دورة سنة 2009 العادية	امتحانات الشهادة الثانوية العامة فرع الإجتماع والإقتصاد	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل: اربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

The table below shows the turnover y_i, expressed in **billions** of LL, of a company during six consecutive years:

Year	2002	2003	2004	2005	2006	2007
Rank of the year: x_i	1	2	3	4	5	6
Turnover: y_i	150	180	200	225	265	300

1) Construct, in a rectangular system, the scatter plot of the points $(x_i ; y_i)$ associated to this distribution.

2) Calculate the coordinates of the center of gravity G of this distribution and plot this point in the same system.

3) Determine an equation of $(D_{y/x})$, the regression line of y in terms of x. Draw this line in the preceding system.

4) Suppose that the above pattern remains valid till the year 2015.
   a- What is the turnover expected to be in 2010?
   b- After which year would the turnover of this company exceed 450 billions of LL for the first time?

II- (4 points)

A factory produces watches. Each watch is tested before being approved for selling.
If the test is positive, that is if the watch functions properly, then the watch is approved for selling.
But if the test is negative then the watch is repaired after which it is tested again. If its second test is positive then it is approved for selling, but if the test is negative then the watch is destroyed.
It is known that:

- for **80%** of the watches produced, the first test is positive;
- for **60%** of the **repaired** watches, the second test is positive.

One watch is chosen randomly from the production.

1) Prove that the probability for this watch to be destroyed is 0.08.

2) Determine the probability that this watch is approved for selling.

3) The cost of production of a watch is 40 000LL with an additional cost of 10 000LL if it needs to be repaired.
   Each watch is sold for 70 000LL.

   Let X be the random variable that is equal to the profit achieved by the factory upon selling a watch.

   a- Verify that the three possible values of X are: –50 000 , 20 000 and 30 000.
   b- Determine the probability distribution for X.
   c- Calculate the expected value E(X).
   d- Suppose that the daily production is 50 watches.
      Estimate the daily profit for this factory.

III- (4 points)

During the year 1990, a factory produced 3500 tons of cement. Afterwards, the production decreased regularly by 15% every year till the end of the year 2000.
Denote by U_n the production of this factory, in tons, during the year (1990 + n), where $U_0 = 3500$.

1) Verify that $U_1 = 2975$ and calculate U_2.

2) a- Show that the sequence (U_n) is a geometric sequence whose common ratio is to be determined.
   b- Express, for $n \leq 10$, U_n in terms of n and calculate the production of this factory during the year 2000.

3) After the year 2000, the production of this factory increased regularly by 15% every year.
   a- Calculate U_{11} .
   b- Given that $U_n = 3500 \times (0.85)^{10} \times (1.15)^{n-10}$, for $n \geq 11$.
   Starting from which year would the annual production of this factory become greater than or equal to the production of the year 1990 ?

IV- (8 points)

Consider the function g defined over $[0,+\infty[$ by $g(x) = (x-1)e^{-x} + 1$.

A-

1) a- Determine $\lim\limits_{x \to +\infty} g(x)$ and calculate g(0), g(2).
   b- Verify that $g'(x) = (2-x)\,e^{-x}$ and set up the table of variations of g.
   c- Show that $g(x) \geq 0$ for all x in the interval $[0,+\infty[$.

2) Let f be the function defined over $[0,+\infty[$ by $f(x) = x + 1 - xe^{-x}$ and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.
   a- Determine $\lim\limits_{x \to +\infty} f(x)$ and show that the straight line (d) of equation $y = x + 1$ is an asymptote to (C).
   b- Determine the coordinates of the point of intersection of (C) and (d) and prove, for $x > 0$, that (C) is below (d).
   c- Show that $f'(x) = g(x)$ and set up the table of variations of f.
   d- Draw (d) and (C).

B-

An industrial company produces each week x **hundreds** of objects $(0 \leq x \leq 9)$
The weekly total cost of production of x **hundreds** of objects is given by $f(x) = x + 1 - xe^{-x}$ expressed in **millions** of LL.

1) Determine the weekly fixed cost of the company.

2) Determine the marginal cost of production of x hundreds of objects.

3) Calculate the marginal cost of production of 700 objects.
   Give an economical interpretation of the value obtained.

4) Find the production for which the marginal cost is maximum.
   Calculate in this case the corresponding weekly total cost.

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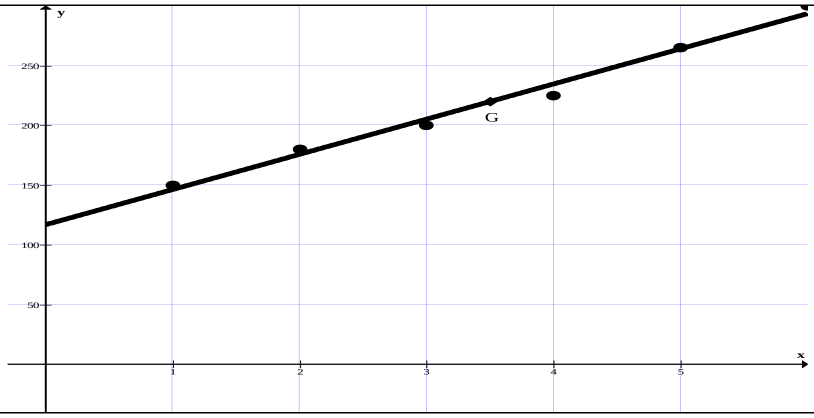
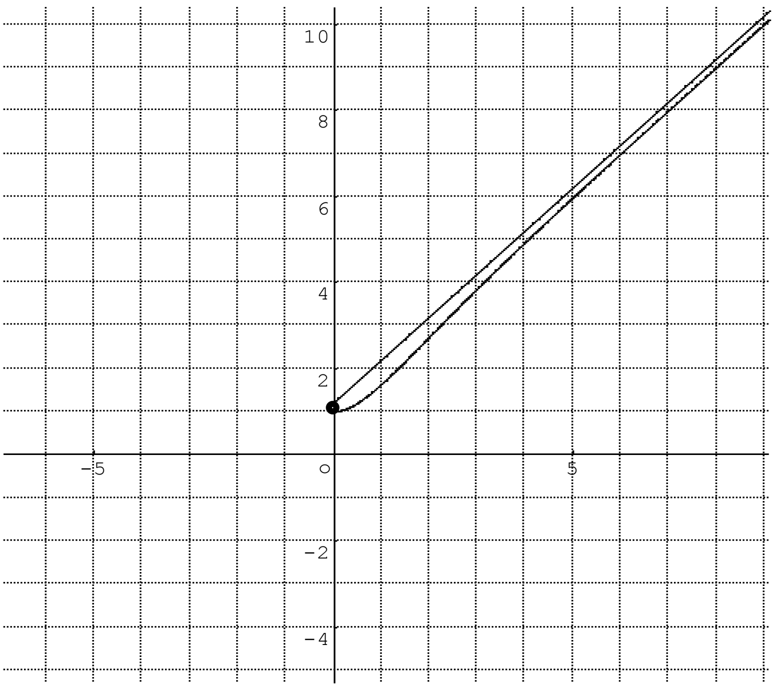
	مادة الرياضيات	مشروع معيار التصحيح

QI	Answer	Mark
1		1
2	$\bar{x} = 3.5$ and $\bar{y} = 220$: G(3.5 :220)	1
3	$y = 29.428x + 117$	1.5
4a	The rank of 2010 is 9. $y = 29.428(9) + 117 = 381.852$; The turnover in 2010 is 381.852 billion LL.	1.5
4b	$29.428x + 117 > 450$; $29.428x > 333$; $x > 11.31$, then $x \geq 12$; but x is an integer so the turnover exceeds 450 billion LL for the first time in the year 2013	2

QII	Answer	Mark
1	Let D be the event: The watch is destroyed. D occurs only when both tests are negative. Thus $P(D) = 0.2 \times 0.4 = 0.08$.	1
2	Let S be the event: The watch is approved for selling. $P(S) = 0.8 + 0.2 \times 0.6 = 0.92$ OR $P(S) = 1 - P(D) = 1 - 0.08 = 0.92$.	1
3a	The possible values of X are: - 40 000–10 000 = – 50 000 (destroyed); 70000 – 50 000= 20 000 (repaired); 70000 – 40000= 30000 (not repaired).	1
3b	<table><tr><td>x_i</td><td>-50000</td><td>20000</td><td>30000</td></tr><tr><td>$P(X = x_i)$</td><td>0.08</td><td>0.12</td><td>0.8</td></tr></table>	2
3c	$E(X) = -50000 \times 0.08 + 20000 \times 0.12 + 30000 \times 0.8 = 22400$.	1
3d	The daily profit is: $22400 \times 50 = 1120000$LL.	1

QIII	Answer	Mark
1	$U_1 = 3500\,(1-0.15) = \mathbf{2975}$. $U_2 = 2975\,(1-0.15) = \mathbf{2528.75}$.	1
2a	(U_n) is a geometric sequence of common ratio $q = \mathbf{0.85}$	1.5
2b	$U_n = 3500\,(0.85)^n$ for $n \leq 10$. $U_{10} = 3500\,(0.85)^{10} = \mathbf{689.06}$ tons.	1.5
3a	$U_{11} = U_{10}\,(1 + 0.15) = 792.41$	1
3b	$3500\,(0.85)^{10} \times (1.15)^{n-10} \geq 3500 \Rightarrow (1.15)^{n-10} \geq (0.85)^{-10}$ $\Rightarrow (n-10)\ln(1.15) \geq -10\ln(0.85)$. So we get $n \geq \mathbf{21.6}$ and thus $n = 22$. So starting from the year **2012**, the production of the factory becomes greater than or equal to that of the year 1990.	2

QIV	Answer	Mark
A1a	$\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty} (xe^{-x} - e^{-x} + 1) = 1$.; g(0) = 0 ; g(2) = e^{-2} + 1= 1.1353.	1
A1b	g '(x) = (2 – x) e^{-x} over [0, + ∞[. x: 0 … 2 … $+\infty$ g ' (x): + 0 – g(x): 0 ↗ $e^{-2}+1$ ↘ 1	1.5
A1c	The function g increases from 0 to e^{-2} + 1 then decreases to 1. Hence, g(x) ≥ 0.	1.5
A2a	$\lim\limits_{x\to+\infty} f(x) = +\infty$; $\lim\limits_{x\to+\infty} [f(x) - y] = \lim\limits_{x\to+\infty} -xe^{-x} = 0$. Then, the straight line (d) is an asymptote to (C).	1
A2b	f(x) – y = $-xe^{-x}$ = 0 for x = 0, (C) cuts (d) at point (0;1) $f(x) - y = -xe^{-x} < 0$ for all $x > 0$. So, (C) is always below (d).	1
A2c	f '(x)=1-e^{-x} +xe^{-x} =(x-1)e^{-x} +1=g(x) . x: 0 … $+\infty$ f ' (x): + f(x): 1 ↗ $+\infty$	1.5
A2d		1.5
B1	f(0) = 1, then the fixed costs of the company add up to 1 000 000 LL in a week.	1
B2	The marginal cost of production of x hundreds of objects is f '(x) = g(x).	1
B3	g(7)=(7– 1)e^{-7} +1. g(7) = 6 e^{-7} +1 = 1.005471. The cost of production of the 8th 100 objects is 1 005 471 LL.	1.5
B4	The marginal cost of production of x hundreds of objects is maximum for x = 2, Hence for a production of 200 objects. f(2) = 3 – 2 e^{-2} = 2.729 ; the corresponding total cost is then 2.729 × 1 000 000 = 2 729 000LL.	1.5