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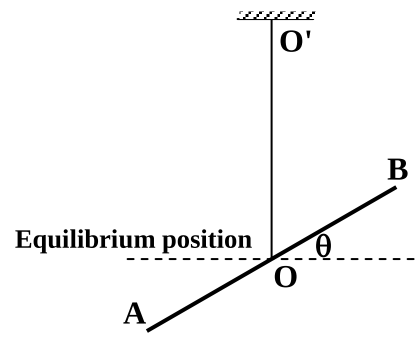
| الدورة العادية للعام 2009 | امتحانات الشهادة الثانوية العامة<br>الفرع : علوم عامة | وزارة التربية والتعليم العالي<br>المديرية العامة للتربية<br>دائرة الامتحانات |
| الاسم:<br>الرقم: | مسابقة في مادة الفيزياء<br>المدة ثلاث ساعات | |

***This exam is formed of four exercises in four pages numbered from 1 to 4.***
***The use of non-programmable calculator is recommended***

## <u>First Exercise:</u> (7 ½ points)

### Torsion pendulum

The object of this exercise is to determine the moment of inertia I of a homogeneous rod AB with respect to an axis perpendicular to the rod at its midpoint and the torsion constant C of a wire OO' of negligible mass.
The rod has a mass M and a length AB = $\ell$ = 60 cm.
A torsion pendulum [P] is obtained by fixing the mid-point of AB to one end O of the wire while the other end O' is fixed to a support .
The rod is shifted, from its equilibrium position, by a small angle $\theta_m$ in the horizontal plane and it is released from rest at an instant $t_0 = 0$ .The rod thus may turn in the horizontal plane about an axis ($\Delta$) passing through OO'.




At an instant t during motion, the angular abscissa of the rod is $\theta$ and its angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$

The horizontal plane containing the rod is taken as a gravitational potential energy reference.
We neglect any force of friction and take $\pi^2 = 10$.

***A* – Theoretical study**

***1)*** Give, at the instant t, the expression of the mechanical energy M.E of the system [(P), Earth] in terms of I, C, $\theta$ and $\dot{\theta}$ .

***2)*** ***a)*** Write the expression of M.E when $\theta = \theta_m$

***b)*** Determine , in terms of C, $\theta_m$ and I, the expression of the angular speed of [P] as it passes through its equilibrium position .

***3)*** Derive the second order differential equation in $\theta$ that governs the motion of [P].

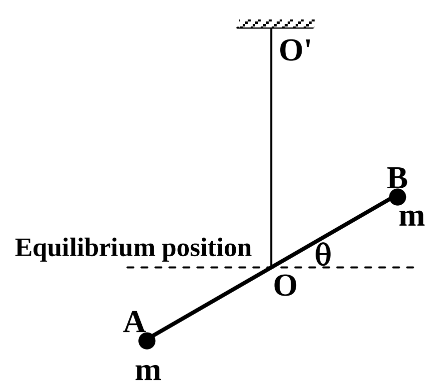
***4)*** Deduce that the motion of [P] is sinusoidal.

***5)*** Determine the expression of the proper period $T_1$ of the pendulum in terms of I and C.

***B* – Experimental study**

***1)*** By means of a stopwatch, we measure the duration $t_1$ of 20 oscillations and we obtain $t_1$ = 20 s. Determine the relation between I and C.

***2)*** At each extremity of the rod we fixe a particle of mass m = 25g .
We thus obtain a new torsion pendulum [P'] whose motion is also rotational sinusoidal of proper period $T_2$.




***a)*** Determine the moment of inertia I' of the system (rod + particles) with respect to the axis ($\Delta$) in terms of I, m, and $\ell$ .

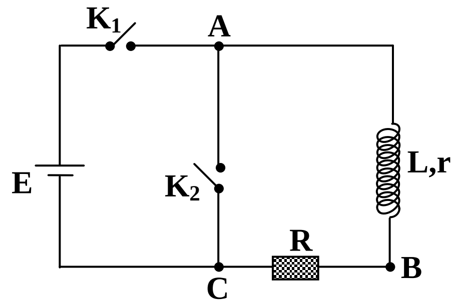
***b)*** Write down the expression of $T_2$ in terms of I , C , m and $\ell$ .

***c)*** By means of a stopwatch, we measure the duration $t_2$ of 20 oscillations and we obtain $t_2$ = 40 s.
Find a new relation between I and C.

***3)*** Calculate the values of I and C.

**<u>Second Exercise:</u> (7 ½ points)**

## The phenomenon of self-induction




The set up represented by the adjacent figure consists of an ideal generator of emf E = 12 V, a coil of resistance r = 10 Ω and of inductance L = 40 mH, a resistor of resistance R = 40 Ω and two switches $K_1$ and $K_2$.

***A*** – At the instant $t_0 = 0$, we close the switch $K_1$ and we leave $K_2$ open.
At an instant t , the circuit carries a current $i_1$ in the transient state.

***1)*** Derive the differential equation that governs the variation of $i_1$ as a function of time.

***2)*** $I_0$ is the current in the steady state. Determine the expression of $I_0$ in terms of E,r and R and calculate its value.

***3)*** The solution of the differential equation is of the form: $i_1 = I_0(1 - e^{\frac{-t}{\tau}})$.

***a)*** Determine the expression of τ in terms of L, r and R and calculate its value.

***b)*** Give the physical significance of τ .

***4)*** **a)** Determine the expression of the self-induced emf $e_1$ as a function of time t.

**b)** Calculate the algebraic value of $e_1$ at the instant $t_0 = 0$.

***B*** – After a few seconds, the steady state being reached, we open $K_1$ and we close $K_2$ at the same instant.
We consider the instant of closing $K_2$ as a new origin of time $t_0 = 0$.
The circuit (L, R, r) thus carries an induced current $i_2$ at an instant t.

***1)*** Determine the direction of $i_2$ .

***2)*** Derive the differential equation that governs the variation of $i_2$ as a function of time .

***3)*** Verify that $i_2 = I_0 e^{\frac{-t}{\tau}}$ is the solution of this differential equation.

***4)*** Calculate the algebraic value of the self-induced emf $e_2$ at the instant $t_0 = 0$.

***C*** – Compare $e_1$ and $e_2$ and deduce the role of the coil in each of the two previous circuits.

**<u>Third Exercise</u>: (7 ½ points)**

## Characteristics of an (R, L, C) circuit

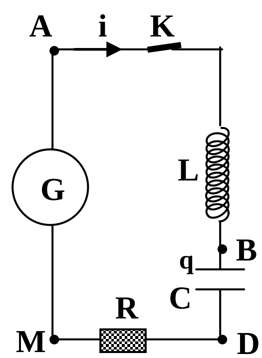



**Fig.1**

In order to determine the characteristics of an (R, L, C) circuit, we connect the circuit represented in figure 1. This circuit is formed of a resistor of resistance R= 650 Ω, a coil of inductance L and of negligible resistance and a capacitor of capacitance C, all connected in series across a function generator (LFG) delivering across its terminals a sinusoidal alternating voltage $u_g$ of the form:

$u_g = u_{AM} = U_m \cos(2\pi f)t$ .

***A*** –The frequency of the voltage $u_G$ is adjusted on the value $f_1$ .
We display, on the screen of an oscilloscope, the variations , as a function of time, of the voltage $u_{AM}$ across the generator on the channel ($Y_1$) and the voltage $u_{DM}$ across the resistor on the channel ($Y_2$) .
The waveforms obtained are represented in figure 2.

Vertical sensitivity on both channels is: 2 V/div.

Horizontal sensitivity is: 0.1 ms /div.

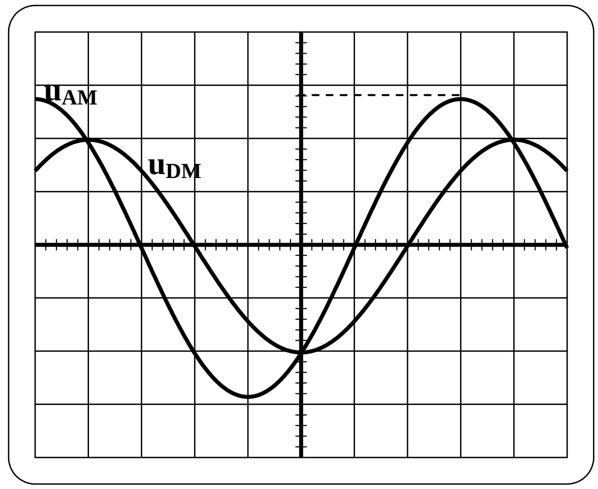



**Fig.2**

***1)*** Redraw figure (1) showing on it the connections of the oscilloscope.

***2)*** Referring to the waveforms, determine:

***a)*** The value of the frequency $f_1$.
***b)*** The absolute value of $\varphi_1$ the phase difference between $u_{AM}$ and $u_{DM}$.

***3)*** The current i carried by the circuit has the form:

$i = I_m \cos(2\pi f_1 t - \varphi_1)$.

***a)*** Write down the expressions of the voltages: $u_{AB}$, $u_{BD}$ and $u_{DM}$ as a function of time.

***b)*** The relation: $u_{AM} = u_{AB} + u_{BD} + u_{DM}$ is valid for any instant t . Show, by giving t a particular value, that:

$$\tan\varphi_1 = \frac{L(2\pi f_1) - \dfrac{1}{C(2\pi f_1)}}{R}$$

***B*** – Starting from the value $f_1$, we decrease continuously the frequency f . We notice that, for $f_0 = 500$ Hz the circuit is the seat a of current resonance phenomenon.

Deduce from what preceded a relation between L, C and $f_0$.

***C*** – We keep decreasing the frequency f . For a value $f_2$ of f we find that the phase difference between $u_{AM}$ and $u_{DM}$ is $\varphi_2$ such that $\varphi_2 = -\varphi_1$.

***1)*** Determine the relation among $f_1$, $f_2$ and $f_0$ .

***2)*** Deduce the value of $f_2$.

***D*** – Deduce from what is preceded the values of L and C.

**<u>Fourth Exercise</u>: (7 ½ points)**

## Energy levels of the hydrogen atom

The energies of the various levels of the hydrogen atom are given by the relation:

$E_n = -\frac{E_0}{n^2}$, where $E_0$ is a positive constant and n is a positive whole number.

**Given:**

Planck's constant $h = 6.62\times10^{-34}$ J.s; 1 eV = $1.6\times10^{-19}$ J; 1 nm = $10^{-9}$ m.
Speed of light in vacuum: $c = 3\times10^{8}$ m/s

***1)*** ***a)*** The energy of the hydrogen atom is quantized. What is meant by "*quantized energy*"?
***b)*** Explain why the absorption or emission spectrum of hydrogen consists of lines.

***2)*** A hydrogen atom, initially excited, undergoes a downward transition from the energy level $E_2$ to the energy level $E_1$. It then emits the radiation of wavelength in vacuum: $\lambda_{2\to1} = 1.216\times10^{-7}$ m.
Determine, in J, the value:
- ***a)*** of the constant $E_0$;
- ***b)*** of the ionization energy of the hydrogen atom taken in its ground state.

***3)*** For hydrogen, we define several series that are named after the researchers who contributed in their study . Among these series we consider that of Balmer, which is characterized by the downward transitions from the energy level $E_p > E_2$ ( p >2) to the energy level $E_2$ ( n = 2).
To each transition $p \to 2$ corresponds a line of wave $\lambda_{p\to2}$.
- ***a)*** Show that $\lambda_{p\to2}$, expressed in nm, is given by the relation:. $\frac{1}{\lambda_{p\to2}} = 1.096\times10^{-2}\left[\frac{1}{4}-\frac{1}{p^2}\right]$
- ***b)*** The analysis of the emission spectrum of the hydrogen atom shows four visible lines.
We consider the three lines $H_\alpha$ , $H_\beta$ and $H_\gamma$ of respective wavelengths in vacuum are
$\lambda_\alpha$ = 656.28 nm ; $\lambda_\beta$ = 486.13 nm and $\lambda_\gamma$ = 434.05 nm.
To which transition does each of these radiations correspond?
- ***c)*** Show that the wavelengths of the corresponding radiations tend ,when $p \to \infty$, towards a limit $\lambda_0$.
whose value is to be calculated.

***4)*** Balmer, in 1885, knew only the lines of the hydrogen atom that belong to the visible spectrum. He wrote the formula**:** $\lambda = K\frac{p^2}{p^2-4}$, where K is a positive constant and p a positive whole number.
Determine the value of K using the numerical values and compare its value with that of $\lambda_0$.

## First exercise (7.5 points)

**A – 1)** $ME = KE + P.E_g + PE_e = \frac{1}{2}I\theta'^2 + 0 + \frac{1}{2}C\theta^2$ **(¾)**

**2) a)** for max. deviation, $\theta = \theta_m$ and $\theta' = 0$.

$E_m = \frac{1}{2}C\theta^2_m$ **(½)**

**b)** At equilibrium position, $E_m = \frac{1}{2} I\theta'^2_m$

$\Rightarrow \theta' = \pm\theta_m\sqrt{\frac{C}{I}}$ **(¾)**

**3)** M.E is conserved since no friction thus the derivative of M.E w.r.t time is zero

$I\theta'\theta'' + C\theta\theta' = 0$, $\theta' \neq 0$ thus $\theta'' + \frac{C}{I}\theta = 0$ **(¾)**

**4)** Equation has the form $\theta'' + \omega^2\theta = 0$

It has a sinusoidal solution where $\omega^2 = \frac{C}{I}$: **(¼)**

**5)** $\omega^2 = \frac{C}{I}$

$\omega = \frac{2\pi}{T}$ thus $T_1 = 2\pi\sqrt{\frac{I}{C}}$ **(¾)**

**B – 1)** $t_1 = 20T_1 = 20s$ thus $T_1 = 1 = 2\pi\sqrt{\frac{I}{C}}$

and $40\frac{I}{C} = 1$ then $C = 40 I$ **(1)**

**2) a)** $I' = I + 2m(\frac{\ell}{2})^2 = I + m\frac{\ell^2}{2} = I + 0.0045$ **(¾)**

**b)** Same law of motion thus $T_2 = 2\pi\sqrt{\frac{I'}{C}} = 2\pi\sqrt{\frac{I + \frac{m\ell^2}{2}}{C}}$ **(½)**

**c)** $T_2 = 2$ thus $10\frac{I'}{C} = 1$ or $C = 10I' = 10(I + 0.0045) = 10I + 0.045$ **(¾)**

**3)** $C = 40I = 10I + 0.045 \Rightarrow I = 1.5\times10^{-3}$ kg.m$^2$ and $C = 0.06$ N×m **(¾)**

## Second exercise (7.5 points)

**A – 1)** $E = ri_1 + L\frac{di_1}{dt} + Ri_1 \Rightarrow E = (r+R)i_1 + L\frac{di_1}{dt}$. **(½)**

**2)** When the steady state mode is established, $i_1$ becomes constant and $\frac{di_1}{dt} = 0$;

the current is then $I_0$ such that: $E = (r+R)I_0 \Rightarrow I_0 = \frac{E}{R+r}$.

$I_0 = \frac{12}{40+10} = 0.24$ A **(1)**

**3) a)** $\frac{di_1}{dt} = I_0/\tau\,(e^{\frac{-t}{\tau}}) \Rightarrow E = (r+R)I_0(1 - e^{\frac{-t}{\tau}}) + L\,I_0/\tau\,(e^{\frac{-t}{\tau}})$

$\Rightarrow L/\tau = (r+R) \Rightarrow \tau = \frac{L}{R+r} = \frac{0.04}{50} = 0.8$ ms. **(1)**

**b)** The time constant characterizes the duration of the growth of the current in a (R+r, L) component **(¼)**

**4) a)** $e_1 = -L\frac{di_1}{dt} = -L\,I_0/\tau\,(e^{\frac{-t}{\tau}}) = -E\,e^{\frac{-t}{\tau}}$. **(½)**

**b)** For $t = 0$, $e_1 = -E = -12$ V. **(¼)**

**B – 1)** According to Lenz law, the coil carries a current $i_2$ of the same direction as that of $i_1$. **(½)**

**2)** $u_{AC} = u_{AB} + u_{BC} \Rightarrow 0 = ri_2 + L\frac{di_2}{dt} + Ri_2 \Rightarrow L\frac{di_2}{dt} + (R+r)i_2 = 0$ **(¾)**

**3)** $\frac{di_2}{dt} = -I_0/\tau\,e^{\frac{-t}{\tau}} \Rightarrow -L\,I_0/\tau\,e^{\frac{-t}{\tau}} + (R+r)I_0\,e^{\frac{-t}{\tau}} = 0$ **(1)**

**4)** $e_2 = -L\frac{di_2}{dt} = -L(-I_0/\tau\,e^{\frac{-t}{\tau}}) = E\,e^{\frac{-t}{\tau}}$.

At $t = 0$, $e_2 = E = 12$ V. **(¾)**

**C –** $e_1 = -e_2$.

When $K_1$ is closed, the self-induced emf opposes the growth of the current in the circuit $\Rightarrow e_1 < 0$ (the coil plays the role of a generator in opposition).

When $K_2$ is closed, the self-induced emf opposes the decay of the current in the circuit $\Rightarrow e_2 > 0$ (the coil plays the role of a generator). **(1)**

**Third exercise (7.5 points)**

**A – 1)** Connections of the oscilloscope. **(¼)**

**2) a)** $T_1 \rightarrow$ 8 div $\Rightarrow T_1 = 0{,}8$ ms

$f_1 = 1/T_1 = 1/\ 0.8 \times 10^{-3} = 1250$ Hz **(½)**

**b)** $|\varphi_1| = 2\pi\ 1/8 = \pi/4$ rad. **(¼)**

**3) a)** $i = I_m\cos(2\pi f_1 \cdot t - \varphi_1)$ ; $u_{AB} = L\, di/dt = -LI_m(2\pi f_1)\sin(2\pi f_1 t - \varphi_1)$

$u_C = 1/C \int i\, dt = I_m/C \int\cos(2\pi f_1 t - \varphi_1)\, dt$

$u_C = (I_m / C.2\pi f_1)\sin(2\pi f_1 t - \varphi_1)$

$u_R = Ri = R\, I_m\cos(2\pi f_1 t - \varphi_1)$ **(1)**

**b)** $U_m\cos 2\pi f_1 t = R\, I_m\cos(2\pi f_1 t - \varphi_1) + (I_m / C.2\pi f_1)\sin(2\pi f_1 t - \varphi_1) -$
$LI_m(2\pi f_1)\sin(2\pi f_1 t - \varphi_1)$

$2\pi f_1 t = \pi/2 \Rightarrow 0 = RI_m\sin\varphi_1 + (I_m / C.2\pi f_1)\cos\varphi_1 - L\, I_m(2\pi f_1)\cos\varphi_1$

$\Rightarrow R\sin\varphi_1 = [L(2\pi f_1) - 1/(C(2\pi f_1))]I_m\cos\varphi_1$

$$\mathrm{tg}\varphi_1 = \frac{L(2\pi f_1) - \dfrac{1}{C(2\pi f_1)}}{R} \quad \textbf{(¾)}$$

**B –** Current resonance $\Rightarrow \varphi = 0 \Rightarrow \mathrm{tg}\varphi = 0 \Rightarrow L2\pi f_0 - 1/C(2\pi f_0) = 0$

$\Rightarrow LC4\pi^2 f_0^2 = 1.$ **(¾)**

**C – 1)** $\mathrm{tg}\varphi_1 = -\mathrm{tg}\,\varphi_2 \Rightarrow$
$$\frac{L(2\pi f_1) - \dfrac{1}{C(2\pi f_1)}}{R} = \frac{\dfrac{1}{C2\pi f_2} - L2\pi f_2}{R}$$

$\Rightarrow L2\pi f_1 + L2\pi f_2 = 1/C\,[1/(2\pi f_1) + 1/(2\pi f_2)]$

$LC = 1/\ 4\pi^2 f_1 f_2 = 1/\ 4\pi^2 f_0^2 \Rightarrow f_0^2 = f_1 f_2$ **(1½)**

**2)** $f_2 = (500^2)/\ 1250 = 250000/1250 = 200$ Hz **(½)**

**D –** $\varphi_1 = \pi/4 \Rightarrow L2\pi(1250) - 1/(C \times 2\pi \times 1250) = 650$

$LC = 1/(4\pi^2 500^2) = 10^{-7} \Rightarrow LC \times 4\pi^2 \times 1250^2 - 1 = 650 \times C \times 2\pi \times 1250$

$\Rightarrow C = 5.25/(650 \times 2\pi \times 1250) = 10^{-6}$ F = 1 µF

$L = 10^{-7}/\ 10^{-6} = 10^{-1}$ H = 0.1 H **(2)**

**Fourth exercise (7.5 points)**

**1) a)** The energies of the hydrogen atom can take only well defined values (discrete) **(½)**

**b)** For an electronic transition p $\rightarrow$ n the emitted photon (or absorbed) has a wavelength:

$\lambda_{p,n} = \dfrac{hc}{E_p - E_n}$. As $E_p$ and $E_N$ are quantized then $(E_p - E_N)$ is quantized too; which means that the $\lambda_{p,n}$ has a well determined value, which corresponds to a line.; **(1)**

**1) a)** $E_2 = -\dfrac{E_o}{4}$ and $E_1 = E_o \Rightarrow E_2 - E_1 = \dfrac{3E_0}{4} = \dfrac{hc}{\lambda_{2,1}}$

$$\Rightarrow E_0 = \frac{4 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.216 \times 10^{-7}} = 2.177 \times 10^{-18} \quad \textbf{(1½)}$$

**b)** $E_i = E_\infty - E_1 = E_0 = 2.177 \times 10^{-18}$ J. **(½)**

**3) a)**

$$E_p - E_2 = -\frac{E_0}{p^2} + \frac{E_0}{4} = \frac{hc}{\lambda_{p,2}} \Rightarrow \frac{1}{\lambda_{p,2}} = \frac{E_0}{hc}\left(\frac{1}{4} - \frac{1}{p^2}\right)$$

$$= \frac{2.177 \times 10^{-18} \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8}\left(\frac{1}{4} - \frac{1}{p^2}\right) = 1.096 \times 10^{-2}\left(\frac{1}{4} - \frac{1}{p^2}\right) \quad \textbf{(1½)}$$

**b)** $\lambda_\alpha = 656.28$ nm $\Rightarrow$ p= 3, then it is the downward transition $3 \rightarrow 2$.

$\lambda_\beta$ ; $4 \rightarrow 2$ and $\lambda_\gamma$ ; $5 \rightarrow 2$. **(¾)**

**c)** when $p \rightarrow \infty \Rightarrow \lambda \rightarrow \lambda_0 = \dfrac{4}{1.096 \times 10^{-2}} = 364.96$ nm **(½)**

**4)** For $\lambda_\alpha = 656.28$ nm, p = 3 ; $K = \lambda\dfrac{p^2 - 4}{p^2} = 364.6$ nm, $K \cong \lambda_0$ **(1¼)**