الدورة الإستثنائية للعام 2009	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة أربع ساعات	عدد المسائل : ست

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات0
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)0

I- (2 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, with justification, the answer corresponding to it.

N°	Questions	Answers		
		a	b	c
1	Let f be the function defined over $IR-\{0\}$ by $f(x)=\frac{1}{x}$ and let g be the function defined over $IR-\{1\}$ by $g(x)=\frac{x}{x-1}$. The domain of definition of $g\circ f$ is:	$IR-\{0\}$	$IR-\{1\}$	$IR-\{0;1\}$
2	$\neg\,(p\Rightarrow q)$ is equivalent to :	$p\wedge(\neg q)$	$(\neg p)\wedge(\neg q)$	$(\neg q)\Rightarrow(\neg p)$
3	A, M and N are three distinct points of respective affixes i, z_1 and z_2. If $z_2=iz_1+1+i$, then, triangle AMN is :	equilateral	semi-equilateral	right isosceles
4	With 10 distinct points situated on a circle, we can determine:	720 triangles	120 triangles	150 triangles
5	The function f defined over $]0;1]$ by $f(x)=\sqrt{\frac{1-x}{x}}$ has an inverse function g defined by :	$g(x)=\sqrt{\frac{x}{1-x}}$	$g(x)=\frac{1}{x^2-1}$	$g(x)=\frac{1}{x^2+1}$
6	If $z=-2\left(\sin\left(\frac{\pi}{3}\right)+i\cos\left(\frac{\pi}{3}\right)\right)$, then $\arg(\bar{z})=$	$-\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$

II- (2 points)

The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$.
Consider the point A(-1 ; 1 ; 0) , the plane (P) of equation $x - 2y + 2z - 6 = 0$ and the straight line (D) defined by the system $x = 2m - 3$; $y = 3m - 2$; $z = 2m - 2$ (m is a real parameter).

1) a- Verify that A does not belong to (P) and calculate the distance from A to (P) .
   b- Prove that (D) passes through A and is parallel to (P).

2) a- Determine a system of parametric equations of the straight line (d) passing through A and perpendicular to (P) .
   b- Determine the coordinates of B, the point of intersection of (d) and (P) .
   c- Determine a system of parametric equations of the straight line (Δ_0) passing through B and parallel to (D) and prove that (Δ_0) lies in (P).

3) Let (Δ) be a straight line, other than (Δ_0), passing through B and lying in (P) .

   a- Prove that (Δ) and (D) are skew (not coplanar).
   b- Prove that (AB) is perpendicular to (Δ) and to (D).

III- (3 points)

In an oriented plane, consider the rectangle ABCD such that:
$\left(\overrightarrow{AB};\overrightarrow{AD}\right)=\frac{\pi}{2}(\mathrm{mod}\,2\pi)$, AB = 4 and AD = 3.
Let H be the orthogonal projection of A on (BD) and *h* be the dilation, of center H, that transforms D to B.

1) a- Determine the image of the straight line (AD) by *h*.

   b- Deduce the image E of point A by *h*. Plot E.

   c- Construct the point F image of B by *h* and the point G image of C by *h*, then determine the image of rectangle ABCD by *h*.

2) Let S be the direct similitude that transforms A onto B and D onto A.

   a- Determine an angle of S .

   b- Determine the image of the straight line (AH) by S and the image of the straight line (BD) by S.

   c- Deduce that H is the center of S.

3) Show that $S(B)=E$ and deduce that $S\circ S(A)=h(A)$.
4) Show that $S\circ S=h$.

IV- (3 points)

An urn contains **three** white balls and **two** black balls.

A player draws **randomly** and **successively** three balls from this urn, respecting the following rule:

In each draw: if the drawn ball is black, he replaces it back in the urn ;

if it is white, he doesn't replace it back in the urn.

1) a- Calculate the probability of drawing, in the following order: one black ball, one black ball then one white ball.

b- Show that the probability of obtaining one white ball only, among the three drawn balls, is equal to $\frac{183}{500}$.

2) Among the three drawn balls, the player marks three points for each white ball drawn and two points for each black ball drawn.

Designate by X the random variable equal to the sum of points marked for the three drawn balls.

a- Show that the possible values of X are: 6, 7, 8 and 9.

b- Determine the probability distribution of X and calculate its expected value.

3) The player now draws **randomly** and **successively** n balls from the urn ($n > 3$) respecting the same rule.

a- Calculate, in terms of n, the probability of the event: "the player draws n black balls".

b- Calculate, in terms of n, the probability P_n of the event:

"the player obtains at least one white ball ".

c- What is the minimum number of balls to be drawn by the player so that $P_n \geq 0.99$?

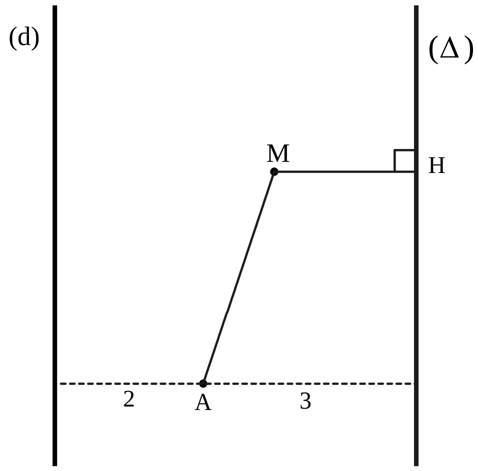
V- (3 points)

In a plane, given two parallel straight lines (d) and (Δ) at a distance from each other equal to 5 cm and a point A situated between (d) and (Δ) at a distance of 3 cm from (Δ).

M is a variable point in the plane and H is its orthogonal projection on (Δ).

1) Show that if MA + MH = 5 cm, then M moves on a parabola (S) of focus A.

In what follows, the plane is referred to a direct orthonormal system $(O;\vec{i},\vec{j})$ *such that* A(1 ; 0).



2) a- Prove that $y^2 = 4x$ is an equation of the parabola (S).

b- Draw (S) .

3) Let E be a point on (S) of **ordinate** a such that $a \neq 0$.

Show that $4x - 2ay + a^2 = 0$ is an equation of the tangent (d_1) to (S) at E .

4) Let G be a point on (S) of ordinate b such that $E\hat{O}G = 90^\circ$.

a- Prove that $ab = -16$.

b- The tangent (d_2) to (S) at G cuts (d_1) at a point L .

Prove that, as E and G vary on (S) such that $E\hat{O}G = 90^\circ$, the point L moves on a straight line to be determined.

VI- (7 points)

A-

Consider the function f defined on IR by $f(x) = e^{2x} - 4e^{x} + 3$.

Designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim_{x\to-\infty} f(x)$, $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to+\infty} \frac{f(x)}{x}$.

b- Solve the equation f(x) = 0.

2) Calculate $f'(x)$ and set up the table of variations of f.

3) Show that O is a point of inflection of (C) .

4) Write an equation of the tangent (T) at O to (C) .

5) Let h be the function defined on IR by $h(x) = f(x) + 2x$.

a- Show that $h'(x) \geq 0$ for every real number x.

b- Deduce, according to the values of x, the relative positions of (C) and (T).

6) Draw (T) and (C) .

7) Calculate the area of the region bounded by (C) , the axis of abscissas and the two lines of equations $x = 0$ and $x = \ln 3$.

8) a- Show that f has, on $[\ln 2 ; +\infty[$, an inverse function f^{-1}.

b- Show that the equation $f(x) = f^{-1}(x)$ has a unique solution α and verify that $1.2 < \alpha < 1.3$.

B-

Let g be the function given by $g(x) = \ln[f(x)]$.

Designate by (Γ) its representative curve in an orthonormal system.

1) Justify that the domain of definition of g is $]-\infty;0[\cup]\ln 3;+\infty[$.

2) Determine $\lim_{x\to-\infty} g(x)$. Deduce an asymptote (D) of (Γ) .

3) Show that the line (d) of equation $y = 2x$ is asymptote to (Γ) at $+\infty$.

4) Determine the coordinates of the points of intersection of (Γ) with (d) and (D).

5) Set up the table of variations of g .

6) Draw (Γ) .

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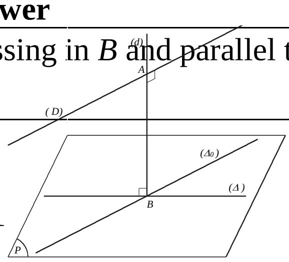
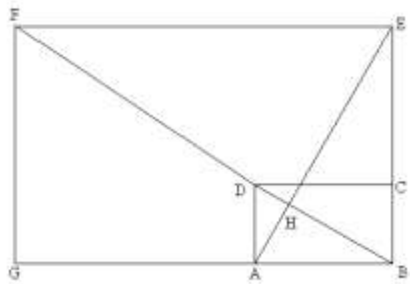
	مسابقة في مادة الرياضيات	مشروع معيار التصحيح

QI	Answer	M
1	$x \in IR-\{0\}$ and $f(x) \neq 1$, so the domain of $g \circ f$ is $IR-\{0;1\}$. (c)	0.5
2	$p \Rightarrow q$ is $\neg p \vee q$ then $\neg(p \Rightarrow q)$ is equivalent to $p \wedge (\neg q)$. (a)	0.5
3	$\frac{z_{\overrightarrow{AN}}}{z_{\overrightarrow{AM}}} = \frac{z_2 - i}{z_1 - i} = \frac{iz_1+1+i-i}{z_1-i} = \frac{i(z_1-i)}{z_1-i} = i$ So $AM = AN$ and $(\overrightarrow{AM};\overrightarrow{AN}) = \frac{\pi}{2}(2\pi)$, the triangle AMN is right isosceles at A. (c)	1
4	With 10 distinct points situated on a circle, we can determine $C_{10}^{3} = 120$ triangles (b)	0.5
5	$y = \sqrt{\frac{1-x}{x}}$ gives $y^2 = \frac{1-x}{x}$ then $x = \frac{1}{y^2+1}$, hence $g(x) = \frac{1}{x^2+1}$. (c)	1
6	$\bar{z} = -2\left(\sin\left(\frac{\pi}{3}\right) - i\cos\left(\frac{\pi}{3}\right)\right) = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$, $\arg(\bar{z}) = \frac{5\pi}{6}$. (b)	0.5

QII	Answer	M
1a	$-1-2-6=-9 \neq 0$, A does not belong to (P) ; $d(A;(P))=3$.	0.5
1b	A is the point of (D) corresponding to $m=1$; $(D)\cap(P)=\phi$ OR $\vec{n}(1;-2;2) \perp (P)$, $\vec{u}(2;3;2) // (D)$ and $\vec{n}\cdot\vec{u}=0$.	0.5
2a	$\vec{n}(1;-2;2)$ is a direction vector of (d) ; (d) : $x=t-1$; $y=-2t+1$; $z=2t$.	0.5
2b	$(d)\cap(P)=\{B(0;-1;2)\}$.	0.5
2c	$\vec{u}(2;3;2)$ is a direction vector of (Δ_0);(Δ_0) : $x=2\lambda$; $y=3\lambda-1$; $z=2\lambda+2$. (D) is parallel to (P) and (Δ_0) passes through the point B of (P) and is parallel to (D); hence, (Δ_0) lies in (P).	0.5
3a	(Δ) is not parallel to (D) since (Δ_0) is parallel to (D) and $(\Delta) \neq (\Delta_0)$. (Δ) and (D) do not intersect since (D) is parallel to (P) and (Δ) is a straight line in (P). Hence (Δ) and (D) are not coplanar .	1
3b	(AB) is perpendicular to (P) at B ;then (AB) is perpendicular to (Δ) and to (Δ_0) at B. But (Δ_0) is parallel to (D); hence (AB) is perpendicular to (D) at A and to (Δ) at B.	0.5

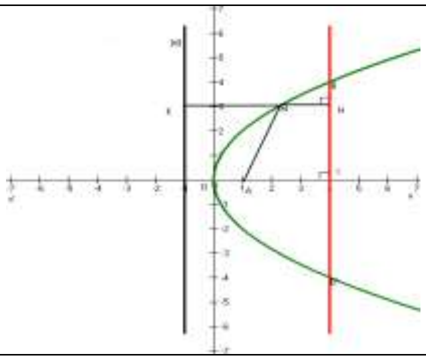
QIII	Answer	M
1a	$h(D)=B$, so the image of (AD) is the line passing in B and parallel to (AD), so $h(AD)=(BC)$.	0.5



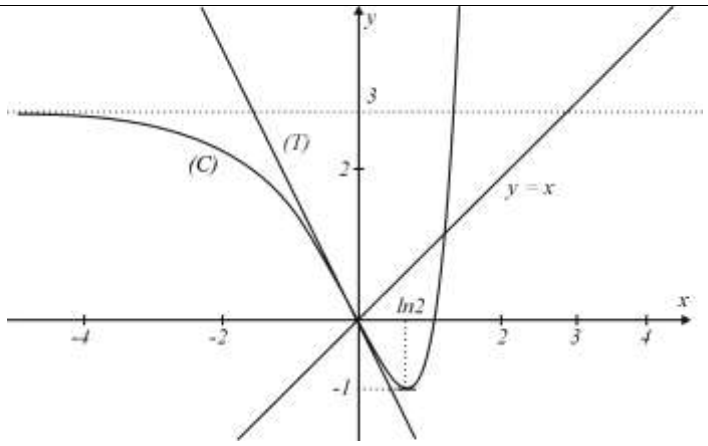
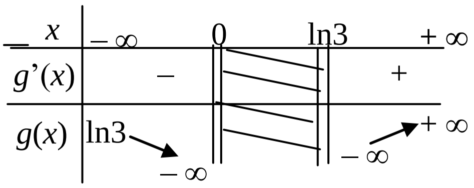
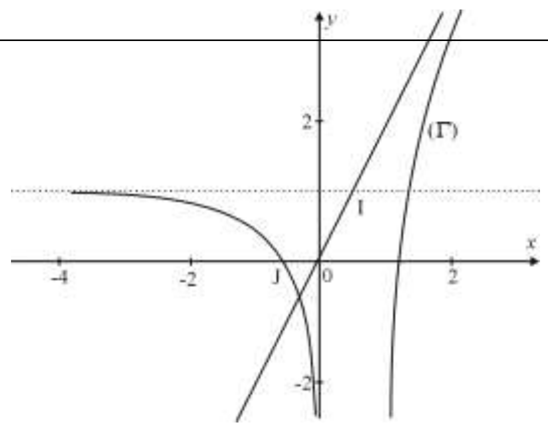
1b	$E\in(AH)$ and $A\in(AD)$, then $E\in(BC)$, thus $\{E\}=(AH)\cap(BC)$. 	0.5
1c	$F\in(BH)$. $B\in(AB)$, so $F\in(d)$ passing through E and parallel to (AB), so $\{F\}=(BH)\cap(d)$. G is the intersection of (HC) and the parallel through B to (DC).The image of rectangle $ABCD$ by h is the rectangle $EFGB$.	1
2a	$\alpha=\left(\overrightarrow{AD};\overrightarrow{BA}\right)=\frac{\pi}{2}(2\pi)$	0.5
2b	$S(A)=B$, so the image of (AH) is line passing through B and perpendicular to (AH), so it is (BD). $S(D)=A$, so the image of (BD) is line passing through A and perpendicular to (BD), so it is (AH).	1.5
2c	$\{H\}=(AH)\cap(BD)$, so $\{S(H)\}=(BD)\cap(AH)$, thus $S(H)=H$ so H is the center of S .	0.5
3	$B\in(BD)$, so $S(B)\in(AH)$ and $S(AB)=(BC)$ thus $S(B)=E$ intersection of the two lines (AH) and (BC). $S\circ S(A)=S(S(A))=S(B)=E=h(A)$.	1
4	SoS is a similitude of center H and angle π, thus SoS is a negative homothecy. Since $\mathrm{S}\circ\mathrm{S}(\mathrm{A})=\mathrm{h}(\mathrm{A})$, then $SoS=h$.	0.5

QIV	Answer	M
1a	$P_r(BBW)=\left(\frac{2}{5}\right)^2\times\frac{3}{5}=\frac{12}{125}$.	0.5
1b	$P_r(BBW)+P_r(BWB)+P_r(BBW)=\frac{3}{5}\times\left(\frac{2}{4}\right)^2+\left(\frac{2}{5}\times\frac{3}{5}\times\frac{2}{4}\right)+\left(\frac{2}{5}\right)^2\times\frac{3}{5}=\frac{75+60+48}{500}=\frac{183}{500}$.	1
2a	In three draws the possible outcomes are 3 black, or 1 white and 2 black, or 2W and 1B, or 3W; Thus the possible values of X are: 6;7;8 and 9.	0.5
2b	$P_r(X=6)=P_r(0\text{white balls})=P_r(BBB)=\left(\frac{2}{5}\right)^3=\frac{8}{125}$ $P_r(X=7)=P_r(1\text{white ball})=\frac{183}{500}$; $P_r(X=8)=P_r(2W)=P_r(WWB)+P_r(WBW)+P_r(BWW)=\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}+\frac{3}{5}\times\left(\frac{2}{4}\right)^2+\frac{2}{5}\times\frac{3}{5}\times\frac{2}{4}=\frac{235}{500}$ $P_r(X=9)=P_r(3\text{white balls})=\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}=\frac{1}{10}$ $E(X)=\frac{6\times32+7\times183+8\times235+9\times50}{500}=7.606$	2

3a	P_r(n black balls in the n draws) = $\left(\frac{2}{5}\right)^n$	0.5
3b	P_r(at least a white ball)=1-$\left(\frac{2}{5}\right)^n$ = P_n	1
3c	$P_n \geq 0.99 \Leftrightarrow 1-\left(\frac{2}{5}\right)^n \geq 0.99 \Leftrightarrow \left(\frac{2}{5}\right)^n \leq 0.01 \Leftrightarrow n \geq \frac{\ln(0.01}{\ln(0.4)} \Leftrightarrow n \geq 5.026$ Hence, the minimum number of balls is 6.	0.5

QV	**Answer**	**M**
1	(MH) is perpendicular to (d) at K and $MA+MH = MK+MH = 5$ then $MA = MK = d(M\ ,(d))$ .Then M moves on a parabola (S) of focus A and directrix (d).	1
2a	$A(1\,;0)$ and $(d): x=-1$ then (S) : $y^2 = 4x$, since the origin is the vertex and p = 2	1
2b		0.5
3	2yy'=4; $y' = \frac{2}{y}$; Equation of (d_1): $y - a = \frac{2}{a}\left(x - \frac{a^2}{4}\right)$; then $(d_1)\ :\ 4x - 2ay + a^2 = 0$.	1
4a	$E(\frac{a^2}{4}\ ;a)$ and $G(\frac{b^2}{4}\ ;b)$. $\overrightarrow{OE}.\overrightarrow{OG} = 0$ gives $ab = -16$.	1
4b	$(d_2)\ :\ 4x - 2by + b^2 = 0$. So $(d_1)\cap(d_2) = \left\{ L(-4\ ;\frac{a+b}{2}) \right\}$. When E and G vary on (S), y_L describes IR and L moves on the line of equation $x = -4$.	1.5

VI	**Answer**	**M**
A1a	$\lim\limits_{x\to-\infty} f(x) = 3$, $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left[e^x\left(e^x-4\right)+3\right] = +\infty$ $\lim\limits_{x\to+\infty}\frac{f(x)}{x} = \lim\limits_{x\to+\infty}\left[\frac{e^x}{x}\left(e^x-4\right)+\frac{3}{x}\right] = +\infty.$	1
A1b	f(x) = 0, $e^x = 1$ or $e^x = 3$; x = 0 or x = ln3	0.5
A2	$f'(x) = 2e^x\left(e^x - 2\right)$ x: $-\infty$; $\ln 2$; $+\infty$ $f'(x)$: — ; 0 ; + $f(x)$: 3 ↘ -1 ↗ $+\infty$	1
A3	$f''(x) = 4e^x\left(e^x - 1\right)$, $f''(x) > 0$ for $x > 0$, $f''(x) < 0$ for $x < 0$ and $f''(x) = 0$ for $x = 0$, $f(0) = 0$, thus O is a point of inflection of (C) .	1

A4	$y-0=f'(0)(x-0)$ and $f'(0)=-2$ then $y=-2x$.	0.5
A5a	$h'(x)=f'(x)+2=2(e^x-1)^2$; then $h'(x)\geq 0$, for every x .	1
A5b	$h(x)=f(x)-(-2x)$, h is strictly increasing and $h(0)=0$ then for $x>0$, $h(x)>0$ so (C) is above (T) and for $x<0$, $h(x)<0$ so (C) is below (T). (C) and (T) intersect at point O.	1
A6	 The line of equation y = 3 is asymptote at $-\infty$. y'y is an asymptotic direction at $+\infty$.	1.5
A7	$S=\int_0^{\ln 3}-f(x)dx=\left[-\frac{1}{2}e^{2x}+4e^x-3x\right]_0^{\ln 3}=(4-3\ln 3)$ units of area.	1
A8a	f is continuous and strictly increasing on $[\ln 2;+\infty[$, so it has an inverse function f^{-1}.	0.5
A8b	The curves of the functions f and f^{-1} intersect on the line with equation $y=x$. The line of equation $y=x$ cuts (C) at one point only of abscissa α. Let $\psi(x)=f(x)-x$, $\psi(1.2)\approx -0.4$, $\psi(1.3)\approx 0.4$ then $\alpha\in]1.2;1.3[$.	1
B1	$f(x)>0$ for $x<0$ or $x>\ln 3$, so the domain of definition of g is $]-\infty;0[\cup]\ln 3;+\infty[$.	0.5
B2	$\lim_{x\to-\infty} g(x)=\ln 3$ thus the line of equation y =ln3 is a horizontal asymptote of (Γ).	0.5
B3	$\lim_{x\to+\infty} g(x)-2x=\lim_{x\to+\infty}\ln(e^{2x}-4e^x+3)-\ln e^{2x}=\lim_{x\to+\infty}\ln(1-4e^{-x}+3e^{-2x})=0$.	0.5
B4	$g(x)=\ln 3$ gives $e^{2x}-4e^x=0$, so $e^x=4$ then $x=\ln 4$, $I(\ln 4;\ln 3)$. $g(x)=2x$ gives $-4e^x+3=0$ then $J\left(\ln\frac{3}{4};2\ln\frac{3}{4}\right)$.	0.5
B5		1
B6		1