عدد المسائل ثلاث مسابقة في الرياضيات الاسم المدة : ساعة واحدة الرقم :

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات. يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (10 points)

Let f be a function defined by $f(x) = px + q - \frac{3}{x+1}$, (p and q are real numbers).

(C) is the representative curve of f in an orthonormal system.

Α-

- 1) Verify that the domain of definition of f is $]-\infty;-1[\cup]-1;+\infty[$.
- 2) Determine p and q in the case where f(-2) = 0 and f(2) = 0.

B-

In this part we take $f(x) = x - 1 - \frac{3}{x + 1}$.

- 1) Calculate $\lim_{\substack{x\to -1\\x<-1}} f(x)$ and $\lim_{\substack{x\to -1\\x>-1}} f(x)$. Deduce an asymptote (D) of (C).
- 2) a- Calculate $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to +\infty} f(x)$.

b- Show that the line (d) of equation y = x - 1 is an asymptote of (C).

- 3) Calculate f'(x) and verify that f'(x) > 0.
- 4) Set up the table of variations of f.
- 5) Draw (D), (d) and (C).
- 6) Solve the inequality $f(x) \ge 0$.

II- (4 points)

A florist sells two kinds A and B of bouquets of flowers each is formed of both red and white flowers. Compositions and their prices are given in the following table:

	Red flower	White flower	Price of the bouquet
Bouquet of kind A	8	12	10 200 LL.
Bouquet of kind B	5	7	6 200 LL.

- 1) What is the price of a red flower and that of a white one?
- 2) During the holidays the price of a red flower increases by 30 % and that of a white flower increases by 20 %.

What is the price of a bouquet of flowers of kind A during the holidays?

III- (6 points)

A library has 100 books textbooks placed on a shelf according to the given table:

The book is written in	Arabic	French	English
Secondary	15	28	20
Intermediate	12	9	16

A- A customer chooses at random a book from this shelf.

Consider the following events:

S: « the chosen book is of the secondary level ».

F: «the chosen book is in French ».

1) Calculate the following probabilities:

$$P(S)$$
, $P(F)$, $P(S \cap F)$ and $P(S \cup F)$.

- 2) Knowing that the chosen book is in French, what is the probability it is of secondary level?
- **B-** In this part, the customer chooses successively, at random and without replacement, two books from the 100 books on the shelf.

What is the probability of getting the first book is English and the second one is French?

LH	[MATH 1 st SESSION	2004
	Q	Short answers	M
	A-1	f is defined iff $x+1\neq 0$, so $D_f=]-\infty$; $-1[\cup]-1$; $+\infty[.$	
	A-2	f(-2) = 0 and $f(2) = 0$ that give : $-2p + q + 3 = 0$ and $2p + q - 1 = 0$	
		We get $p = 1$ and $q = -1$.	
	D 4	$\lim_{x \to -1} f(x) = +\infty \text{ and } \lim_{x \to -1} f(x) = -\infty.$	
	B-1	x<-1	
	D 2	Consequently the line (D) of equation $x = 1$ is an asymptote of (C).	
	B-2 a	$\lim_{x \to -\infty} f(x) = -\infty \text{ and } \lim_{x \to +\infty} f(x) = +\infty.$	
	a	3	
	D 2	$f(x) - (x-1) = -\frac{5}{x+1}$	
	B-2 b	$\lim_{x \to 1} [f(x) - (x-1)] = 0$ and $\lim_{x \to 1} [f(x) - (x-1)] = 0$. Thus (d) is an	
		$f(x) - (x - 1) = -\frac{3}{x + 1}$ $\lim_{x \to -\infty} [f(x) - (x - 1)] = 0 \text{ and } \lim_{x \to +\infty} [f(x) - (x - 1)] = 0. \text{ Thus (d) is an}$ asymptote of (C)	
		asymptote of (C). $f'(x) = 1 + \frac{3}{(x+1)^2}$, $f'(x) > 0$.	
	B-3	$f'(x) = 1 + \frac{3}{x^2}, f'(x) > 0.$	
		$(x+1)^2$	
		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
	B-4	f'(x) + $+$	
	5 -	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
		$-\infty$	
I	B-5	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
	B-6	$f(x) \ge 0$ when (C) is above $(x'x)$ then $-2 \le x < -1$ or $x \ge 2$.	

II	x :the price of a red rose; y :the price of a white rose.	

	$\begin{cases} 8x + 12 \text{ y} = 10\ 200 \\ 5x + 7\text{y} = 6\ 200 \end{cases}$ $x = 750 \text{ and } y = 350.$ The price of a red rose is 750 LL and that of a white one is 350LL.	
2	Price of a red rose : $750 + 750 \times 0.3 = 750 \times 1.3 = 975$ LL. Price of a white rose : $350 \times 1.2 = 420$ LL. Price of a bouquet of kind A : $12 \times 975 + 8 \times 420 = 15$ 060LL.	

		$P(S) = \frac{15 + 28 + 20}{100} = 0.63.$	
	A-1	$P(F) = \frac{28+9}{100} = 0.37.$	
III		$P(S \cap F) = \frac{28}{100} = 0.28.$	
		$P(S \cup F) = P(S) + P(F) - P(S \cap F) = 0.63 + 0.37 - 0.28 = 0.72.$	
	A-2	$P(S/F) = \frac{28}{28+9} = \frac{28}{37}.$	
	В	P(E, F) = $\frac{20+16}{100} \times \frac{28+9}{100-1} = \frac{36}{100} \times \frac{37}{99} = \frac{37}{275}$.	