الدورة الإستثنائية للعام 2009	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل : اربع

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)0

I-(4 points)

The following table shows the relationship between the number of years of experience and the monthly salary, in hundred thousands of LL, of the employees in a company.

(Number of years of experience): X_i	2	4	6	8	10
(Salary in hundred thousands of LL.): Y_i	4.5	6	9	10	12

1) Calculate the means $\overline{X}$ and $\overline{Y}$ of the two variables X and Y respectively.

2) Represent graphically the scatter plot of the points (X_i; Y_i) as well as the center of gravity $G(\overline{X};\overline{Y})$ in a rectangular system.

3) Determine an equation of the regression line $D_{Y/X}$, of y in terms of x, and draw it in the preceding system.

4) Suppose that the above pattern remains valid for 20 years.
   a- Estimate the salary of an employee with 15 years of experience.
   b- An employee started working at this company at the age of 25.
   At what age would his salary become 2 000 000 LL?
   c- At the age of 44, this employee opens a savings account in which he deposits 500 000 LL at the end of each month. The annual interest rate is 6%, compounded monthly.
   Calculate the total amount that would be in his account when he retires after 20 years.

II- (4 points)

Consider the sequence (U_n) defined by $U_0 = 1600$ and by $U_{n+1} = 1.05\, U_n - 40$, for every natural integer n, and let (V_n) be the sequence defined by: $V_n = U_n - 800$.

1) Prove that (V_n) is a geometric sequence. Specify its common ratio and its first term.

2) Calculate V_n in terms of n. Deduce U_n in terms of n.

3) Let $T = V_0 + V_1 + ... + V_{10}$ and $S = U_0 + U_1 + ... + U_{10}$. Calculate T and deduce S.

4) On October 1, 2006, a school had 1600 students. Every year, before the first of October, the number of students increases by 5 % and 40 students definitely leave the school.
   a- Determine the number of students in this school on October 1, 2007.
   b- 50% of the students in this school are in the elementary division. Knowing that the number of students in each classroom is 30, what is the number of classrooms needed in the elementary division on October 1, 2011?

III- (4 points)

In order to encourage students to improve reading habits, a teacher uses two urns A and B such that:
The urn A contains 6 white balls and 5 red balls.
The urn B contains 4 red balls and 7 green balls.
He proposes the following game:
The student draws at random one ball from the urn A:

- If the drawn ball is white, then the student does not get anything.

- If the ball is red, the student draws randomly a ball from urn B:
  - If it is red, the student gets a gift of 10 books.
  - If it is green, he again draws, without replacing the ball in B, another ball from B:
    If this last ball is red, then he gets 5 books; if not, he does not get anything.

Consider the following events:
F: « The student gets 10 books ».
E: « The student gets 5 books ».
N: « The student does not get anything ».

1) What is the probability of the event: « the student does not get anything for the draw from urn A».
2) Calculate the probability p(F) and show that $p(E) = \dfrac{14}{121}$.
3) Calculate P(N).
4) Designate by X the random variable that is equal to the number of books received by the student.
Find the expected value E(X).

IV- (8 points)

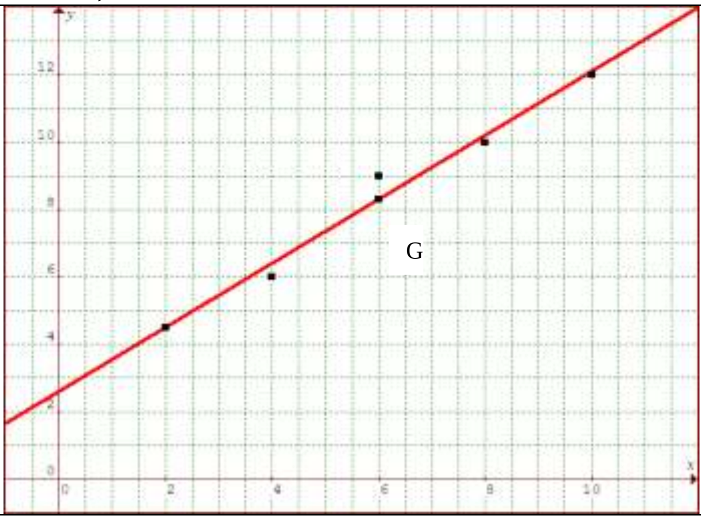
A- Let f be the function defined on $[0\,;+\infty[$ by $f(x) = (2x + 1)\,e^{-x}$ and (C) be its representative curve in an orthonormal system $(O;\ \vec{i}\ ,\ \vec{j}\ )$.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$.
2) Show that $f'(x) = (-2x+1)e^{-x}$.
3) Set up the table of variations of f.
4) Calculate, to the nearest 10^{-2} , f (2) and f (3).
5) Draw (C).

B- The demand function of a certain article is modeled, in thousands of articles, by
$f(x) = (2x + 1)\,e^{-x}$ where x is the price of an article, expressed in thousands LL. ($0.5 \le x \le 10$)

1) Determine the demand when the price of an article is 3000 LL.
2) Determine the elasticity of the demand in terms of the price.
3) Is the demand elastic for x = 2? Justify the answer.
   Give an economical interpretation of the value found.
4) The management of the factory that produces this article notices that the supply is modeled by the function h defined over [0.5 ; 10] by $h(x) = (3x - 1)\,e^{-x}$.
   This management wants to stock a certain quantity in advance for the high season.
   What are the prices that fulfill the condition $h(x) > f(x)$?

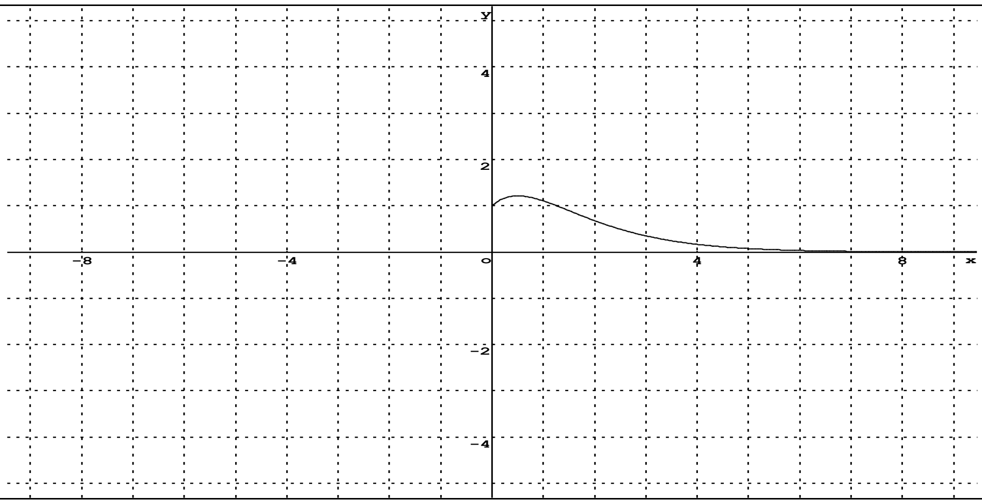
وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد	الدورة الإستثنائية للعام 2009
مشروع معيار التصحيح	مسابقة في مادة الرياضيات المدة ساعتان	

QI	Answer	M
1	$\overline{X} = 6$; $\overline{Y} = 8.3$.	0.5
2		1
3	y = 0.95x+2.6 Drawing on the graph.	1.5
4 a	For x=15 , y=16.85 ; The salary is 1 685 000 L.L.	1
4b	0.95x+2.6 = 20 gives x = 18.316 After 19 years the salary becomes 2 000 000 LL. The age of the employee is 44 years.	1.5
4c	$n = 20\times12 = 240$; $i = \frac{6}{100\times12} = 0.005$ $V_n = a\frac{(1+i)^n - 1}{i} = 500\ 000\ \frac{(1.005)^{240} - 1}{0.005} = 231\ 020\ 447.6LL.$ The total amount is 231 020 447.6LL.	1.5

QII	Answer	M
1	$V_{n+1} = U_{n+1} - 800 = 1.05\ U_n - 840 = 1.05(\ U_n - 800) = 1.05\ V_n$. Thus (V_n) is a geometric sequence of ratio q = 1.05 and first term $V_0 = U_0 - 800 = 1600 - 800 = 800$.	1.5
2	$V_n = V_0.\ q^n = 800\ (1.05)^n$ and $U_n = V_n + 800 = 800(1.05)^n + 800$.	1.5
3	$T = V_0.\frac{1-q^{11}}{1-q} = 800\frac{1-(1,05)^{11}}{1-1,05} = 11365$ $U_0 = V_0 + 800$; $U_1 = V_1 + 800$ --------------- $U_{10} = V_{10} + 800$ $S = T + 11\times800 = 20165$	1.5
4 a	The number of students is : 1600(1+ 0.05) - 40 = 1640 students.	1
4 b	$U_5 = 800(1.05)^5 + 800 = 1821$. [1821÷2]÷30 = 30.25 ; that is 31 classes.	1.5

QIII	Answer	M
1	The probability to get nothing from the first draw is the probability that the student draws a white ball from the urn A and which is $\frac{6}{11}$.	1

2	$p(F) = \frac{5}{11} \times \frac{4}{11} = \frac{20}{121} = 0.165$; $p(E) = \frac{5}{11} \times \frac{7}{11} \times \frac{4}{10} = \frac{14}{121} = 0.12$	3
3	The events E, F and N form a partition Hence $p(N) = 1 - p(E) - p(F) = 1 - \frac{20}{121} - \frac{14}{121} = \frac{87}{121} = 0.72$	2
4	The values of X are: 0, 5 and 10.The expected value is : $E(X) = 0 \times 0.72 + 5 \times 0.12 + 10 \times 0.165 = 2.25$	1

QIV	Answer	M
A1	$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} (2x + 1)\, e^{-x} = \lim\limits_{x \to +\infty} \left(\frac{2x}{e^{x}} + e^{-x} \right) = 0$ The line of equation y = 0 is an asymptote to (C).	1.5
A2	$f'(x) = 2(e^{-x}) + (-e^{-x})(2x + 1) = (-2x+1)e^{-x}$	1
A3	$f'(x) = 0$ for $x = \frac{1}{2}$ since $e^{-x} > 0$ $\begin{array}{c\|ccccc} x & 0 & & 1/2 & & +\infty \\ \hline f'(x) & & + & 0 & - & \\ \hline f(x) & 1 & \nearrow & 2e^{-1/2} & \searrow & 0 \end{array}$	2
A4	f (2) = 0.67 ; f (3) = 0.34	1
A5		1.5
B1	f(3) = 0.34 ; The demand is 0.34×1000 = 340 articles.	1.5
B2	$E(x) = -\ x \times \frac{d'(x)}{d(x)} = -\ x \times \frac{(-2x+1)e^{-x}}{(2x+1)e^{-x}} = \frac{x(2x-1)}{(2x+1)}$	2
B3	$E(2) = -\frac{2(-4+1)}{4+1} = \frac{6}{5} =$ 1.2 > 1, then the demand is elastic for x = 2. Since E(2) > 1. Interpretation: At a price of 2000LL, a raise in the price by 1 % will cause a decrease in demand by 1.2 %.	2
B4	h(x) > f(x) when $(3x-1)e^{-x} > (2x + 1)\, e^{-x}$ then $3x - 1 > 2x + 1$ so, $x > 2$. So $2 < x \leq 10$. So, the price should be between 2000 and 10 000LL or equal to 10 000LL	1.5