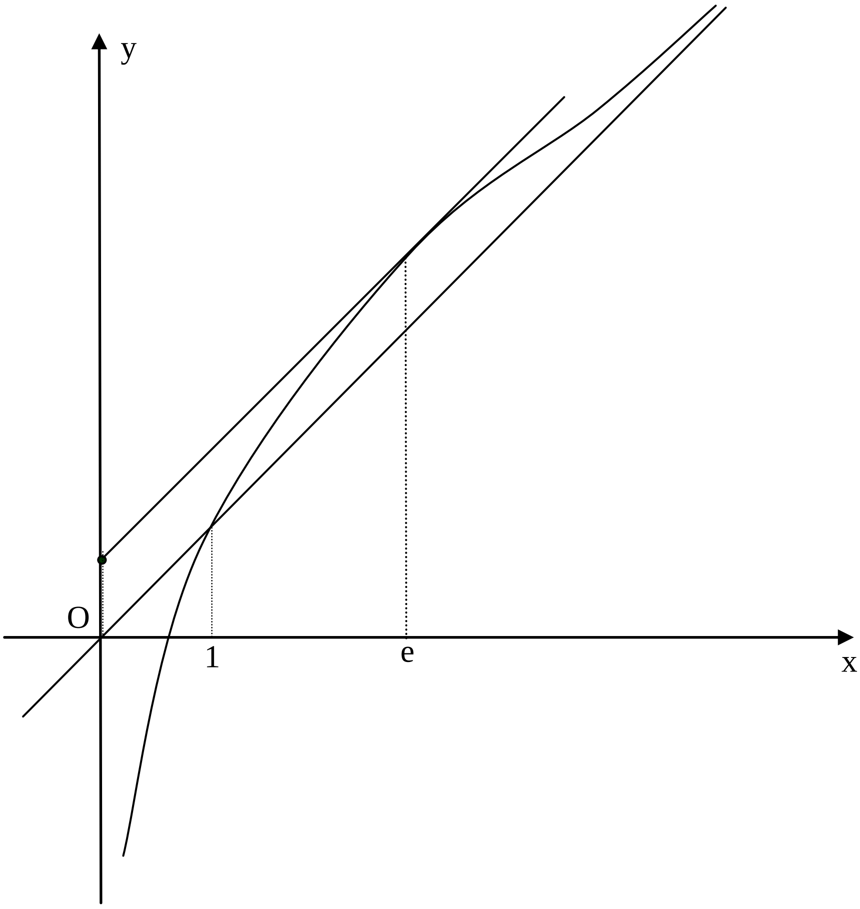
2) Let q designate the product of the too numbers that are marked on the two drawn balls. .

   a - Show that the probability P(q = 0) is equal to $\dfrac{8}{15}$.

   b – Calculate the probability P(q < 4) .

Q		Answers	M
I	1-a	$z_B - z_A = \frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i\frac{\pi}{3}}$	
	1-b	$(\vec{u}\,;\vec{AB}) = \arg(Z_{\vec{AB}}) = \arg(z_B - z_A) = \frac{\pi}{3}$	
	1-c	$AB = \lvert z_B - z_A \rvert = 1$ then B belongs to (C).	
	2-a	$\bar{z}(z'-1) = \bar{z}\left(\frac{\bar{z}+2}{\bar{z}} - 1\right) = \bar{z}\left(\frac{2}{\bar{z}}\right) = 2$.	
	2-b	If M' moves on (C) then AM' = 1 and $\lvert z'-1 \rvert = 1$ hence $\lvert \bar{z} \rvert = 2$ then $\lvert z \rvert = 2$ and M moves on the circle of center O and radius 2.	

II	1	$\vec{V}(1\,;2\,;1)$ and $\vec{V'}(2\,;-1\,;1)$; $\vec{V}$ and $\vec{V'}$ are not collinear, then (d) and (d') are not parallel. Study of the intersection of (d) and (d') : t + 1 = 2m ; 2t = – m + 1 ; t – 1 = m + 1 Take 2t = – m + 1 ; t – 1 = m +1, we get t = 1 and m = –1, these values do not verify t + 1 = 2m. Hence (d) and (d') are skew ►Or : Let L (1 ; 0 ; –1) be a point of (d) and J (2 ; 0 ; 2) be a point of (d') ; $\vec{LJ}.(\vec{V}\wedge\vec{V'}) = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix} = -12 \neq 0$	
	2-a	By verification : O is a point of (P) (d) lies in (P) because t + 1 –2t + t – 1 = 0 for every real number t. ►Or : M(x ; y ; z) belongs to (P) iff $\vec{OM}.(\vec{OL}\wedge\vec{V}) = 0$ which gives x – y + z = 0	
	2-b	2m + m –1 + m + 1 = 0 ; m = 0 then E (0 ; 1 ; 1).	
	2-c	(OE) is a line in plane (P), (OE) and (D) are coplanar and they are not parallel ($\vec{OE}$ and $\vec{V}$ are not collinear), therefore they intersect. ►Or : Determine a system parametric equations of (OE) and then prove that it cuts (d).	
	3-a	distance (O/ (d)) = ….. = $\sqrt{2}$.	
	3-b	OE = $\sqrt{2}$ = distance (O/ (d)) ; then (C) is tangent to (d).	

III	1-a	$\lim\limits_{x\to 0} \ln x = -\infty$ then $\lim\limits_{x\to 0} f(x) = -\infty$; y'y is an asymptote of (C).	
	1-b	$\lim\limits_{x\to +\infty} \frac{\ln x}{x} = 0$ then $\lim\limits_{x\to +\infty} f(x) = +\infty$; $\lim\limits_{x\to +\infty} [f(x) - x] = 0$ hence the line (d) of equation y = x is an asymptote of (C) at $+\infty$.	
	1-c	$f(x) - x = 2\frac{\ln x}{x}$. For x = 1, (C) cuts (d). For 0 < x < 1, f(x) – x < 0 then (C) is below (d). For x > 1, (C) is above (d).	
	2-a	$f'(x) \geq 1 - \frac{1}{e^3} > 0$ then f is strictly increasing. 	
	2-b	$y = f'(e)(x-e) + f(e)$; $y = x - e + e + \frac{2}{e} = x + \frac{2}{e}$	
	2-c	f ”(x) vanishes for x = $e\sqrt{e}$ and changes sign, then (C) has a point of inflection L of abscissa $e\sqrt{e}$.	
	2-d	f is continuous and changes sign on its domain, f(x) = 0 has at least a root α, moreover f is strictly increasing, then α is unique. f (0.75) ×f (0.76) = –0.017×0.377 < 0, then 0.75 < α < 0.76.	
	3		
	4	A= $\int_1^e 2\frac{\ln x}{x}\,dx = \left[\ln^2 x\right]_1^e = 1\,u^2$, then A = 4 cm^2.	

<table>
<tr><td rowspan="5">IV</td><td>A-1</td><td>To get a product equal to 0 it's enough to draw from U a ball numbered 0, therefore the probability is equal to $\frac{3}{5}$.
►Or : Number of possible draws is equal to $5 \times 5 = 25$
$P(X = 0) = \frac{3\times 5}{5\times 5} = \frac{3}{5}$.</td><td></td></tr>
<tr><td>A-2</td><td><table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>p_i</td><td>3/5</td><td>2/25</td><td>2/25</td><td>2/25</td><td>2/25</td><td>2/25</td></tr></table></td><td></td></tr>
<tr><td>B-1</td><td>$C_{10}^{2} = 45$.</td><td></td></tr>
<tr><td>B-2 a</td><td>To get a product equal to 0 we must obtain one of the following outcomes:
Two balls numbered 0
or
{0 ; a} with a = 1, 2, 3, 4, 5.
Number of favorable cases is $C_3^2 + C_3^1 \times C_7^1 = 24$
$P(q = 0) = \frac{24}{45} = \frac{8}{15}$</td><td></td></tr>
<tr><td>B-2 b</td><td>$P(q < 4) = P(q = 0) + P(q = 1) + P(q = 2) + P(q = 3)$
$= \frac{8}{15} + \frac{C_3^2 + C_3^1 \times C_1^1 + C_3^1 \times C_1^1}{45} = \frac{33}{45} = \frac{11}{15}$.</td><td></td></tr>
</table>