

الدورة الإستثنائية للعام 2012	امتحانات الشهادة الثانوية العامة الفرع : آداب و إنسانيات	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعة	عدد المسائل: ثلاث

**ملاحظة:** - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.  
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

### I- (5 points)

Farid wants to save money in order to secure his child's university education.

He has two offers A and B:

Offer **A** : Depositing, in a savings account , an amount of 20 000 000 LL for a period of 10 years  
at an annual interest rate of 5% , compounded quarterly.

Offer **B**: Depositing, in a savings account, at the end of every month, an amount of 200 000 LL for a  
period of 10 years at an annual interest rate of 6%, compounded monthly.

- 1) a- If Farid chooses offer **A**, calculate the future value at the end of the tenth year.  
b- If Farid chooses offer **B**, calculate the future value at the end of the tenth year.
- 2) Which of the two offers is more advantageous for the child ?

### II- (5 points)

A gardener has 170 flower bulbs. These bulbs may bloom to give irises or dahlias as indicated in the table below:

Nature	Color of the flower	Red	White	Yellow
	Irises	40	10	30
Dahlias	60	20	10	

The gardener chooses randomly one of these bulbs to plant.

- 1) Calculate the probability of each of the following events:

A: « the chosen bulb gives a red flower when it blooms».

B: « the chosen bulb gives a white iris when it blooms».

C: « the chosen bulb gives a non- red dahlia when it blooms».

- 2) The chosen bulb gives a yellow flower, what is the probability that this yellow flower is a dahlia?

### III- (10 points)

Consider the function  $f$  defined, over  $] -\infty; 0[ \cup ] 0; +\infty [$ , by  $f(x) = -x + 3 - \frac{4}{x}$

and denote by  $(C)$  its representative curve in an orthonormal system  $(O; \vec{i}, \vec{j})$ .

1) Calculate  $\lim_{\substack{x \rightarrow 0 \\ x < 0}} f(x)$  and  $\lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x)$ . Deduce an asymptote to  $(C)$ .

2) a- Calculate  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow +\infty} f(x)$ .

b- Show that the line  $(D)$  with equation  $y = -x + 3$  is an asymptote to  $(C)$ .

3) Show that  $f'(x) = \frac{-x^2 + 4}{x^2}$ .

4) The table below represents the table of variations of  $f$ . Copy and complete this table:

$x$	$-\infty$	$-2$	$0$	$2$	$+\infty$
$f'(x)$	$-$	$0$		$+$	$0$
$f(x)$					

5) Draw  $(D)$  and  $(C)$ .

6) a- Calculate the abscissas of the points of intersection of  $(C)$  with the line with equation  $y = 8$ .

b- Solve, graphically, the inequality  $f(x) > 8$ .

I	Solution	Grade
1a	$A : F=P(1+i)^n , F= 20\,000\,000\left(1+\frac{0.05}{4}\right)^{40} = 32\,872\,389.27 .$ <p>The future value is LL 32 872 389.27.</p>	2
1b	$B : S = R \frac{(1+i)^n - 1}{i} = 200000 \frac{\left(1 + \frac{0.06}{12}\right)^{120} - 1}{\frac{0.06}{12}} = 32775869.36.$ <p>The future value is 32 775 869.36 LL.</p>	2
2	Offer A is more advantageous for the child.	1

II	Solution	Grade
1	$p(A) = \frac{100}{170} = 0.588. \quad p(B) = \frac{10}{170} = 0.0588. \quad p(C) = \frac{30}{170} = 0.1764$	3.5
2	$P(\text{a dahlia knowing that it is yellow}) = \frac{10}{40} = 0.25$	1.5

III	Solution	Grade
1	$\lim_{x \rightarrow 0^-} f(x) = 0 + 3 + \infty = +\infty \quad \text{and} \quad \lim_{x \rightarrow 0^+} f(x) = 0 + 3 - \infty = -\infty .$ <p>Then, the line with equation : <math>x = 0</math> is an asymptote to (C).</p>	1
2a	<ul style="list-style-type: none"> <li>• <math>\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} (-x+3) = -\infty .</math></li> <li>• <math>\lim_{x \rightarrow -\infty} f(x) = +\infty</math></li> </ul>	0.5

2b	$\lim_{x \rightarrow -\infty} [f(x) - (-x+3)] = \lim_{x \rightarrow -\infty} \left(-\frac{4}{x}\right) = 0^+$ $\lim_{x \rightarrow +\infty} [f(x) - (-x+3)] = \lim_{x \rightarrow +\infty} \left(-\frac{4}{x}\right) = 0^-$ <p>Hence, (D) is an asymptote to (C) at <math>-\infty</math> and at <math>+\infty</math>.</p>	0.5
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3	$f'(x) = -1 + \frac{4}{x^2} = \frac{-x^2 + 4}{x^2}$	1
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4	<table border="1" style="width: 100%; text-align: center;"> <tr> <td style="width: 15%;">x</td> <td style="width: 15%;">-∞</td> <td style="width: 15%;">-2</td> <td style="width: 15%;">0</td> <td style="width: 15%;">2</td> <td style="width: 15%;">+∞</td> </tr> <tr> <td>f'(x)</td> <td>-</td> <td>0</td> <td>+</td> <td>+</td> <td>0</td> <td>-</td> </tr> <tr> <td>f(x)</td> <td>+∞</td> <td>↘</td> <td>↗</td> <td>+∞</td> <td>↘</td> <td>-∞</td> </tr> <tr> <td></td> <td></td> <td>7</td> <td></td> <td>-∞</td> <td>-1</td> <td></td> </tr> </table>	x	-∞	-2	0	2	+∞	f'(x)	-	0	+	+	0	-	f(x)	+∞	↘	↗	+∞	↘	-∞			7		-∞	-1		2
x	-∞	-2	0	2	+∞																								
f'(x)	-	0	+	+	0	-																							
f(x)	+∞	↘	↗	+∞	↘	-∞																							
		7		-∞	-1																								

5		2
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6a	$f(x) = 8$ then $-x^2 + 3x - 4 = 8x$ which gives $x^2 + 5x + 4 = 0$ which is true for $x = -1$ or $x = -4$ .	1.5
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6b	$f(x) > 8$ when $x < -4$ or $-1 < x < 0$	1.5
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