دورة سنة 2009 العادية	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل : أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points M and M' of respective affixes z and z', where $z' = \left(1+i\sqrt{3}\right)z$.

1) In this part, suppose that z = 2i.

a- Determine the exponential form of z'.

b- Calculate $\left|\frac{z'}{z}\right|$ and $\arg\left(\frac{z'}{z}\right)$.

c- Show that triangle OMM' is right angled at M.

2) Assume in this part that $z = (1+i)^3$.

a- Write the exponential form and the algebraic form of z.

b- Write the exponential form and the algebraic form of z'.

c- Deduce the exact value of $\cos\frac{13\pi}{12}$.

II- (4 points)

Consider two bags B_1 and B_2 such that:

B_1 contains **six** cards numbered 1, 2, 3, 4, 5, 6.

B_2 contains **five** cards numbered 0, 1, 2, 4, 5.

A-

One card is drawn randomly from bag B_1:

- if it carries one of the numbers 1 or 2, then **three** cards are drawn randomly and simultaneously from bag B_2.
- But if it carries one of the numbers 3, 4, 5 or 6, then **two** cards are drawn randomly and simultaneously from bag B_2.

Consider the following events:

K: « the card drawn from bag B_1 carries the one of the numbers 1 or 2 ».

L: « the card drawn from bag B_1 carries the one of the numbers 3, 4, 5 or 6 ».

E: « The product of numbers shown on the cards drawn from bag B_2 is zero ».

1) a- Calculate the probabilities p(K) and p(L).

b- Show that $p(E\cap K) = \frac{1}{5}$.

c- Calculate $p(E\cap L)$ and deduce p(E).

2) Knowing that the product of the numbers shown on the cards drawn from bag B_2 is zero, calculate the probability that **three c**ards were drawn from B_2.

B-

In this part we use only the bag B_2 and **three** cards are drawn randomly and simultaneously from this bag. Let X be the random variable that is equal to the biggest number among those shown on the **three** drawn cards, thus the possible values of X 2, 4 and 5.

Prove that $p(X=4) = \frac{3}{10}$, and determine the probability distribution of X.

III- (4 points)

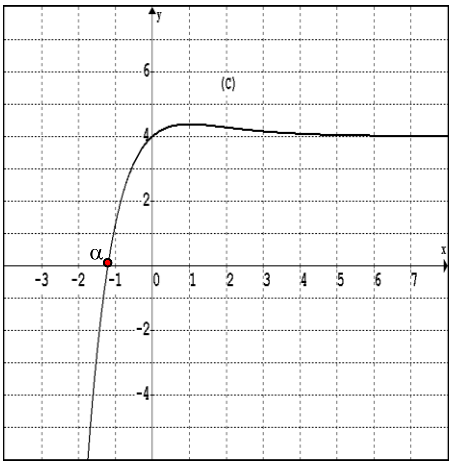
In the space referred to a direct orthonormal system $\left(O\,;\vec{i}\,,\vec{j}\,,\vec{k}\right)$, consider the point A(1;0;1) and the two planes (P) and (Q) with equations $2x - y - 2 = 0$ and $x + 2y - z = 0$ respectively.

1) a- Verify that point A is a common to (P) and (Q).
   b- Determine a system of parametric equations of (d), the line intersection of (P) and (Q).

2) a- Determine a system of parametric equations of the line (D) that is perpendicular to (P) at A.
   b- Determine the coordinates of a point E on (D) such that $AE = \sqrt{5}$.

3) a- Show that the points B(0;–2;0) and C(2; 2;t) belong to (P). (t is a real number).
   b- Calculate t so that the triangle ABC is right at B and find in this case the volume of the tetrahedron EABC.

III- (8 points)



A-Consider the function f defined on IR by $f(x) = 4 + x\,e^{-x}$ whose representative curve (C) is shown in the adjacent figure.

(C) cuts the axis of abscissas in one point of abscissa α.

1) Use (C) to study the sign of f(x).

2) Use integration by parts to calculate $\int_0^2 x\,e^{-x}dx$, then calculate the area of the region bounded by the axis of ordinates, the axis of abscissas, the curve (C) and the straight line with equation x = 2 .

B- In all what follows, let $\alpha = -1.2$.

Consider the function g defined on IR, by $g(x) = 4x - 3 - (x+1)e^{-x}$ and designate by (G) its representative curve in an orthonormal system $(O;\vec{i}\,,\,\vec{j})$.

1) Verify that $\lim\limits_{x\to-\infty} g(x) = +\infty$ and determine g(-2.5) to the nearest 10^{-2}.

2) Calculate $\lim\limits_{x\to+\infty} g(x)$ and verify that the straight line (D) with equation y = 4x-3 is an asymptote of (G).

3) Determine the coordinates of A, the point of intersection of (G) with its asymptote (D), and study the position of (G) with respect to (D).

4) a- Verify that $g'(x) = f(x)$.

   b- Set up the table of variations of g.

5) Draw (D) and (G).

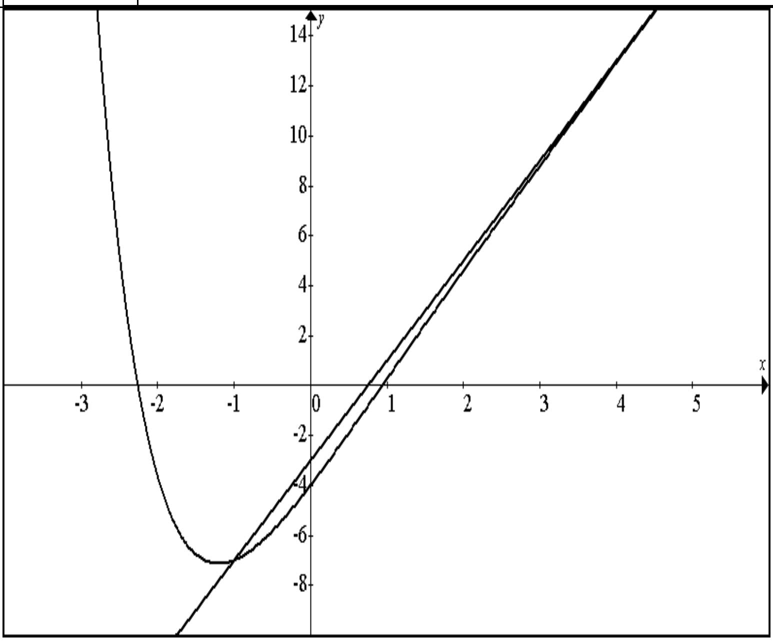
وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	دورة سنة 2009 العادية
مشروع معيار التصحيح	مسابقة في مادة الرياضيات المدة ساعتان	

QI	Answer	M
1a	$1+i\sqrt{3}=2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=2e^{i\frac{\pi}{3}}$;$2i=\ 2e^{i\frac{\pi}{2}}$; $z'=4e^{i\frac{5\pi}{6}}$.	0.5
1b	$\left\lvert\frac{z'}{z}\right\rvert=2$; $\arg\left(\frac{z'}{z}\right)=\frac{\pi}{3}[2\pi]$.	0.5
1c	OM=$\lvert z\rvert=2$; $OM'=\lvert z'\rvert=4$; $MM'=\lvert z'-z\rvert=2\sqrt{3}$ so $OM'^2=OM^2+MM'^2$ **OR** : $z'=(1+i\sqrt{3})2i=-2\sqrt{3}+2i$ M and M' have the same ordinate and M belongs to y-axis so OMM' is right at M.	1
2a	$(1+i)^3=2\sqrt{2}\ e^{i\frac{3\pi}{4}}$; $(1+i)^3=-2+2i$	0.5
2b	<u>Exponential form</u>: $z'=(1+i\sqrt{3})(1+i)^3=2e^{i\frac{\pi}{3}}\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^3=4\sqrt{2}e^{i\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)}=4\sqrt{2}e^{i\left(\frac{13\pi}{12}\right)}$. <u>Algebraic form:</u> $z'=\left(1+i\sqrt{3}\right)(1+i)^2(1+i)=\left(1+i\sqrt{3}\right)(2i-2)=2\left(-1-\sqrt{3}\right)+2\left(1-\sqrt{3}\right)i$	0.5
2c	Comparing the exponential and the algebraic forms of z': $4\sqrt{2}\cos\left(\frac{13\pi}{12}\right)=2\left(-1-\sqrt{3}\right)$, hence $\cos\frac{13\pi}{12}=\frac{-(\sqrt{2}+\sqrt{6})}{4}$	1

QII	Answer	M
A1a	$p(K)=\frac{2}{6}=\frac{1}{3}$; $p(L)=\frac{2}{3}$	0.5
A1b	$p(E\cap K)=p(K)\times p(E/K)=\frac{1}{3}\times\frac{C_1^1\times C_4^2}{C_5^3}=\frac{1}{5}$.	0.5
A1c	$p(E\cap L)=p(L)\times p(E/L)=\frac{2}{3}\times\frac{C_1^1\times C_4^1}{C_5^2}=\frac{4}{15}$. $p(E)=p(E\cap K)+p(E\cap L)=\frac{1}{5}+\frac{4}{15}=\frac{7}{15}$.	1
A2	$p(K/E)=\frac{p(E\cap K)}{p(E)}=\frac{\frac{1}{5}}{\frac{7}{15}}=\frac{3}{7}$.	0.5
B	The number of possible cases is $C_5^3=10$ $p(X=4)=p(0,1,4 \text{ or } 0,2,4 \text{ or } 1,2,4)=\frac{3}{10}$. $X=x_i$: 2, 4, 5 ; p_i: $\frac{1}{10}$, $\frac{3}{10}$, $\frac{6}{10}$	1.5

$X=x_i$	2	4	5
p_i	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10}$

QIII	Answer	M
1a	2-0-2=0 and 1+0-1=0.	0.5
1b	(d): x = m+1; y=2m; z = 5m+1	0.5
2a	(D): $x=2t+1$; $y=-t$; $z=1$	0.5
2b	$\vec{AE}(2t,-t,0)$; $AE=\sqrt{5t^2}=\sqrt{5}$, hence $t=\pm1$. For t = 1, E(3,-1,1).	0.5
3a	0+2-2=0; 4-2-2=0 then B and C belong to P.	0.5
3b	$\vec{AB}(-1,-2,-1)$; $\vec{BC}(2,4,t)$; $\vec{AB}.\vec{BC}=0$ so $t=-10$ Area of ABC = $\frac{\sqrt{6}\times\sqrt{120}}{2}=6\sqrt{5}$ Volume of EABC = $\frac{\text{area(ABC)}\times EA}{3}=\frac{6\sqrt{5}\times\sqrt{5}}{3}=10u^3$. ***OR*** Calculate the mixed product	1.5

QIV	Answer	M
A1	$f(x) = 0$ for $x = \alpha$; $f(x) > 0$ for $x > \alpha$; $f(x) < 0$ for $x < \alpha$	0.5
A2	Let $U = x; V' = e^{-x}$ then $U' = 1; V = -e^{-x}$ $\int_0^2 xe^{-x}dx = \left[-xe^{-x}-e^{-x}\right]_0^2 = -3e^{-2}+1$ Area = $\int_0^2 4dx + \int_0^2 xe^{-x}dx = \left[4x\right]_0^2 + \left[-xe^{-x}-e^{-x}\right]_0^2 = (-3e^{-2}+9)$ u^2	1.5
B1	$\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} \frac{4xe^x - 3e^x - x - 1}{e^x} = +\infty$ $\lim_{x\to-\infty} 4xe^x = 0.$ and $\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} \frac{-x-1}{e^x}$ g(-2.5)=5.27.	1
B2	$\lim_{x\to+\infty} g(x) = \lim_{x\to+\infty} (4x - 3 - xe^{-x} - e^{-x}) = +\infty$ $\lim_{x\to+\infty} xe^{-x} = 0$ $\lim_{x\to+\infty} (g(x) - (4x-3)) = \lim_{x\to+\infty} -(x+1)e^{-x} = \lim_{x\to+\infty} (-xe^{-x} - e^{-x}) = 0$ then the straight line with equation y = 4x - 3 is an asymptote of (G).	1
B3	$g(x) - (4x-3) = -(x+1)e^{-x}$ (G) intersects (D) at x = -1 thus A(-1;-7) If x < -1, $-(x+1)e^{-x} > 0$, then (G) is above (D) If x > -1, $-(x+1)e^{-x} < 0$, then (G) is below (D)	1
B4a	$g'(x) = 4 - e^{-x} + (x+1)e^{-x} = 4 + xe^{-x} = f(x)$	0.5
B4b	x: $-\infty$ -1.2 $+\infty$ g'(x): - 0 + g(x): $+\infty$ ↘ -7.1 ↗ $+\infty$	1
B5		1.5