وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع العلوم العامة

دورة سنة ٢٠٠٤ العادية

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مسابقة في الرياضيات
المدة : أربع ساعات

الاسم :
الرقم :

ملاحظة : يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (2 points)

Let f be the function defined, on [–3 ; 3], by $f(x)=\frac{2}{3}\sqrt{9-x^2}$ and (C) be its representative curve in an orthonormal system $(O\,;\vec{i}\,,\vec{j})$.

1) Calculate f '(x) and set up the table of variations of f.
2) Draw the curve (C).
3) Consider the ellipse (E) of equation $\frac{x^2}{9}+\frac{y^2}{4}=1$.
   Show that (C) is a part of (E) and draw (E).
4) Using the change of variable $x = 3\cos\theta$, we get :
   $\int_{-3}^{3} f(x)\,dx = 6\int_{0}^{\pi}\sin^2\theta\,d\theta$ (It is not required to prove this equality).
   Deduce, from this equality, the area of the region bounded by (E).

II - (3points)

The space is referred to a direct orthonormal system $(O\,;\vec{i}\,,\vec{j}\,,\vec{k})$.
Consider the lines (d) and (d') defined by :

$$(d):\begin{cases}x=2t+1\\ y=-2t-1\\ z=t+2\end{cases} \quad\text{and}\quad (d'):\begin{cases}x=m\\ y=2m-3\\ z=2m\end{cases}$$

(t and m are two real parameters) .

1) Show that the lines (d) and (d') intersect at the point A (1 ; –1 ; 2) and that they are perpendicular.
2) Write an equation of the plane (P) determined by (d) and (d') .
3) Consider, in plane (P), the line (D) defined by:

$$(D):\begin{cases}x=3\lambda-1\\ y=-1\\ z=3\lambda\end{cases}$$

(λ is a real parameter) .

   a- Prove that the line (D) is a bisector of one of the angles formed by (d) and (d').
   b- E (-1 ; -1 ; 0) is a point on (D) ; designate by (C) the circle in plane (P) , with center E , that is tangent to (d) at T and to (d') at S.
   Determine the nature of the quadrilateral ATES and calculate the length AT.
   c- Write an equation of the mediator plane of [AE] and deduce a system of parametric equations of the straight line (TS).

III - (2.5 points)

A tourist agency offers its customers two choices of 7-day voyages:
full-board or half-board.
The agency published the following advertisement:

Choice / Destination	Full-board	Half-board
France	1 500 000 LL	1300 000 LL
Italy	1 250 000 LL	1100 000 LL
Turkey	800 000 LL	700 000 LL

This agency estimates that 25 % of its customers choose France, 35 % choose Italy, and the others choose Turkey, and that out of the customers to any destination, 60 % choose full-board.
A customer is questioned at random.

Consider the following events:
F : « the questioned customer has chosen France ».
I : « the questioned customer has chosen Italy ».
T : « the questioned customer has chosen Turkey ».
B : « the questioned customer has chosen full-board ».

1) a- Calculate the following probabilities :
$P(B \cap F)$; $P(B \cap I)$; $P(B \cap T)$ and $P(B)$.
b- The questioned customer had chosen full-board, what is the probability that he chose Italy?

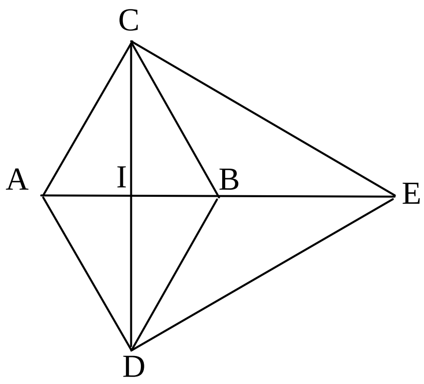
2) Let X be the random variable that is equal to the amount paid to the agency by a voyager.
a- Determine the probability distribution of X .
b- Calculate the mean (expected value) E(X). What does the number thus obtained represent?
c- Estimate the sum received by the agency when it serves 200 voyagers.

IV - (3points)

In the adjacent figure,
ABC , ADB and CDE are three direct equilateral triangles
such that $(\overrightarrow{AB};\overrightarrow{AC}) = \frac{\pi}{3} \quad (2\pi)$.

Designate by I the midpoint of [AB] .

1) Show that AE = 2AB.



Let S **be the direct similitude, with center** W, **ratio** k **and angle** θ , **that transforms** A **onto** B **and** E **onto** D.

2) Determine k and verify that $\theta = \frac{-2\pi}{3} \quad (2\pi)$.

3) Designate by (T) the circle circumscribed about triangle ACE.
Prove that the image of (T), under S, is the circle (T ') of diameter [BD] and deduce that the image of point C under S is point J, the midpoint of [DE].

4) The complex plane is referred to a direct orthonormal system $(A;\vec{u},\vec{v})$ such that $\vec{u} = \overrightarrow{AI}$.
   a- Determine the affixes of the points B , C , D and E.
   b- Find the complex form of S and specify the affix of its center W.

5) Let S' be the direct similitude with center W, ratio 2 and angle $\frac{-\pi}{3}$.
   a- Determine the nature and the elements of the transformation S'oS .
   b- Calculate the affix of point A' , the image of A under S'oS .

V - (2.5points)

In the plane referred to an orthonormal system $(O;\vec{i},\vec{j})$ (unit 3 cm),
consider the parabolas (P) and (P') of equations $y^2 = 2x - 1$ and $x^2 = 2y - 1$ respectively .

1) Determine the vertex, the focus and the directrix of each of these two parabolas.
2) Verify that the point A (1 ; 1) is common to (P) and (P') , and prove that (OA) is a common tangent to these two parabolas.
3) Prove that the line (d), perpendicular to (OA) at O , is a common tangent to (P) and (P').
4) Draw (d) , (P) and (P').
5) The area of the region bounded by (P), the axis of abscissas and the line of equation x = 1 is equal to 3 cm^2.
   Deduce the area, in cm^2, of the region bounded by (P), (P'), the axis of abscissas and the axis of ordinates.

VI - (7 points)

A- Consider the differential equation (E) : $y'' + 3y' + 2y = 2$.
Let $z = y - 1$.

1) Form a differential equation (E_1) satisfied by z , and solve (E_1).

2) Deduce the general solution of (E) and find the particular solution of (E) whose representative curve, in an orthonormal system $(O;\ \vec{i},\ \vec{j})$, is tangent at O to the axis of abscissas .

B- Let f be the function defined, on IR, by $f(x) = e^{-2x} - 2e^{-x} + 1$ and (C) be its representative curve in the system $(O;\ \vec{i},\ \vec{j})$.

1) Calculate $\lim\limits_{x \to +\infty} f(x)$ and deduce an asymptote (d) of (C).

2) Calculate $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to -\infty} \dfrac{f(x)}{x}$.

3) Find f '(x) and set up the table of variations of f.

4) Prove that (C) has a point of inflection I whose coordinates are to be determined.

5) Determine the coordinates of the point of intersection of (C) with its asymptote (d).

6) Draw (d) and (C).

7) Calculate the area of the region bounded by the curve (C) , its asymptote (d) and the axis of ordinates.

8) Let g be the function given by $g(x) = \ln(f(x))$, and let (G) be its representative curve .

a- Justify that the domain of the definition of g is $]-\infty ; 0[\cup]0 ; +\infty[$, and set up its table of variations.

b- Prove that the line (D) of equation $y = -2x$ is an asymptote of (G) .

c- Solve each of the equations $g(x) = 0$ and $g(x) = -2x$.

d- Draw (D) and (G) in a new system of axes.

GENERAL SCIENCES	MATH	1st SESSION(2004)

Q		Short answers	M

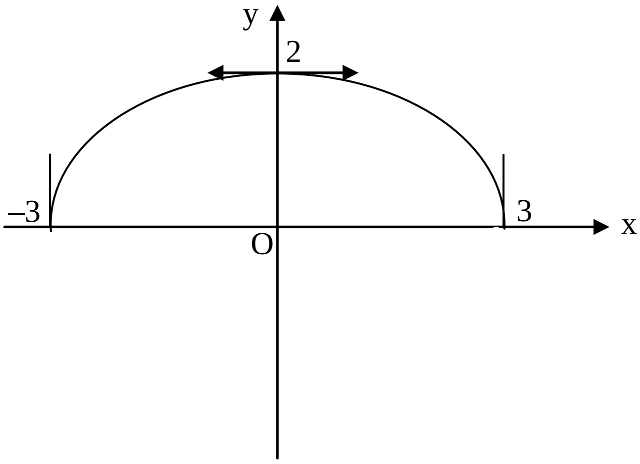
I

1

$f'(x) = \dfrac{-2x}{3\sqrt{9-x^2}}$

x	-3		0		3
f '(x)	$+\infty$	$+$	0	$-$	$-\infty$
f (x)	0	↗	2	↘	0

2



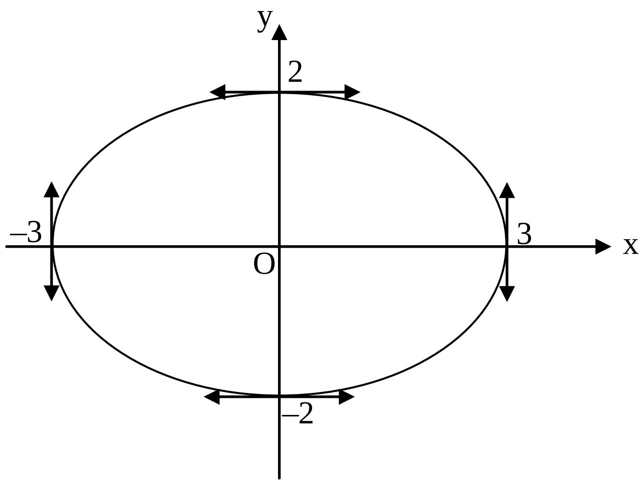
3

(E) : $\dfrac{y^2}{4} = 1 - \dfrac{x^2}{9}$, $y^2 = \dfrac{4}{9}(9 - x^2)$

$y = \dfrac{2}{3}\sqrt{9-x^2}$ or $y = -\dfrac{2}{3}\sqrt{9-x^2}$

Thus (C) is the part of (E) located above the axis of abscissas.

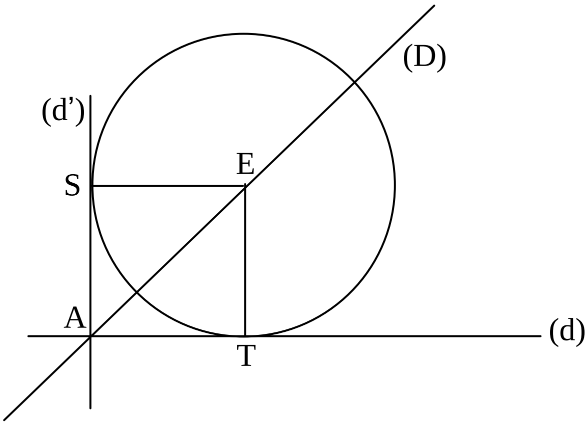
(E) =(C) $\cup$ (C') where (C') is the symmetric of (C) with respect to the axis of abscissas.



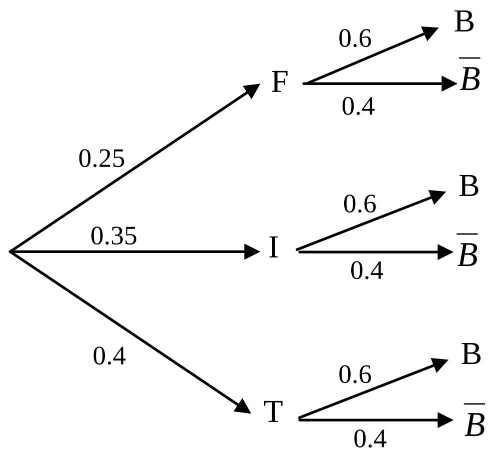
4

$A = 2\int_{-3}^{3} f(x)\,dx = 12\int_{0}^{\pi} \sin^2\theta\, d\theta = 6\int_{0}^{\pi}(1-\cos 2\theta)\,d\theta$

$= 6\left[\theta - \dfrac{1}{2}\sin 2\theta\right]_0^{\pi} = 6\pi\ u^2$

II	1	A is the point of (d) corresponding to t = 0 . A is the point of (d') corresponding tom = 1 . $\vec{V}_d$ (2 ; – 2 ; 1) and $\vec{V}_{d'}$ (1 ;2 ;2) ; $\vec{V}_d \cdot \vec{V}_{d'} = 0$, thus (d)$\perp$ (d')	
	2	M(x,y,z) is a point on (P) iff $\vec{AM}.(\vec{V}_d \wedge \vec{V}_{d'}) = 0$; So (P) : 2x + y – 2z + 3 = 0	
	3-a	A is a point on (D) corresponding to $\lambda = \frac{2}{3}$. ►$\cos(\vec{V}_d \ ; \ \vec{V}_D) = \frac{\vec{V}_d . \vec{V}_D}{\Vert \vec{V}_d \Vert \times \Vert \vec{V}_D \Vert} = \frac{\sqrt{2}}{2}$. Thus one of the angles formed by (d) and (D) is equal to 45°. ►Or : $\vec{V}_D$ (3 ;0 ;3) ; $\vec{V}_D = \vec{V}_d + \vec{V}_{d'}$ where $\Vert \vec{V}_d \Vert = \Vert \vec{V}_{d'} \Vert = 3$ ►Or : Let E(–1 ; –1 ; 0) be a point on (D) ; d(E ;(d)) = d(E,(d')) = 2.	
	3-b	 $\hat{A} = \hat{T} = \hat{S} = 90^o$ and ES = ET ,then ATES is a square, consequently AT = $\frac{AE}{\sqrt{2}} = 2$. Or AT = ES = 2.	
	3-c	Let L be the mid point of [AE] ; L(0 ; – 1 ; 1) (Q) : $\vec{LM}.\vec{AE} = 0$ where $\vec{AE}$ (– 2 ; 0 ;– 2) (Q) : x + z – 1 = 0 T and S are two points belonging to (Q) and to(P) , thus (TS) is the line of intersection of (Q) and (P). (TS) : $\begin{cases} x + z - 1 = 0 \\ 2x + y - 2z + 3 = 0 \end{cases}$ (TS) : x = – α + 1 , y = 4α – 5 , z = α .	

III



1-a
P(B ∩ F) = 0.25 × 0.6 = 0 .15 ; P(B ∩ I) = 0.35 × 0.6 = 0.21
P(B ∩ T) = 0.4 × 0.6 = 0.24; P(B) = P(B ∩ F)+P(B ∩ I)+P(B ∩ T) = 0.6
► P(B) = 0.6 because 60% of customers choose full-board.

1-b
$P(I/B) = \dfrac{P(I \cap B)}{P(B)} = \dfrac{0.21}{0.6} = 0.35.$

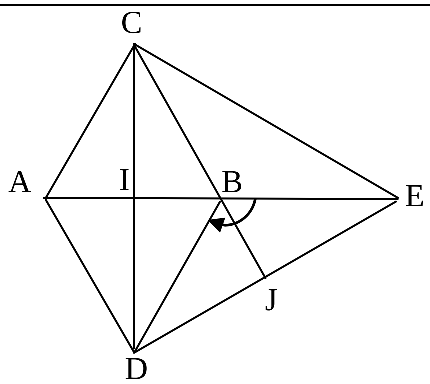
2-a

x_i	700 000	800 000	1100 000	1250 000	1300 000	1500 000
p_i	0.16	0.24	0.14	0.21	0.1	0.15

2-b
E(X) = 1 075 500
1,075,500LL is the average amount paid by a voyager to the agency.

2-c
If the agency serves 200 voyagers, it estimates to receive:
200 × E(X) = 215 ,100, 000 LL

IV



1
(CI) : bisector of $A\hat{C}B$ then $I\hat{C}B = 30^o$.
$I\hat{C}E = 60^o$ then (CB) is a bisector of $I\hat{C}E$.
Similarly (DB) : bisector of $I\hat{D}E$, then B is the center of the equilateral triangle DEC ; BE = 2 BI = AB and consequently AE = 2AB.

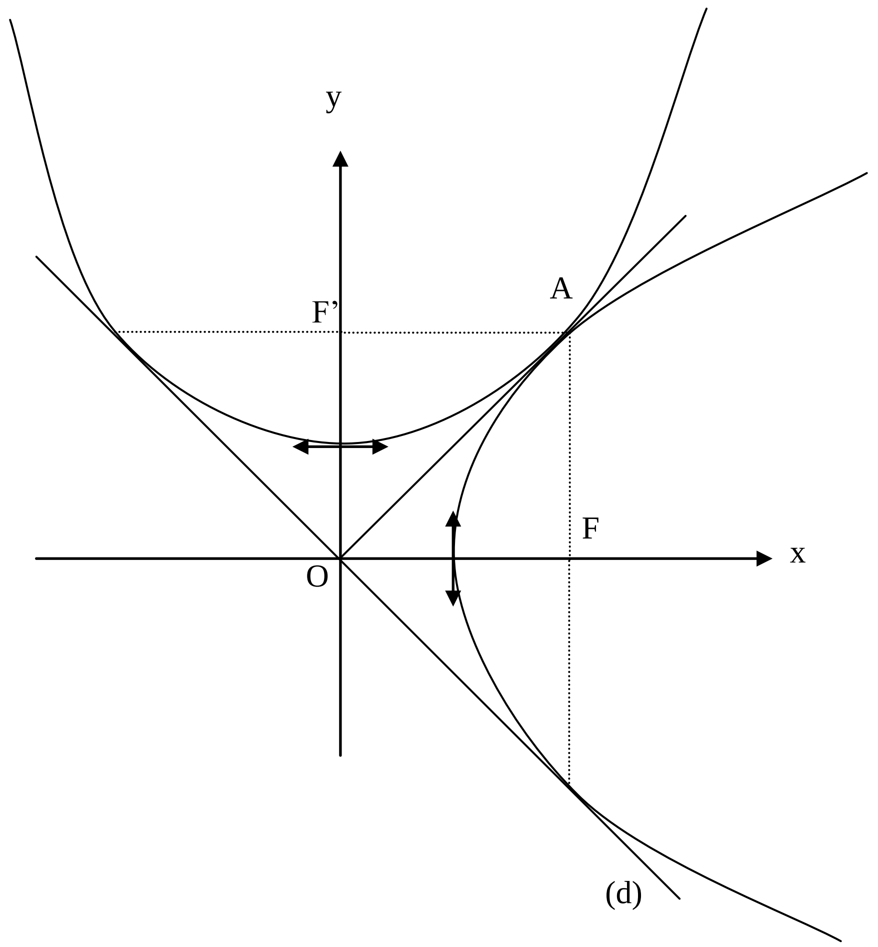
2
S(A) = B and S(E) = D then $k = \dfrac{BD}{AE} = \dfrac{AB}{AE} = \dfrac{1}{2}$

and $\theta = (\overrightarrow{AE}, \overrightarrow{BD})\,(2\pi) = (\overrightarrow{BE}, \overrightarrow{BD})\,(2\pi) = -\dfrac{2\pi}{3}\ (2\pi)$

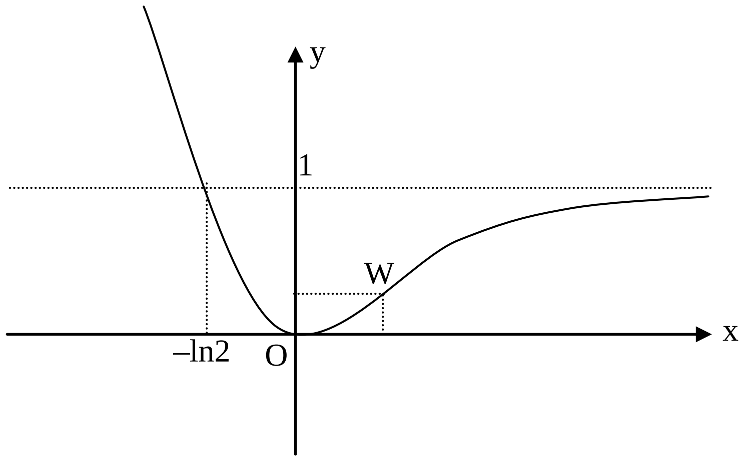
IV 3
The triangle ACE is right at C , the circle (T) has as diameter [AE], thus (T’) is the circle of diameter [BD] = S([AE]) .
J: mid point of [ED] and BDE is isosceles then $B\hat{J}D = 90^o$ and J ∈ (T ’).

		AEC is a direct semi - equilateral triangle and BDJ is direct semi-equilateral triangle, thus S(C) = J.	
	4-a	$z_B = 2$; $z_C = 1 + i\sqrt{3}$; $z_D = 1 - i\sqrt{3}$ and $z_E = 4$.	
	4-b	$z' = az + b = \frac{1}{2}e^{-i\frac{2\pi}{3}}z + b = -\frac{1}{4}(1+i\sqrt{3})z + b$ S(A) = B then b = 2 so $z' = -\frac{1}{4}(1+i\sqrt{3})z + 2$ ►Or : S(A) = B and S(E) = D give 2= 0 + b and $1 - i\sqrt{3} = a(4) + b$ b = 2 and $a = -\frac{1}{4}(1+i\sqrt{3})$. $z_W = \frac{b}{1-a} = \frac{2}{7}(5-i\sqrt{3})$	
	5-a	S'oS is similitude of center W , of ratio $\frac{1}{2} \times 2 = 1$ and angle $-\frac{\pi}{3} - \frac{2\pi}{3} = -\pi$. S'oS is a central symmetry with center W.	
	5-b	$z_{A'} = 2z_W = \frac{4}{7}(5-i\sqrt{3})$	

V		(P) : $y^2 = 2x - 1 = 2(x - \frac{1}{2})$ and (P') : $x^2 = 2y - 1$.	
	1	(P) has vertex $S(\frac{1}{2},0)$, focus F(1 ;0) and diretrix (y'y). (P') has vertex $S'(0, \frac{1}{2})$, focus F'(0 ;1) and directrix (x'x)	
	2	The coordinates of A satisfy the equations of (P) and of (P') , thus A is a common point to these parabolas. ►(OA) : y = x (OA) $\cap$ (P) : $x^2 = 2x - 1$; $(x-1)^2 = 0$, x' = x'' = 1(double root) (OA) is tangent to (P) at A. (OA) $\cap$ (P') : $y^2 = 2y - 1$; $(y-1)^2 = 0$, y' = y'' = 1(double root) (OA) is tangent to (P') at A. ►Or : 2yy' = 2 , $y' = \frac{1}{y}$ and $y_A' = 1$; the equation of the tangent at A to (P) is y – 1 = 1(x – 1) ; y = x which is (OA) Similarly for (P'). ►Notice that (OA) is the bisector of $F\hat{A}F'$.	
	3	(d) : y = –x . (d) $\cap$ (P) : $x^2 = 2x - 1$; double root $x' = x'' = x_A$ (d) $\cap$ (P') : $y^2 = 2y - 1$; double root $y' = y'' = y_A$ ►Or (d) is the symmetric of (OA) with respect to the focal axis (axis of symmetry) of (P) thus (d) is tangent to (P) .Similarly (d) is the symmetric of (OA) with respect to the focal axis (y'y) of (P').	

V	4		
	5	Let S be the required area. The area of the domain bounded by (P) , (x'x) and the line x = 1 is equal to the area limited by (P') , (y'y) and the line y = 1 . S = (area of square OFAF') – 2×3 = 9 – 6 = 3 cm^2	

VI	A-1	y'' + 3y' + 2y = 2 ; z = y – 1 (E_1) : z'' + 3z' + 2z = 0 Characteristic equation of (E_1) : $r^2 + 3r + 2 = 0$; r = – 1 or r = – 2 Then $z = C_1e^{-x} + C_2e^{-2x}$	
	A-2	$y = z + 1 = C_1e^{-x} + C_2e^{-2x} + 1$ y(0) = 0 and y'(0) = 0 give $C_1 + C_2 = -1$ and $-C_1 - 2C_2 = 0$; $C_1 = -2$ and $C_2 = 1$ Then $y = -2e^{-x} + e^{-2x} + 1$	
	B-1	$f(x) = e^{-2x} - 2e^{-x} + 1$ $\lim_{x\to+\infty} f(x) = 0 + 1 = 1$, the line (d) of equation y = 1 is an asymptote of (C) .	

VI	B-2	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} e^{-x}(e^{-x} - 2 + e^{x}) = +\infty$ $\lim\limits_{x\to-\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to-\infty} \dfrac{e^{-x}(e^{-x} - 2 + e^{x})}{x} = -\infty(+\infty) = -\infty$ y'y an asymptotic direction .	
	B-3	f '(x) = $-2e^{-2x} + 2e^{-x}$ $= 2e^{-x}(1 - e^{-x})$, x: $-\infty$; 0 ; $+\infty$ f '(x): – ; 0 ; + f (x): $+\infty$ ↘ 0 ↗ 1	
	B-4	f ''(x) = $4e^{-2x} - 2e^{-x} = 2e^{-x}(2e^{-x} - 1)$ f''(x) vanishes at x = ln2 and changes sign ;moreover f(ln2) = $\frac{1}{4}$ Then the point W(ln2 ; $\frac{1}{4}$) is a point of inflection of (C).	
	B-5	(C) cuts (d) ; $e^{-2x} - 2e^{-x} + 1 = 1$; $e^{-x}(e^{-x} - 2) = 0$; $e^{-x} = 2$; x = – ln2 (C) cuts (d) at (– ln2 ; 1).	
	B-6		
	B-7	$S = \int_{-\ln 2}^{0}(1 - f(x))dx = \int_{-\ln 2}^{0}(-e^{-2x} + 2e^{-x})dx = \left[\frac{1}{2}e^{-2x} - 2e^{-x}\right]_{-\ln 2}^{0} = \frac{1}{2}u^2$	
	B-8 a	g(x) = ln(f(x)) g is defined for f(x) > 0 which corresponds to $D_g =]-\infty;0[\cup]0;+\infty[$ g(x) = ln(f(x)) and ln is strictly increasing , therefore g and f have the same sense of variation. x: $-\infty$; 0 ; $+\infty$ g(x): $+\infty$ ↘ $-\infty$ ‖ $-\infty$ ↗ 0 ► Or $g'(x) = \dfrac{f'(x)}{f(x)}$ then g'(x) has the same sign as f'(x) on D_g. And $\lim\limits_{x\to-\infty} f(x) = +\infty$ *then* $\lim\limits_{x\to-\infty} g(x) = +\infty$ $\lim\limits_{x\to+\infty} f(x) = 1$ *then* $\lim\limits_{x\to+\infty} g(x) = 0$	

VI	B-8 b	$g(x)-(-2x)=\ln(e^{-2x}-2e^{-x}+1)+\ln(e^{2x}) = \ln(1-2e^{x}+e^{2x})$ $\lim_{x\to-\infty}[g(x)+2x] = \ln 1 = 0$; (D) is an asymptote of(G) at $-\infty$.	
	B-8 c	g(x) = 0 is equivalent to ln(f(x)) = 0 ; f(x) = 1 ; x = – ln2. g(x) = – 2x is equivalent to $\ln(1-2e^{x}+e^{2x}) = 0$; $-2e^{x}+e^{2x} = 0$; $e^{x} = 2$;x = ln2	
	B-8 d