وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع الاجتماع والاقتصاد

دورة سنة ٢٠٠٤ العادية

عدد المسائل: اربع

مسابقة في الرياضيات
المدة ساعتان

الاسم:
الرقم:

ملاحظة : يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (4points).

A – A factory manufactures a certain product.
The table below shows the demand Y of this product, in thousands of units, in terms of X where X is the price of one unit expressed in thousands LL.

X_i	1.5	3	5	8	11
Y_i	12	11	10	9	8

1) Calculate $\overline{X}$ and $\overline{Y}$, the means of the two variables X and Y respectively.

2) Represent graphically the scatter plot of the points (X_i ; Y_i) as well as the center of gravity $G(\overline{X};\overline{Y})$ in a rectangular system.

3) Determine an equation of the regression line $(D_{Y/X})$ and draw it in the preceding system.

4) Suppose that the above pattern remains valid as price increases.
Find an estimation of the demand corresponding to a unit price equal to 14 500LL.

B - The table below shows the supply Z of this product, in thousands of units, in terms of the price X, in thousands LL.

X_i	1.5	3	5	8	11
Z_i	6	8	8.5	9	10

The regression line $(d_{Z/X})$, of Z in terms of X, cuts the line $(D_{Y/X})$ at the point L (7.87 ; 9.1) .
Give an economical interpretation of the coordinates of L.

II- (4points)

Zahi deposits a capital C_o = 10 000 000 LL in an investment company.

At the end of every year, this company transfers into Zahi's account an interest of 5% together with a supplementary amount of 200 000LL.

Designate by C_n the balance in his account at the end of the $n^{th.}$ year.

1) Verify that C_1 = 10 700 000LL .

2) Prove that $C_{n+1} = (1 .05)C_n + 200\ 000$.

3) Consider the sequence (S_n) defined by $S_n = C_n + 4\ 000\ 000$; ($n \geq 0$).
   a- Prove that (S_n) is a geometric sequence of ratio 1.05 and calculate S_o .
   b- Write S_n in terms of n , and deduce C_n in terms of n.
   c- Find the number of years needed for the balance in Zahi's account , in this company ,to exceed 17 000 000 LL for the first time?

III - (4points)

In a library, two boxes A and B contain 200 calculators (graphic or non-graphic).
The box A contains calculators manufactured in the year 2004 , and the box B contains calculators manufactured in 2000.
These calculators are distributed as shown in the table below:

Type / Box	graphic	non- graphic
A	50	40
B	30	80

A customer chooses randomly one calculator from **each** box.

1) Consider the events:
   E : « the customer chooses two graphic calculators ».
   F : « the customer chooses one graphic calculator and another non-graphic one ».

   Prove that the probability P(E) is equal to $\frac{5}{33}$ and calculate P(F) .

2) The prices of the calculators are given in the following table:

Type / Box	graphic	non-graphic
A	120 000 LL	36 000 LL
B	100 000 LL	30 000 LL

Designate by X the random variable equal to the sum of prices paid by this customer for the two calculators chosen.

a- Find the four values of X .
b- Determine the probability distribution of X.

IV - (8points).

A- Consider the function f defined , on $\left]\frac{1}{e};+\infty\right[$, by $f(x)=\frac{4}{1+\ln x}$.

(C) is the representative curve of f in an orthonormal system $(O\,;\vec{i}\,,\,\vec{j})$.

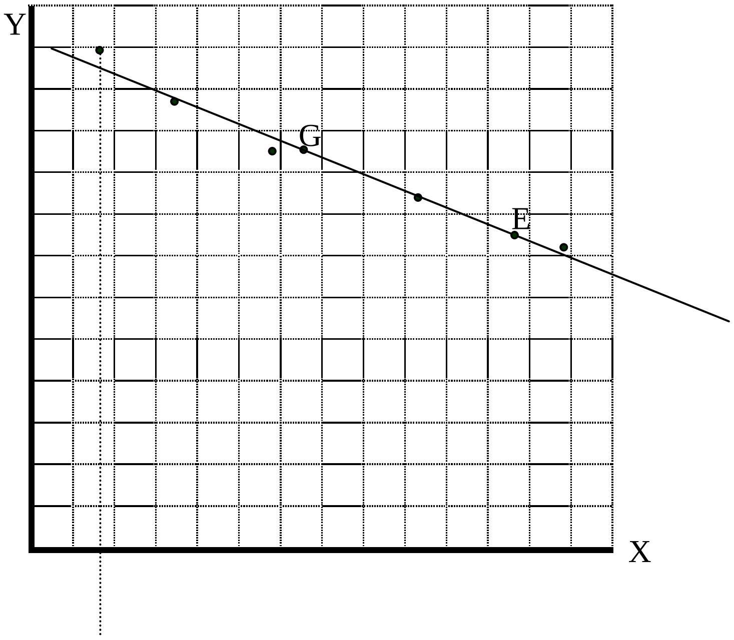
1) Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to\frac{1}{e}} f(x)$. Deduce the asymptotes of (C).
2) Verify that f '(x) < 0 and set up the table of variations of f .
3) Calculate f(1) and give the values of f(2) and f(3) correct to two decimal places.
4) Write an equation of the line (d) tangent to (C) at the point of abscissa 1.
5) Draw (d) and (C) .

B – A company produces batteries having a unit price p expressed in thousands LL; ($0.5 \le p \le 8$).

The demand f(p) of this product, expressed in thousands of units, is given by $f(p)=\frac{4}{1+\ln p}$.

1) Calculate the number of demanded batteries for a unit price of 1000 LL.
2) a- Find the elasticity E(p) of the demand in terms of the price.
   b- Calculate E(3); give an economical interpretation for the value thus obtained.
   Is f elastic for p = 3?

Barème

ES		Math	1st SESSION 2004
Q		**Short answers**	M
I	A-1	$\overline{X} = 5.7$ and $\overline{Y} = 10$.	
	A-2	G(5.7 ; 10) 	
	A-3	$Y = – 0.41X + 12.32$(using a calculator) . $(D_{Y/X})$ passes through G and a point E(10 ; 8.32). ► $(D_{Y/X})$: $Y = aX + b$; $a = \dfrac{\text{cov}(X,Y)}{V(X)} = \dfrac{-4.8}{11.76} = - 0,41$. $b = \overline{Y} – a\overline{X}$; $b = 12.32$. Then $Y = - 0.41X + 12.32$.	
	A-4	14 500 corresponds to $X = 14.5$ gives $Y =(– 0.41\times14.5) + 12.32$ $Y = 6.375$. The demand is 6375 units. ►Or : graphically.	
	B	For a unit price equal to 7870 LL and a demand equal to 9100 units, we have market equilibrium.	

II	1	$C_1 = C_0(1 + 0.05) + 200\ 000 = 10\ 700\ 000$ LL.	
	2	$C_{n+1} = C_n(1 + 0.05) + 200\ 000 = 1.05C_n + 200\ 000$.	
	3-a	$S_{n+1} = C_{n+1} + 4\ 000\ 000 = 1.05\ C_n + 4\ 200\ 000$. $= 1.05(C_n + 4\ 000\ 000\ ) = 1.05\ S_n$. Then (S_n) is a gemeotric sequence of common ration $r = 1.05$. $S_o = C_o + 4\ 000\ 000 = 14\ 000\ 000$.	
	3-b	$S_n = S_o \times r^n = 14\ 000\ 000\ (1.05)^n$. $C_n = S_n – 4\ 000\ 000 = 14\ 000\ 000(1.05)^n – 4\ 000\ 000$.	
	3-c	$C_n > 17\ 000\ 000$ then $(1.05)^n > \dfrac{21\ 000\ 000}{14\ 000\ 000}$; $(1.05)^n > 1.5$ $n\times\ln(1.05) > \ln(1.5)$ with $\ln(1,05) > 0$, $n > 8.31$ Therefore after 9 years the balance in Zahi's account exceeds 17 000 000 LL for the first time.	

III	1	$P(E) = \dfrac{50}{50+40} \times \dfrac{30}{30+80} = \dfrac{15}{99} = \dfrac{5}{33}$.	

<table>
<tr><td></td><td></td><td>P(F) = $\frac{50}{50+40}\times\frac{80}{30+80}+\frac{30}{30+80}\times\frac{40}{50+40}=\frac{52}{99}$.</td><td></td></tr>
<tr><td></td><td>2-a</td><td>The four values of X are :
66000 , 136000, 150000 and 220000.</td><td></td></tr>
<tr><td></td><td>2-b</td><td><table>
<tr><td>x_i</td><td>66000</td><td>136000</td><td>150000</td><td>220000</td></tr>
<tr><td>P_i</td><td>$\frac{4}{9}\times\frac{8}{11}=\frac{32}{99}$</td><td>$\frac{3}{11}\times\frac{4}{9}=\frac{12}{99}$</td><td>$\frac{5}{9}\times\frac{8}{11}=\frac{40}{99}$</td><td>$\frac{5}{9}\times\frac{3}{11}=\frac{15}{99}$</td></tr>
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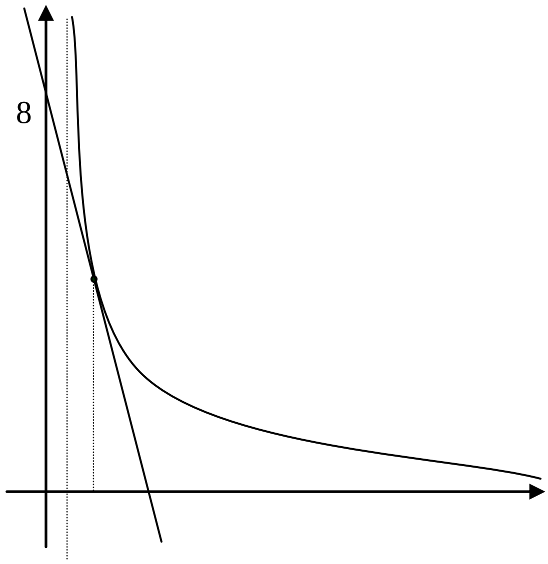
<tr><td>IV</td><td>A-1</td><td>$\lim\limits_{x\to+\infty} f(x)=0$; the line of equation y = 0 is an asymptote of (C).
$\lim\limits_{x\to\frac{1}{e}} f(x)=+\infty$; the line of equation $x=\frac{1}{e}$ is an asymptote of (C).</td><td></td></tr>
<tr><td></td><td>A-2</td><td>$f'(x)=\frac{-4}{x(1+\ln x)^2}$;
f ’(x) < 0 over its domain.<table>
<tr><td>x</td><td>$\frac{1}{e}$</td><td>$+\infty$</td></tr>
<tr><td>f ’(x)</td><td>‖</td><td>—</td></tr>
<tr><td>f(x)</td><td>‖ $+\infty$</td><td>↘ 0</td></tr>
</table></td><td></td></tr>
<tr><td></td><td>A-3</td><td>f(1)= 4 ; f(2) = 2.36 and f(3) = 1.90 .</td><td></td></tr>
<tr><td></td><td>A-4</td><td>(d) : y – f(1) = f ’(1)(x – 1) ; y = – 4x + 8.</td><td></td></tr>
<tr><td></td><td></td><td> </td><td></td></tr>
</table>

	B-1	Price = 1 000 LL if p = 1. $f(1)=\frac{4}{1+\ln 1}=4$; the number of batteries is 4 000.	
IV	B-2 a	$e(p)=-p\frac{f'(p)}{f(p)}=\frac{1}{1+\ln p}$.	
	B-2 b	$e(3)=\frac{1}{1+\ln 3}=0.47$. An increase of 1 % in the price, when the unit price is 3 000 LL will	

		cause a decrease of 0.47 % in demand. f is inelastic for $p = 3$. Since $0 < E(3) < 1$.	