وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	دورة سنة 2009 العادية
عدد المسائل: ست	مسابقة في مادة الرياضيات المدة أربع ساعات	الاسم: الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الألتزام بترتيب المسائل الوارد في المسابقة).

I-(2 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give**, *with justification***, the answer that corresponds to it.

N^o	Questions	Answers			
		a	b	c	d
1	t and m are real numbers; $(d):\begin{cases}x=-5t\\y=t-1\\z=t+1\end{cases}$ and $(d'):\begin{cases}x=10m\\y=8m\\z=-7m+8\end{cases}$ The lines (d) and (d') are:	confounded	intersecting	parallel	skew
2	The solution of the differential equation: $Y''+4Y'+4Y=0$ verifying $Y'(0)=Y(0)=1$ is :	$(2x+1)e^{2x}$	$(-3x+1)e^{-2x}$	$(3x+1)e^{-2x}$	$(-x+1)e^{2x}$
3	A solution of the equation: $\cos(\arcsin\frac{1}{x})=\frac{\sqrt{3}}{2}$ is :	$\frac{-2}{\sqrt{3}}$	1	2	–1
4	$h(x)=\frac{1}{\sqrt{1-x^2}}$ where $-1<x<1$. An anti-derivative H of h is :	arccos (x –1)	arcsin (1–x)	$\arcsin(1-x^2)$	$\arctan\frac{x}{\sqrt{1-x^2}}$
5	The imaginary part of z such that $\left\|\frac{z-2i}{z+i}\right\|=1$ is :	$\frac{1}{2}$	$-\frac{3}{2}$	0	– 2

II- (2 points)

The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$.

Consider the points A(4 ; 3 ; 2) , B(–8 ; –1 ; 6), and the plane (P) of equation $x - y - z + 4 = 0$.

1) a- Determine a system of parametric equations of the line (AB) .
   b- Determine the coordinates of I, the point of intersection of (AB) and (P).
   c- Prove that A and B lie on opposite sides with respect to (P).

2) Let (d) be the set of points in (P) which are equidistant from A and B.
   a- Find an equation of (Q), the mediator plane of [AB].
   b- Prove that (d) is the line defined by the system of parametric equations
   $x = m - \frac{3}{2}$; $y = -m - 1$; $z = 2m + \frac{7}{2}$, (m is a real number).

3) Let J be the orthogonal projection of A on (d).
   Find the coordinates of J and prove that (d) is perpendicular to the plane (ABJ).

III- (3 points)

The management of a football team proposes to their supporters new season-tickets for either 6, 8 or 10 matches.

The supporters, who bought these season-tickets, are distributed as follows:

- 45 % of them chose the ticket for 6 matches,
- 35 % chose the ticket for 8 matches,
- the other supporters chose the ticket for 10 matches.

A supporter who bought a ticket is randomly chosen and is interviewed.

1) The ticket for 6 matches costs n LL, that for 8 matches costs (n + 4 000) LL, and that for 10 matches costs (n + 6 000) LL.
   Let Y be the random variable that is equal to the amount paid by the interviewed supporter to buy his season-ticket.
   a- Calculate n so that the expected value of Y is equal to 22 600.
   b- For the value of n obtained, draw the graph of the distribution function of Y.

2) 85% of the supporters who bought tickets are males and among these males 40 % chose the ticket for 6 matches.
   Consider the following events:
   M: « The interviewed supporter is a male».
   A : « The interviewed supporter chose the ticket for 6 matches».
   a- Verify that the probability $P(M \cap A)$ is equal to 0.34 then calculate the probability $P(M \cap \overline{A})$.
   b- Calculate P(M/A).

IV- (3 points)

In the complex plane referred to a direct orthonormal system $\left(O\ ;\ \vec{u}\ ,\ \vec{v}\right)$, we associate to each point M of affix z, the point M' of affix z' such that $z' = f(z) = z^2 - (4+5i)z + 7i - 1$.

1) a- Calculate the square roots of the complex number $-5+12i$.
   b- Solve the equation $f(z) = 0$.

2) Let $z = x + iy$ and $z' = x' + iy'$.
   Show that $x' = x^2 - y^2 - 4x + 5y - 1$ and $y' = 2xy - 5x - 4y + 7$.

3) Prove that when M varies on the line of equation $y = x$, then M' varies on a parabola (P) whose parameter, focus and directrix are to be determined.

4) a- Show that when M' varies on the axis of ordinates, then the point M varies on a hyperbola (H) whose equation, asymptotes and vertices are to be determined. Draw (H).
   b- Let $L(1\ ;\ 1)$ be a point on (H). Write an equation of the tangent at L to (H).

V- (3 points)

ABCD is a square of side 2 and of center O such that $(\overrightarrow{AB}, \overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$.

E and F are the midpoints of [AB] and [BC] respectively and G is the midpoint of [BF].
Let S be the direct plane similitude that transforms A onto B and D onto E.

1) Calculate an angle and the ratio of S.

2) Verify that S(B) = F, and determine S(E).

3) Let h = SoS.
   a- Show that h is a dilation and precise its ratio.
   b- Prove that the center I of S is the point of intersection of (AF) and (DG).
   c- Determine the image by S of the square ABCD and deduce the nature of triangle OIC.

4) Let (A_n) be the sequence of points defined by: $A_o = A$ and $A_{n+1} = S(A_n)$ for all natural integers n.
   a- Let $L_n = A_nA_{n+1}$ for all n.
   Prove that (L_n) is a geometric sequence whose common ratio and first term are to be determined.
   Calculate $S_n = L_0 + L_1 + \ldots\ldots + L_n$ and $\lim\limits_{n\to+\infty} S_n$.
   b- Calculate $\left(\overrightarrow{IA}, \overrightarrow{IA_n}\right)$ in terms of n and prove that if n is even then the points I, A and A_n are collinear.

VI- (7 points)

Consider the function h defined on IR by: $h(x) = e^{2x} + 2e^{x} - 2$.

A –

1) a- Solve the equation $h(x) = 0$.
   b- Calculate $\lim_{x\to+\infty} h(x)$ and $\lim_{x\to-\infty} h(x)$.
2) a- Set up the table of variations of h.
   b- Draw the representative curve (H) of h in an orthonormal system.
   c- Calculate the area of the region bounded by the curve (H), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

B –

Let g be the function defined on IR by $g(x) = \dfrac{e^{2x}+2}{e^{x}+1}$ and f be the function given by $f(x) = \ln(g(x))$.

Designate by (C) the representative curve of f in the plane referred to a new orthonormal system $(O ; \vec{i}, \vec{j})$; (graphical unit: 2 cm).

1) a- Show that f is defined for every real number x.
   b- Calculate $\lim_{x\to-\infty} f(x)$ and deduce an asymptote (d) of (C).
2) a- Show that $f(x) = x + \ln\left(\dfrac{1+2e^{-2x}}{1+e^{-x}}\right)$.
   b- Calculate $\lim_{x\to+\infty} f(x)$ and prove that the line (d') of equation $y = x$ is an asymptote to (C).
   c- Study, according to the values of x, the relative positions of (C) and (d ').
3) a- Prove that $g'(x) = \dfrac{e^{x}\left(h(x)\right)}{\left(e^{x}+1\right)^{2}}$.
   b- Show that f '(x) and h(x) have the same sign and set up the table of variations of f.
   c- Find the abscissa of the point on the curve (C) at which the tangent to (C) is parallel to (d').
4) Draw (d), (d') and (C).

C -

Designate by f^{-1} the inverse function of f on the interval $[0 ; +\infty[$ and by (C') the representative curve of f^{-1}.

1) Draw (C') in the system $(O ; \vec{i}, \vec{j})$.
2) Write an equation of the tangent to (C') at the point of abscissa ln2.

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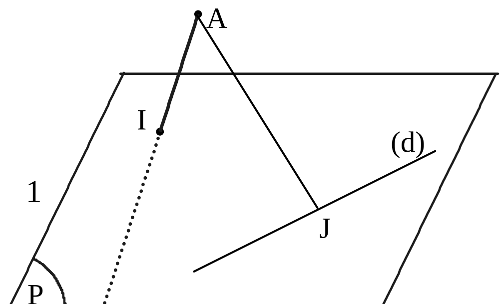
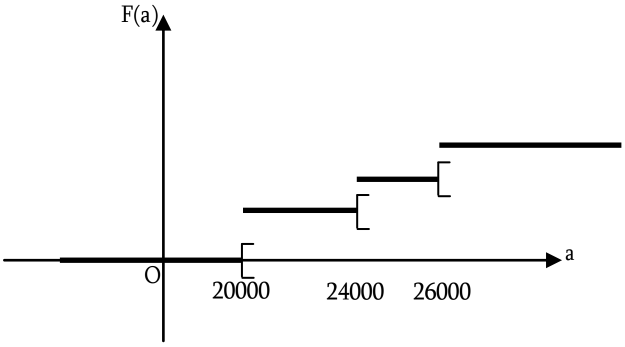
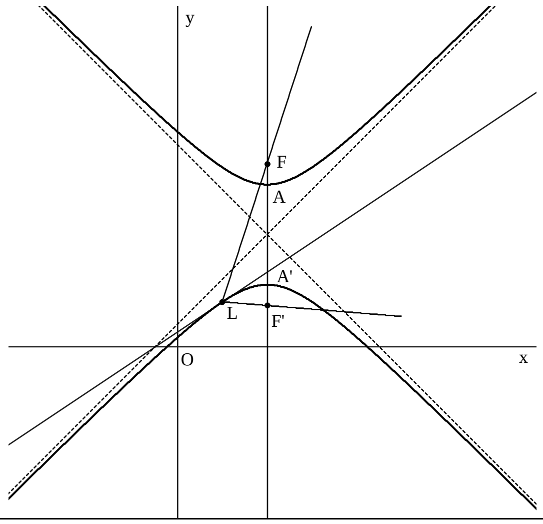
	مسابقة في مادة الرياضيات	مشروع معيار التصحيح

Q_1	Answer	Mark
1	(d) and (d') are not parallel nor confounded The system – 5t = 10m ; t – 1 = 8m and t + 1 = – 7n + 8 has no solution In fact : – 5t = 10m and t = 8m + 1 give t = 1/5 and m = –1 /10 with t + 1 = 6/5 and – 7m + 8 = 87/10. ***d***	1
2	The characteristic equation is $(r + 2)^2 = 0$; $Y= (ax + b)e^{-2x}$ But Y(0) = Y'(0) = 1 then a = 3 and b =1. **c**	0.5
3	$\cos(\arcsin\frac{1}{x}) = \frac{+\sqrt{3}}{2}$ is true for x = 2 only. ***c***	1
4	The derivatives of the first three functions are not equal to h(x). ***d***	1
5	A(2i) and B(-i) ; $\left\|\frac{z-z_A}{z-z_B}\right\| = 1$ Then AM = BM and so M belong the perpendicular bisector of [AB] : y = ½ , so Im(z) = ½ . ***a***	0.5

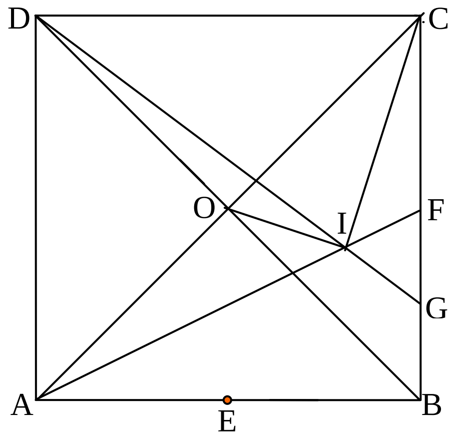
Q_{II}	Answer	Mark
1a	(AB) : $x = 3t + 4$; $y = t + 3$; $z = -t + 2$.	0.5
1b	$(AB) \cap (P) = \{I(1\,;\,2\,;3)\}$.	0.5
1c	$\overrightarrow{IA}(3\,;1\,;-1)$ and $\overrightarrow{IB}(-9\,;-3\,;3)$;then $\overrightarrow{IB} = -3\overrightarrow{IA}$; A and B are on opposite sides with respect to the plan (P).	0.5
2a	$MA^2 = MB^2$ is equivalent to 3x + y – z + 9 = 0.	0.5
2b	(d)$\subset$(P) and (d) $\subset$(Q) then (d) = (P)$\cap$(Q). (d) : $x = m - \frac{3}{2}$; $y = -m - 1$; $z = 2m + \frac{7}{2}$. ***OR :*** We prove that the given line lies in the two planes (P) and (Q). .	0.5
3	J is a point on (d) ; J(m – $\frac{3}{2}$; – m –1 ; 2m + $\frac{7}{2}$) $\vec{AJ}\cdot\vec{V_d} = 0$ gives us m = – $\frac{1}{4}$ and $J\left(-\frac{7}{4},-\frac{3}{4},3\right)$ $\overrightarrow{AB}\times\overrightarrow{AJ} = 11\vec{i} - 11\vec{j} + 22\vec{k}$ That is parallel to (d). Therefore (d) is perpendicular to (ABJ). ***OR*** $(AB)\perp(Q)$ and $(d)\subset(Q)$; then $(d)\perp(AB)$. also $(d)\perp(AJ)$. Therefore $(d)\perp(ABJ)$	1.5

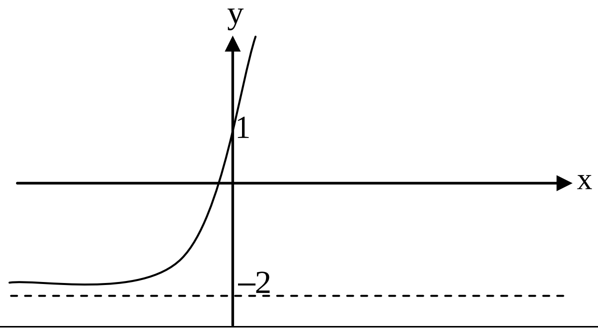


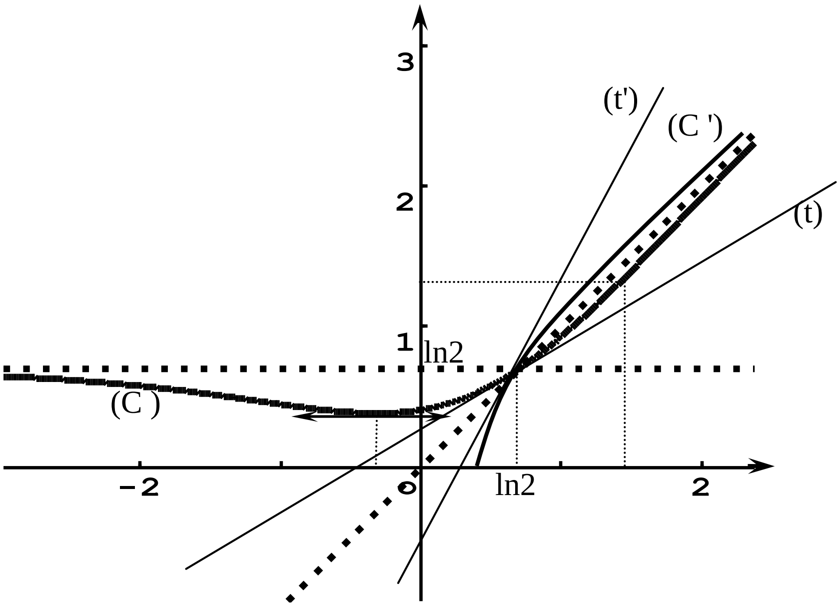
Q_{III}	Answer	Mark
1a	E(Y) = 0.45n + 0.35(n + 4000) + 0.2(n + 6000) = 22600 ; n = 20 000	1
1b	For every real number we have F(a) = P(Y≤ a) ♦ a < 20 000 ; F(a) = 0 ♦ 20 000≤ a < 24 000 ; F(a) = 0.45 ♦ 24 000≤ a < 26 000 ; F(a) = 0.45 + 0,35 = 0.8 ♦ 26 000 ≤ a ; F(a) = 0.8 + 0.2 = 1  	2
2a	P(G∩A) = P(A/G) × p(G) = 0,4 × 0.85 = 0.34. P(G) = P(G∩A) + P(G∩ $\overline{A}$) ; 0.85 = 0.34 + P(G∩ $\overline{A}$) ; P(G∩ $\overline{A}$) = 0 .51.	2
2b	P(G/A) = $\dfrac{P(G\cap A)}{P(A)} = \dfrac{0.34}{0.45} = \dfrac{34}{45}$	1

Q_{IV}	Answer	Mark
1a	$-5+12i=(2+3i)^2$; The square roots of -5 +12i are $2+3i$ et $-2-3i$.	0.5
1b	$\Delta=-5+12i$; $z'=1+i$ and $z''=3+4i$.	0.5
2	$x'=x^2-y^2-4x+5y-1$, $y'=2xy-4y-5x+7$.	0.5
3	$y=x$ give $x'=x-1$ and $y'=2x^2-9x+7$. Therefore $y'=2x'^2-5x'$. The point M' varies on the parabola (P) of equation $y=2(x-\frac{5}{4})^2-\frac{25}{8}$. Parameter: $p=\frac{1}{4}$, focal axis $x=\frac{5}{4}$; Focus $\left(\frac{5}{4};-3\right)$, Directrix: $y=-\frac{13}{4}$.	1.5
4a	$x'=0$ gives $x^2-y^2-4x+5y-1=0$ The point M' varies on a hyperbola (H) of equation $\left(y-\frac{5}{2}\right)^2-(x-2)^2=\frac{5}{4}$ $A\left(2;\frac{\sqrt{5}}{2}+\frac{5}{2}\right)$ $A'\left(2;-\frac{\sqrt{5}}{2}+\frac{5}{2}\right)$ $y=x+\frac{1}{2}$ and $y=-x+\frac{9}{2}$  	2
4b	An equation of the tangent is $\left(y-\frac{5}{2}\right)\left(y_L-\frac{5}{2}\right)-(x-2)(x_L-2)=\frac{5}{4}$;	1

	$2x-3y+1=0$.	

Q_V	Answer	Mark
1	S(A) = B and S(D) = E then an angle de S is $\left(\overrightarrow{AD},\overrightarrow{BE}\right)=\left(\overrightarrow{AD},\overrightarrow{BA}\right)=\frac{\pi}{2}(2\pi)$ and the ratio of S is $\frac{EB}{AD}=\frac{1}{2}$.	0.5
2	S(B) = F since $\left(\overrightarrow{AB},\overrightarrow{BF}\right)=\frac{\pi}{2}[2\pi]$ et BF=½ AB S(A)=B and S(B)=F and E is the midpoint of [AB] hence S(E) is the midpoint of [BF] therefore S(E)= G. 	1
3a	h is the similitude of center , angle $\frac{\pi}{2}+\frac{\pi}{2}=\pi$ and ratio ½×½ = ¼ . It is then the negative dilatation of center I and ratio -¼ .	0.5
3b	h (A) = SoS(A) = S(B) = F then I$\in$(AF) and h(D) = SoS(D) = S(E) = G hence I$\in$(DG) Therefore I is the intersection of (AF) and (DG).	0.5
3c	The image by S of the square ABCD is the square BFOE since S(A) = B ,S(B) = E and S(D) = E. Hence , the image of C by S is O, Consequently $\left(\overrightarrow{IC},\overrightarrow{IO}\right)=\frac{\pi}{2}(2\pi)$ and triangle OIC is right at I.	1
4a	$L_{n-1}=A_{n-1}A_n$ and $S(A_{n-1})=A_n$ and $S(A_n))=A_{n+1}$ then $A_nA_{n+1}=½ A_{n-1}A_n$ so $L_n=½L_{n-1}$ then (L_n) is a geometric sequence of r=½ and whose first term is $L_0=A_0A_1=AB=2$. $S_n= l_0\times\frac{1-q^{n+1}}{1-q}=2\times\frac{1-(\frac{1}{2})^{n+1}}{\frac{1}{2}}=4\left[1-\left(\frac{1}{2}\right)^{n+1}\right]$ or $\lim\limits_{n\to+\infty}\left(\frac{1}{2}\right)^{n+1}=0$ then $\lim\limits_{n\to+\infty} S_n=4$	1.5
4b	$\left(\overrightarrow{IA},\overrightarrow{IA_n}\right)=\left(\overrightarrow{IA},\overrightarrow{IA_1}\right)+\left(\overrightarrow{IA_1},\overrightarrow{IA_2}\right)+\ldots\ldots\ldots\ldots+\left(\overrightarrow{IA_{n-1}},\overrightarrow{IA_n}\right)=\frac{n\pi}{2}[2\pi]$. If n is even , then $\left(\overrightarrow{IA},\overrightarrow{IA_n}\right)=k\pi$ therefore $A_n\in$(IA)	1

Q_{VI}	**Answer**	**Mark**
A1a	$e^{2x}+2e^{x}-2=0$ $e^{x}=-1-\sqrt{3}$ or $e^{x}=-1+\sqrt{3}; x=\ln(-1+\sqrt{3})$ □ $-0,312$	0.5
A1b	$\lim\limits_{x\to+\infty} h(x)=+\infty$, $\lim\limits_{x\to-\infty} h(x)=-2$	0.5
A2a	$h'(x)=2e^{2x}+2e^{x}$. x | $-\infty$ ... $+\infty$ h'(x) | + h(x) | -2 ↗ $+\infty$	1
A2b	The line with equation $y=-2$ is asymptote when $x\to-\infty$. $\lim\limits_{x\to+\infty}\frac{h(x)}{x}=+\infty$ then the axis of ordinates is an asymptotic direction. 	1
A2c	$A=\int_0^1(e^{2x}+2e^{x}-2)dx=[\frac{1}{2}e^{2x}+2e^{x}-2x]_0^1=\frac{1}{2}e^{2}+2e-\frac{9}{2}=4,63u^{2}$.	1
B1a	$g(x)=\frac{e^{2x}+2}{e^{x}+1}>0$ for any x; so f is defined for any x	0.5
B1b	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\ln(\frac{e^{2x}+2}{e^{x}+1}) = \ln 2$; (d) : y = ln2 asymptote.	1
B2a	$x+\ln\frac{1+2e^{-2x}}{1+e^{-x}}=\ln e^{x}+\ln\frac{1+2e^{-2x}}{1+e^{-x}}=\ln\frac{e^{x}(1+2e^{-2x})}{1+e^{-x}}=\ln(g(x))=f(x)$	0.5
B2b	$\lim\limits_{x\to+\infty} f(x)=+\infty$ and $\lim\limits_{x\to+\infty}[f(x)-x]=0$ So (d') : y = x is asymptote to (C) as x tends to $+\infty$	1
B2c	$f(x)-x=\ln\frac{1+2e^{-2x}}{1+e^{-x}}$. $\frac{1+2e^{-2x}}{1+e^{-x}}=1$; x = ln2 ; (C) cuts (d′) at (ln2 ; ln2). $\frac{1+2e^{-2x}}{1+e^{-x}}\succ 1; x\prec\ln 2$; (C) is above (d′).	1
B3a	$g'(x)=\frac{2e^{2x}(e^{x}+1)-e^{x}(e^{2x}+2)}{(e^{x}+1)^{2}}=\frac{e^{x}h(x)}{(e^{x}+1)^{2}}$.	0.5
B3b	$f'(x)=\frac{g'(x)}{g(x)}=\frac{e^{x}(h(x))}{(e^{x}+1)(e^{2x}+2)}$ x | $-\infty$ ... -0.31 ... $+\infty$ f'(x) | – 0 + f(x) | ln2 ↘ 0.38 ↗ $+\infty$ f '(x) has same sign as h(x) * The minimum : ≈ ln(1,46) ≈ 0,38	1.5
B3c	f '(x) = 1 is equivalent to $e^{2x}-4e^{x}-2=0$;so : $x=\ln(\sqrt{6}+2)$.	1

B4		1.5
C1	The graph (C') is symmetric of the part of (C) corresponding to $0 \le x$; with respect to the first bisector. *** ***See figure*** ***	0.5
C2	(C') cuts (C) at (ln2 ; ln2). $f^{-1}(\ln 2) = \frac{1}{f'(\ln 2)} = \frac{3}{2}$. $y = \frac{3}{2}x - \frac{\ln 2}{2}$.	1