الدورة الإستثنائية للعام 2012	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل: أربع

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (4 points)

A factory produces household articles. The table below shows the total cost y_i, expressed in millions LL, of the production of x_i hundreds of articles.

Number of articles x_i	0.6	0.8	1.1	1.2	1.5	2
Total cost y_i	1.4	1.5	1.8	2.1	2.5	3

In this exercise, give your answers rounded to the nearest 10^{-3}.

1) Let (D) be the regression line of y in terms of x in an orthogonal system of axes x'Ox, y'Oy. Write an equation of (D).
2) Estimate the total cost corresponding to a production of 220 articles.
3) The selling price of an article is 25 000LL, but only 80% of the produced articles are sold.
   a- Prove that the revenue is given by $R(x) = 2x$.
   b- Estimate the profit achieved by this factory for the production of 220 articles.
   c- In the previous system, consider the line (D') with equation $y = 2x$.
   (D) and (D') intersect at a point S.
   Calculate the abscissa of S and give an economical interpretation to the value thus obtained.

II- (4 points)

Rami's store sells T-shirts and jackets of two different sizes: small and large.

- 70% of the articles are T-shirts
- 40% of the T-shirts are of small size
- 50% of the jackets are of large size.

A client chooses randomly an article from this store.

Consider the following events:
T: « The chosen article is a T-shirt»
J: « The chosen article is a jacket»
S: « The chosen article is of small size».

A- 1) a- Verify that the probability $p(S\cap T)$ is equal to 0.28 and calculate $p(S\cap J)$.
b- Deduce $p(S)$.
2) Knowing that the chosen article is of small size, calculate the probability that it is a T-shirt.

B- The price of a T-shirt of small size is 30 000 LL and that of of a T-shirt of large size is 50 000 LL.
The price of a jacket of small size is 40 000 LL and that of a jacket of large size is 50 000 LL.
Let X be the random variable equal to the sum paid by the client for the purchase of the chosen article.
1) Determine the probability distribution of X.
2) Calculate the expected value E(X).
3) Denote by N the number of articles sold in Rami's store .
   a- Give, in terms of N, an estimation of the revenue of Rami's store.
   b- If Rami is planning to make a revenue that exceeds 6 000 000 LL, what is the minimum number of T-shirts and that of jackets that he should sell?

III- (4 points)

At the beginning of a certain year, Fadi deposits a capital of 100 million LL in a bank, at an annual interest rate of 8% , compounded yearly.

At the end of every year, Fadi withdraws 10 million LL from his account to pay for a trip.

Denote by U_n the amount, in millions LL, that Fadi has in his account at the end of the n^{th} year after withdrawing the 10 million LL. ($U_0=100$).

1) Justify that $U_{n+1} = 1.08U_n - 10$.
2) Verify that the sequence (U_n) is not geometric.
3) For all natural numbers n, let $V_n = U_n + \alpha$.
   Calculate α so that (V_n) is a geometric sequence with common ratio 1.08.

In what follows, take $\alpha = -125$.

4) Calculate V_n and then U_n in terms of n.
5) Prove that (U_n) is decreasing.
6) In how many years will Fadi not be able, for the first time, to pay for his trip using this account?

IV- (8 points)

A- Consider the function f, defined over]-2, 5], by $f(x) = -x + 7 - \ln(2 + x)$, and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Determine $\lim_{x \to -2} f(x)$ and deduce an asymptote (d) to (C).
2) Calculate f(–1), f(0) and f(5).
3) Find f '(x) and set up the table of variations of f.
4) Draw (d) and (C).
5) a-Prove that the function F defined over]-2; 5] by F(x) = $-\frac{x^2}{2} + 8x - (x+2)\ln(x+2)$ is an antiderivative of f.
   b- Deduce the area of the region bounded by (C), the axis of abscissas and the two lines with equations x = 0 and x = 1.

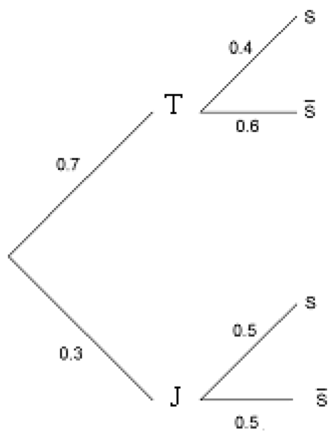
B- A company manufactures files with a unit price x expressed in thousands LL with $0.3 \le x \le 5$.

The demand, expressed in thousands of units, is modeled by f(x) .

The supply g(x), expressed in thousands of units, is given by $g(x) = \frac{3}{4}x + 1$.

1) Calculate the demand corresponding to a unit price of 2 000LL.
2) Draw the graphical representation (G) of g in the same system as that of (C).
3) (G) intersects (C) at a point of abscissa α. Verify that $2.5 < \alpha < 2.6$.
4) In all what follows, suppose that $\alpha = 2.55$.
   a- Give an economical interpretation to this value of α.
   b- Determine the market equilibrium quantity.
   c- Determine the value of the revenue corresponding to the equilibrium price.

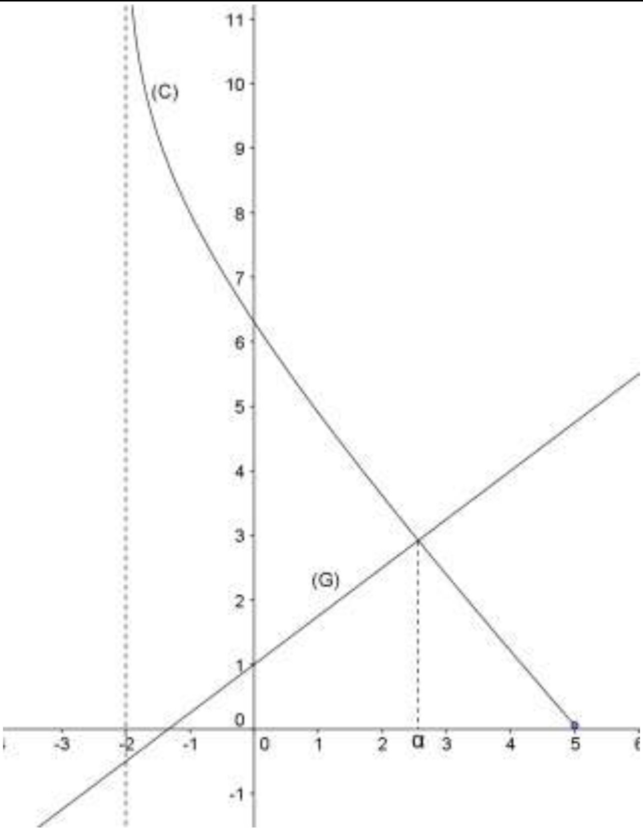
I	Solution	Grade
1	y = 1.214x+0.593.	1
2	x = 2.2 gives y =3.263 that is 3263000LL.	1
3a	If x hundreds of articles are produced, the number of articles sold is 0.8x hundreds or 80*x* articles. The corresponding revenue is $80x \times 25000 = 2000000x = 2x$ million LL. OR: R(x) = $25000 \times \frac{80}{100} \times \frac{100x}{1000000}$ =2x.	1.5
3b	Profit= revenue– cost = 4.4 – 3.263=1.137that is 1137000 LL.	1.5
3c	2x =1.214x+0.593; x = 0.754 .For a production of 75 articles, the profit is negative and the company starts to achieve profit for a production of 76 articles.	2

II		Solution	Grade
	A1a	$P(S\cap T) = P(T)\times P(S/T) = 0.7 \times 0.4 = 0.28$. $P(S\cap J) = P(J)\times P(S/J) = 0.3 \times 0.5 = 0.15$.	1
	A1b	$P(S) = P(S\cap T) + P(S\cap J) = 0.28 + 0.15 = 0.43$.	0.5
	A2	$P(T/S) = \frac{P(T\cap S)}{P(S)} = \frac{0.28}{0.43} = \frac{28}{43} = 0.651$.	1

	Solution	Grade
B1	The possible values of X are: 30 000, 40 000, 50 000. X_i: 30 000, 40 000, 50 000 $P(X_i)$: 0.28, 0.15, 0.57	1.5
B2	E(X) = 30 000 × 0.28 + 40 000 × 0.15 + 50 000 × 0.57 = 42 900.	0.5
B3a	The revenue is E(X)× N = 42 900 N.	1
B 3b	42 900 N > 6 000 000 so N > 139. 8. Consequently, a minimum of 140 articles should be sold . But , since there are 70% of T- shirts and 30% of jackets. Hence, Rami should sell a minimum of 98 T- shirts and 42 jackets.	1.5

X_i	30 000	40 000	50 000
$P(X_i)$	0.28	0.15	0.57

III	Solution	Grade
1	$U_{n+1} = U_n + 0.08\times U_n - 10 = 1.08U_n - 10$.	0.5
2	$U_1 = 1.08 \times 100 - 10 = 98$ $U_2 = 1.08 \times 98 - 10 = 95.84$. Since $\frac{U_2}{U_1} \neq \frac{U_1}{U_o}$, then (U_n) is not a geometric sequence.	1
3	$\frac{V_{n+1}}{Vn} = \frac{U_{n+1}+\alpha}{U_n+\alpha} = \frac{1.08U_n - 10 + \alpha}{U_n + \alpha} = 1.08$; $0.08\alpha = -10$; $\alpha = -125$.	1.5
4	$V_n = V_o \times q^n$ with $V_o = U_o - 125 = 100 - 125 = -25$ $V_n = -25\times (1.08)^n$. $U_n = V_n + 125 = -25\times(1.08)^n + 125$	1
5	$U_{n+1} - U_n = -25\times(1.08)^{n+1} + 25\times(1.08)^n = 25\times(1.08)^n \times (-0.08) < 0$. (U_n) is decreasing .	1
6	$U_n < 10$; $-25\times(1.08)^n + 125 < 10$; $n(\ln 1.08) > \ln\frac{115}{25}$; $n > 19.8$. Hence, after 20 years.	2

IV	Solution	Grade
A1	$\lim\limits_{x\to -2} f(x) = +\infty$. The line (d) with equation x= –2 is an asymptote to (C).	1
A2	f(-1) = 8 ; f(0) = 5.616 ; f(5) = 2 – ln7 = 0.05.	1
A3	$f'(x) = -1 - \frac{1}{2+x} < 0$ $\begin{array}{c\|lcr} x & -2 & & 5 \\ \hline f'(x) & & - & \\ \hline f(x) & +\infty & \searrow & 0.05 \end{array}$	1.5
A4		1
A5a	$F'(x) = -x + 8 - \ln(x+2) - \frac{x+2}{x+2} = -x + 7 - \ln(x+2) = f(x)$	1
A5b	$A = \int_0^1 f(x)dx = \left[-\frac{x^2}{2} + 8x - (x+2)\ln(x+2) \right]_0^1 = \frac{15}{2} - 3\ln 3 + 2\ln 2 = 5.59\ u^2$	1
B1	For a unit price of 2 000LL ; x = 2; f(2) = 5 – ln 4 = 3.613 that is 3613 files.	1.5
B2	See figure	1
B3	Let h(x) = g(x) – f(x) then, $h(x) = \frac{7}{4}x - 6 + \ln(2 + x)$ h(2.5) = – 0.12 < 0 ; h(2.6) = 0.07 > 0 ; consequently 2.5 < α<2.6.	1.5
B4a	For a unit price of 2550LL, the market is in equilibrium.	1
B4b	g(2.55) = 2.912 then the market equilibrium quantity is 2912 files.	1
B4c	R(2.55) = 2.55× 2.912 = 7.4256. Thus, the revenue corresponding to equilibrium is 7 425 600 LL.	1.5