الدورة الإستثنائية للعام 2009	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل: اربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the plane (P) of equation: x – y + z + 2 = 0, and the two straight lines (D) and (D') defined by the parametric equations:

$$(D)\begin{cases} x = t \\ y = -t+1 \\ z = 2t-1 \end{cases} \quad \text{and} \quad (D')\begin{cases} x = -5m-10 \\ y = 5m+11 \\ z = -2m-5 \end{cases}$$ where t and m are real parameters.

1) Show that (D) and (D') intersect at the point A(0; 1; –1) and verify that A belongs to plane (P).

2) Write an equation of the plane (Q) that contains the two straight lines (D) and (D').

3) Determine a system of parametric equations of the straight line (d), the intersection of (P) and (Q) .

4) Verify that the point B(1; 0; –3), which is on the straight line (d), is equidistant from the two straight lines (D) and (D'), and deduce that (d) is a bisector of the angle between (D) and (D').

II- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, M and M' of respective affixes 2, $-i$, z and z' where $z' = \dfrac{iz-1}{z-2}$. $(z \neq 2)$.

1) Find the coordinates of M when $z' = 1+2i$.

2) Give a geometric interpretation for $|z-2|$ and for $|iz-1|$ and determine the set of points M such that $|z-2| = |iz-1|$.

3) Let $z = x + iy$ and $z' = x' + iy'$ (x, y, x' and y' are real numbers).

   a- Calculate x' and y' in terms of x and y.

   b- Show that if z' is pure imaginary, then M moves on a straight line whose equation is to be determined.

   c- Show that if z is real, then M' moves on a straight line whose equation is to be determined.

III- (4 points)

The following table represents the distribution of the ages of 26 men and 24 women.

Age in years	$[20;25[$	$[25;30[$	$[30;35]$
Number of men	8	8	10
Number of women	5	9	10

3 persons are randomly chosen, from these 50 people, to form a committee.
Consider the following events:

M: « the committee is formed of three men ».
F : « the committee is formed of three women ».
A: « the committee is mixed (formed of men and women) ».
B: « the age of each member of the committee is less than 30 years ».

1) Calculate each of the probabilities p(M), p(F) and p(A).

2) a- Calculate p(B) and show that $p\left(B\cap\bar{A}\right)=\frac{33}{700}$. Deduce $p\left(B\cap A\right)$.

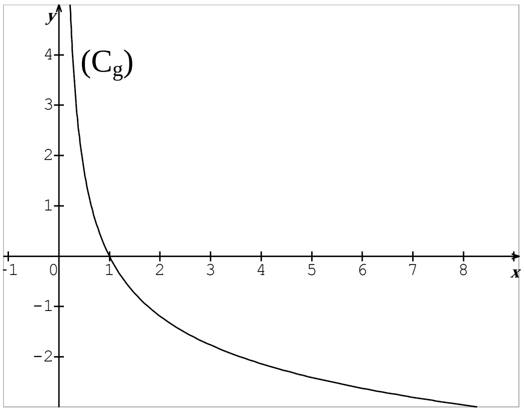
b- Calculate $p\left(B/A\right)$.

3) Designate by X the random variable that is equal to the number of women in the committee who have an age less than 25 years.
Determine the probability distribution of X.

IV- (8 points)

Consider the function f defined, on $]0;+\infty[$, by $f(x)=\frac{1+\ln x}{e^x}$ and let (C) be its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

The curve (C_g), shown in the adjacent figure, is the representative curve in an orthonormal system, of the function g defined on $]0;+\infty[$ by $g(x)=\frac{1}{x}-1-\ln x$.



1) Calculate the area of the region bounded by the curve (C_g), the axis of abscissas and the line of equation x = 2.

2) Show that $f'(x)=\frac{g(x)}{e^x}$ and deduce the sign of $f'(x)$ according to the values of x.

3) Calculate $\lim_{x\to 0} f(x)$ and $\lim_{x\to+\infty} f(x)$ and determine the asymptotes of the curve (C).

4) Set up the table of variations of f.

5) Solve the equation f(x) = 0.

6) Find an equation of the tangent to the curve (C) at the point of abscissa $\frac{1}{e}$.

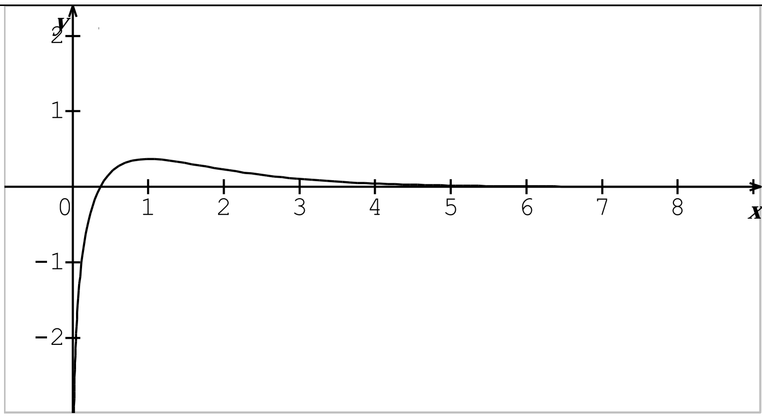
7) Draw (C).

8) Discuss, according to the values of the real number m, the number of solutions of the equation $\ln x = me^x - 1$.

QI	Corrigé	Note
1	x = 0 , donc t = 0 ; y =1 et z = -1 et m= -2 ; y =1 et z = -1. A(0,1,-1) est le point d'intersection des deux droites. (P) : x – y + z + 2 = 0. 0 – 1 – 1 + 2 = 0, donc A appartient au plan (P).	1
2	$\overrightarrow{V_{(D)}}(1,-1,2) \quad \overrightarrow{V_{(D')}}(-5,5,-2)$ et $A(0,1,-1)$ M(x, y, z) est un point du plan (Q) si et seulement si $\det(\overrightarrow{AM},\overrightarrow{V_{(D)}},\overrightarrow{V_{(D')}}) = \begin{vmatrix} x & y-1 & z+1 \\ 1 & -1 & 2 \\ -5 & 5 & -2 \end{vmatrix} = -8x-8y+8 = 0$ une equation de (Q) : $x+y-1=0$	1
3	(P): x – y + z + 2 = 0 et (Q): x + y - 1 = 0. $(d)\begin{cases} x = \alpha \\ y = -\alpha+1 \\ z = -2\alpha-1 \end{cases}$	0.5
4	B (1 , 0 , –3) appartient à (d) et A(0 , 1 , - 1) appartient à (D) $\overrightarrow{BA}\wedge\overrightarrow{V_{(D)}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 4\vec{i}+4\vec{j}$. $d(B,(D)) = \dfrac{\Vert \overrightarrow{BA}\wedge\overrightarrow{V_{(D)}} \Vert}{\Vert \overrightarrow{V_{(D)}} \Vert} = \dfrac{\sqrt{32}}{\sqrt{6}} = \sqrt{\dfrac{16}{3}}$. A(0 , 1 , - 1) appartient à (D') , $\overrightarrow{BA}\wedge\overrightarrow{V_{(D')}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ -5 & 5 & -2 \end{vmatrix} = -12\vec{i} -12\vec{j}$ $d(B,(D')) = \dfrac{\Vert \overrightarrow{BA}\wedge\overrightarrow{V_{(D')}} \Vert}{\Vert \overrightarrow{V_{(D')}} \Vert} = \dfrac{\sqrt{288}}{\sqrt{54}} = \sqrt{\dfrac{16}{3}}$. donc B est équidistant de (D) et (D'). A est l'intersection de(D) et (D'), donc A est équidistant de (D) et (D') La droite (d) est contenue dans le plan (Q) et passe par A et B donc (d) est une bissectrice de l'angle de (D) et (D').	1.5

QII	Corrigé	Note
1	$1 + 2i = \frac{iz-1}{z-2}$; $(1 + i)z = 1 + 4i$; $z = \frac{5}{2} + \frac{3}{2}i$; $M(\frac{5}{2}$; $\frac{3}{2})$.	0.5
2	$\lvert z-2\rvert = AM$ et $\lvert iz-1\rvert = \lvert i.(z+i)\rvert = \lvert i\rvert.\lvert z-(-i)\rvert = \lvert z-(-i)\rvert = BM$. $\lvert z-2\rvert = \lvert iz-1\rvert$; AM=BM ; l'ensemble des points M est la médiatrice du segment [AB].	1
3a	$x' + iy' = \frac{-y-1+ix}{x-2+iy} = \frac{2y-x+2+i(x^2+y^2-2x+y)}{(x-2)^2+y^2}$ $x' = \frac{2y-x+2}{(x-2)^2+y^2}$ et $y' = \frac{x^2+y^2-2x+y}{(x-2)^2+y^2}$	0.5
3b	z' est imaginaire pur si x'= 0 et $z' \neq 0$; $2y - x + 2 = 0$ avec $z \neq -i$ et $z \neq 2$. M se déplace sur la droite (d) d'équation : $y = \frac{x}{2} - 1$	1
3c	z est un réel si y = 0 : $x' = \frac{-1}{x-2}$ et $y' = \frac{x}{x-2}$; $y' = -2x'+1$ et M' se déplace sur la droite d'équation : $y = -2x + 1$.	1

QIII.	Corrigé	Note
1	$P(M) = \frac{C_{26}^3}{C_{50}^3} = \frac{2600}{19600} = \frac{13}{98}$. ; $P(F) = \frac{C_{24}^3}{C_{50}^3} = \frac{2024}{19600} = \frac{253}{2450}$. $P(A) = 1 - P(M) - P(F) = \frac{936}{1225} = 0{,}764$.	1
2a	$P(B) = \frac{C_{30}^3}{C_{50}^3} = 0{,}207$. ; $P(B \cap \overline{A}) = \frac{C_{16}^3 + C_{14}^3}{C_{50}^3} = \frac{33}{700} = 0{,}047$. $p(B \cap \overline{A}) = 0{,}047$, $p(B) = p(B \cap A) + p(B \cap \overline{A})$ Donc $p(B \cap A) = 0{,}207 - 0{,}047 = 0{,}16$.	1.5
2b	$p(B/A) = \frac{p(B \cap A)}{p(A)} = 0{,}21$.	0.5
3	Les valeurs possibles de X : 0,1,2,3 X= x_i : 0 ; 1 ; 2 ; 3 p_i : $\frac{C_5^0 \times C_{45}^3}{C_{50}^3} = \frac{1419}{1960}$; $\frac{C_5^1 \times C_{45}^2}{C_{50}^3} = \frac{99}{392}$; $\frac{C_5^2 \times C_{45}^1}{C_{50}^3} = \frac{9}{392}$; $\frac{C_5^3}{C_{50}^3} = \frac{1}{1960}$	1

<table>
<tr><th>QIV</th><th>Corrigé</th><th>Note</th></tr>
<tr><td>1</td><td>$A=-\int_1^2 g(x)dx=-\left[\ln x-x-x\ln x+x\right]_1^2=\ln 2$ u.a.</td><td>1</td></tr>
<tr><td>2</td><td>$f'(x)=\dfrac{\frac{1}{x}e^x-e^x(1+\ln x)}{(e^x)^2}=\dfrac{g(x)}{e^x}$ et $e^x>0$ alors le signe de f´(x) est celui de g(x) donc f '(x) > 0 pour 0< x < 1 , f '(1) = 0 et f '(x) < 0 pour x >1</td><td>1.5</td></tr>
<tr><td>3</td><td>$\lim\limits_{x\to 0^+} f(x)=\dfrac{-\infty}{1}=-\infty$ alors y´y est une asymptote de (C).
$\lim\limits_{x\to+\infty} f(x)=\dfrac{+\infty}{+\infty}$ ind. (Hop) = $\lim\limits_{x\to+\infty}\dfrac{1}{xe^x}=\dfrac{1}{+\infty}=0$ alors y = 0 est une asymptote de (C).</td><td>1</td></tr>
<tr><td>4</td><td><table>
<tr><td>x</td><td>0</td><td>1</td><td>$+\infty$</td></tr>
<tr><td>$f'(x)$</td><td>|| +</td><td>0</td><td>−</td></tr>
<tr><td>$f(x)$</td><td>|| $-\infty$ ↗</td><td>$\frac{1}{e}$</td><td>↘ 0</td></tr>
</table></td><td>1</td></tr>
<tr><td>5</td><td>f (x) = 0 ; 1+lnx = 0 ; x= $\frac{1}{e}$.</td><td>0.5</td></tr>
<tr><td>6</td><td>x= $\frac{1}{e}$ alors $f(\frac{1}{e})=0$ et $f'(\frac{1}{e})=e^{1-\frac{1}{e}}$ une équation de la tangente :
y = $e^{1-\frac{1}{e}}(x-\frac{1}{e})$</td><td>1</td></tr>
<tr><td>7</td><td>
</td><td>1</td></tr>
<tr><td>8</td><td>lnx = me^x – 1 est équivalente à f(x) = m.
pour m≤ 0 une solution .
pour 0 < m < $\frac{1}{e}$ deux solutions
pour m = $\frac{1}{e}$ une solution double.
Pour m > $\frac{1}{e}$ pas de solutions.</td><td>1</td></tr>
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