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الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ساعتان	

<u>This exam is formed of three exercises in three pages numbered from 1 to 3.</u>
<u>The use of a non-programmable calculator is recommended.</u>

<u>First exercise</u>: (7 points) Horizontal elastic pendulum

The aim of this exercise is to study the effect of the mass on the motion of a horizontal elastic pendulum.
This pendulum is formed of:

- an elastic spring (R), of negligible mass and of stiffness k = 400 N/m ,wound around a horizontal rod;
- a solid (B) considered as a particle of mass m = 100 g.

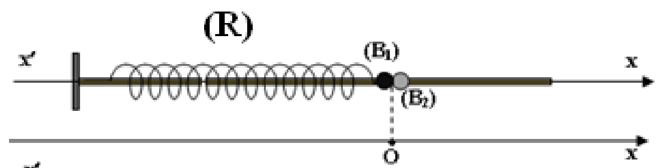

Fig. 1

The solid (B) is formed of two particles (B_1) and (B_2) stuck together and of respective masses m_1 = 25 g and m_2 = 75 g.
The solid (B) can slide, without friction, on the rod (Fig. 1).
At equilibrium, (B) is at O, taken as an origin of abscissas of the axis x′x. (B) is displaced by a distance X_m, from O, in the positive direction, and then released without initial velocity at the instant $t_0 = 0$.
The horizontal plane through (B) is taken as a gravitational potential energy reference.
At the end of two complete oscillations, (B_2) is detached from (B_1) and the system [(R), (B_1)] continues its oscillations. Figure 2 represents the variation of the abscissa x of the moving solid as a function of time in the two intervals [0, 0.2 s] and [0.2s, 0.35 s]. Take $\pi^2 = 10$.

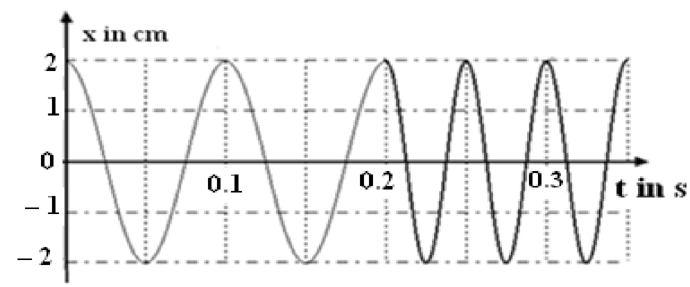

Fig. 2

A – Graphical study

Referring to figure 2, give in each of the intervals [0, 0.2 s] and [0.2s, 0. 35 s]:

1) the value of the amplitude of the motion;
2) the type of oscillations performed by the oscillator;
3) the value of the proper period of the oscillations.

B –Theoretical study of the oscillations of (B)

Consider the system [(R), (B), Earth].

1) Calculate, at $t_0 = 0$, the value of the mechanical energy of the system.
2) At an instant t, (B) has an abscissa x and a velocity $\vec{v}$ of algebraic value $v = \dfrac{dx}{dt}$.
   Write, at an instant t, the expression of the mechanical energy of the system in terms of k, m, x and v.
3) **a)** Derive the second order differential equation in x that describes the motion of (B).
   b) Deduce the expression of the proper period T of the oscillations.
   c) Calculate the value of T, and then compare it to the result obtained in part **(A – 3).**
4) The time equation of motion of (B) is of the form: $x = X_m \sin(\dfrac{2\pi}{T} t + \varphi)$.
   Determine the value of the constant φ.

C – Theoretical study of the oscillations of (B_1)

Consider the system [(R), (B_1), Earth].

1) Referring to figure 2, give the instant at which (B_2) is detached from (B_1).
2) The mechanical energy of the system [(R), (B_1), Earth] is equal to that of the system [(R), (B), Earth]. Justify.
3) When (B) passes through O, its speed is V and when (B_1) passes through O, its speed is V_1 . Show that $V_1 = 2$ V.

<u>Second exercise</u>: (7 points) Study of discharging a capacitor

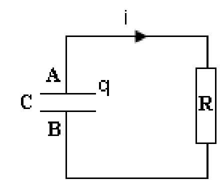

Fig.1

A capacitor of capacitance C is initially charged under a voltage E.
At $t_0 = 0$, we connect across the terminals of the capacitor a resistor of resistance $R = 1\ k\Omega$ (Fig.1).
At an instant t, the armature A carries the charge $q > 0$ and the circuit carries a current i.

A – Theoretical study

1) Write the relation between i and q.
2) Show that the differential equation of the voltage $u_C = u_{AB}$ across the capacitor is $\frac{du_C}{dt} + \frac{1}{RC}u_C = 0$.
3) The solution of this differential equation is $u_C = D\,e^{\frac{-t}{\tau}}$.
   Determine the expressions of the constants D and τ in terms of E, R and C.
4) Show that, after a time $t = \tau$, the voltage across the capacitor attains 37% of its maximum value E.

B – Determination of the capacitance C

In order to determine the value of C, we use a convenient apparatus, which traces, during the discharging of the capacitor, the curves representing $u_C = g(t)$ (Fig.2) and $\ell n(u_C) = f(t)$ (Fig.3)

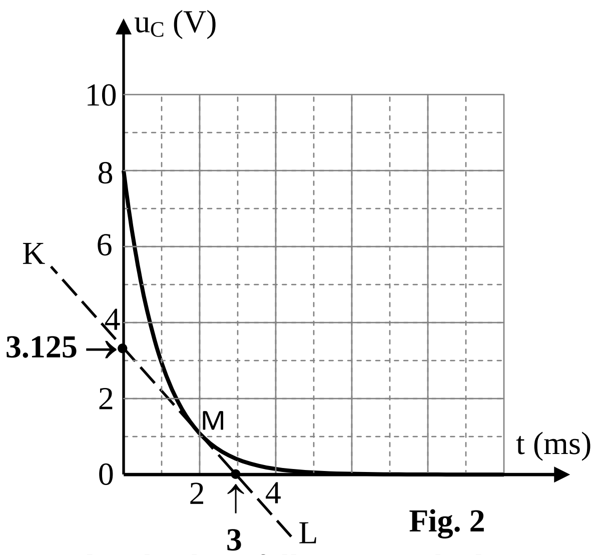

Fig. 2

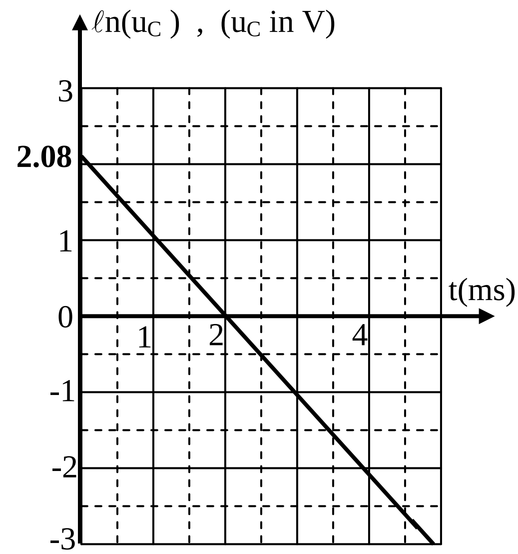

Fig.3

We proceed in the three following methods:

1) First method

Referring to the curve of figure 2:

a) give the value of E;
b) using the result of part (A – 4) determine the value of τ and deduce the value of C.

2) Second method

The figure 2 shows also the tangent KL to the curve at point M (2 ms, 1V).

a) Referring to this figure determine the slope of the tangent at point M.

b) Determine the value of C.

3) Third method:

a) Determine the expression of ℓn (u_C) in terms of E, R, C and t.

b) Show that the shape of the curve in figure 3 is in agreement with the obtained expression of the function ℓn (u_C) = f(t).

c) Referring to the curve of figure 3, determine again the values of E and C.

<u>Third exercise</u>: (6 points) **Iodine 131**

The aim of this exercise is to show evidence of some characteristics of iodine 131.

Iodine 131 ($^{131}_{53}I$) is radioactive and is a β^- emitter. Its radioactive period (half-life) is 8 days.

Given: Mass of an electron: $m_e = 5.5\times10^{-4}$ u; 1 MeV = 1.6×10^{-13} J; 1 u = 931.5 MeV/c^2.

Element	Iodine ($^{131}_{53}I$)	Cesium($^{137}_{55}Cs$)	Xenon($^{131}_{54}Xe$)
Mass of nucleus	130.8770 u	136.8773 u	130.8754 u

A – Disintegration of iodine 131

1) Write down the equation of the disintegration of iodine 131 and identify the daughter nucleus.

2) The disintegration of iodine 131 nucleus, is often, accompanied with the emission of γ rays. Due to what is this emission?

3) Calculate the radioactive constant λ of iodine 131 in day^{-1} and in s^{-1} .

4) Show that the energy liberated by the disintegration of one nucleus of iodine 131 is $E_{lib.} = 1.56\times10^{-13}$ J.

B – Application in medicine

During a medical examination of a thyroid gland of a patient, we inject this gland with a solution of iodine 131. The thyroid of this patient captures from this solution a number N = 10^{11} of iodine nuclei.

1) Calculate, in Bq, the activity A corresponding to these N nuclei knowing that A = λN.

2) Calculate, in J, the energy liberated by the disintegration of these N nuclei.

3) Deduce, in J/kg, the value of the dose absorbed by the thyroid gland knowing that its mass is 25 g.

C – Contamination

On the 26th of April 1986, an accident took place in the nuclear power plant of Chernobyl that provoked an explosion in one of the reactors. One of the many radioactive elements that were ejected to the atmosphere is the iodine 131. This element spread on the ground, absorbed by cows and contaminated their milk and then captured by the thyroid gland of consumers.

Every morning, a person drank a certain quantity of milk containing $N_0 = 2.6\times10^{16}$ nuclei of iodine 131. We suppose that all these nuclei were captured by the thyroid of that person, and that the person drank the first quantity at the instant $t_0 = 0$.

1) Determine , in terms of N_0 and λ (expressed in day^{-1}), the number of iodine 131 nuclei that remained in the thyroid, at the instant:

- **a)** t_1 = 1 day, (just after drinking the 2nd quantity of milk);
- **b)** t_2 = 2 days, (just after drinking the 3rd quantity of milk).

2) Deduce, at the instant t_3 = 3 days just after drinking the 4th quantity of milk that the number N_3 of the iodine 131 nuclei that remained in the thyroid is: $N_3 = N_0\,(1 + e^{-\lambda} + e^{-2\lambda} + e^{-3\lambda})$ where λ is expressed in day^{-1}.

3) Serious troubles in the thyroid gland will take place if the activity of the iodine 131 exceeds 75×10^9 Bq. Show that at the instant t_3, the person was in danger.

First exercise: Horizontal elastic pendulum		7 points
Part of the Q.	**Answer**	**Mark**
A.1	X_m of B is 2cm and X_m of B_1 is 2cm also	½
A.2	The oscillations in each part are free un-damped oscillations.	½
A.3	The periods are: $T_B = 0.1$ s and $T_{B1} = 0.05$ s.	¾
B.1	M.E is conserved since the amplitude of the oscillations are constant $M.E = K.E + P.E_e + P.E_g = ½ m v^2 + ½ kx^2 + 0$, for $x = X_m$,v = 0 thus $M.E = P.E_{max} = ½ kX^2_m$. $M.E = 200\times(0.02)^2 = 0.08$ J.	¾
B.2	$M E = ½mv^2 + ½kx^2$	¼
B.3.a	$M.E = ½ m v^2 + ½ kx^2$. Being constant , its derivative is zero thus: $0 = mvv' + kxx'$ $v = x' \neq 0$ and $v' = x''$ we get $x'' + (\frac{k}{m})x = 0$	1
B.3.b	The differential equation is of the form: $x'' + \omega^2 x = 0$ $\Rightarrow \omega^2 = \frac{k}{m} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$	½
B.3.c	$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{20\pi} = 0.1$ s ; which is in agreement with the result obtained in part A.3	¼ ¼
B.4	The time equation is $x = X_m \sin(\frac{2\pi}{T}t + \varphi)$, at $t_0 = 0$, $x = X_m$ $X_m = X_m \sin(\varphi) \Rightarrow \sin(\varphi) = 1 \Rightarrow \varphi = \frac{\pi}{2}$rd;	½
C.1	B_2 detached at t = 0.2 s	¼
C.2	M.E is the same in the two intervals since the $M.E = [P.E_{el}]$ max $= ½ k X_m^2$, same K and same X_m	½
C.3	At O, The M.E = kinetic energy becuase x = 0, the elastic potential energy is zero ($P.E_{el} = 0$). For B : $0.08 = ½ mV^2$ and for B_1 : $0.08 = ½ m_1 V_1^2$, $m_1 = 25$ g and $m = 100$ g $= 4m_1 \Rightarrow 4m_1 v^2 = m_1 V_1^2 \Rightarrow$ $V_1^2 = 4 v^2$ and $V_1 = 2v$.	1

Second exercise: Study of discharging a capacitor		7 points
Part of the Q	**Answer**	**Mark**
A.1	$i = -\frac{dq}{dt}$ –	¼
A.2	$u_C = Ri = -R\frac{dq}{dt} = -RC\frac{du_C}{dt}$; $u_C + RC\frac{du_C}{dt} = 0$	½
A.3	$\frac{du_C}{dt} = -\frac{D}{\tau}e^{\frac{-t}{\tau}} \Rightarrow -\frac{D}{\tau}e^{\frac{-t}{\tau}} + \frac{1}{RC}\times D e^{\frac{-t}{\tau}} = 0 \Rightarrow \tau = RC$. For t = 0, u_C = E=D.	1
A.4	$u_C = E e^{\frac{-t}{\tau}}$, for $t = \tau \Rightarrow u_C = E\,e^{-1} = 0.37\,E$	¾
B.1.a	E = 8 V	¼
B.1.b	$u_C = 0.37\,E = 0.37\times 8 = 2.96\text{ V} \approx 3\text{V}$. Graphically, we find for u_C = 3V, t = τ =1ms = 10^{-3}s. $\tau = RC = 10^3 C \Rightarrow C = 10^{-6}$F.	1
B.2.a	slope = $\frac{du_C}{dt} = -\frac{3.125}{0.003} = -1041.6$ V/s	½
B.2.b	The differential equation is: $\frac{du_C}{dt} + \frac{1}{RC}u_C = 0 \Rightarrow \frac{du_C}{dt} = -\frac{1}{RC}u_C$ $-\frac{1}{RC}u_C = -\frac{1}{10^3 C}\times 1 \Rightarrow 1041.6 = \frac{1}{10^3 C} \Rightarrow C = 0.96\times 10^{-6}$F.	¾
B.3.a	$\ln(u_C) = \ln(E e^{\frac{-t}{\tau}}) \Rightarrow \ln(u_C) = \ln E - \frac{t}{RC}$.	¾
B.3.b	ln (u_C) = f(t) is a function of time: the shape of the curve is a straight line decreasing and not passing through the origin.	¼
B.3.c	For t =0, we have: $\ln(u_C) = 2.08 = \ln E \Rightarrow E = 8$ V. And $\ln(u_C) = 0$, for t = 2ms $\Rightarrow \ln E = \ln 8$ thus E = 8 V Now $\ln 8 = 2.08 = \frac{2\times 10^{-3}}{10^3\times C} \Rightarrow C = 0.96\times 10^{-6}$F.	1

Third exercise: Iodine 131		6 points
Part of the Q.	**Answer**	**Mark**
A.1	${}^{131}_{53}I \rightarrow {}^{A}_{Z}X + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$ conservation of mass number : A = 131 conservation of charge number : 53 = Z - 1⇒ Z = 54 Daughter nucleus is Xenon ${}^{131}_{54}Xe$	1
A.2	The daughter nucleus ${}^{131}_{54}Xe$ is in excited state and when it drops to the ground state (lower state) emits the γ ray (photon)	¼
A.3	$\lambda = \frac{\ln 2}{T} = \frac{0.693}{8} = 0.087$ days^{-1} and $\lambda = \frac{0.087}{24\times 3600} = 10^{-6}$ s^{-1}.	¾
A.4	$\Delta m = m_{before} - m_{after} = 130.8770 - 130.8754 - 5.5\times 10^{-4} = 1.05\times 10^{-3}$ u $\Delta m = 1.05\times 10^{-3} \times 931.5 = 0.978$ MeV $\times 1.6\times 10^{-13} = 1.56\times 10^{-13}$ J	1
B.1	$A = \lambda N = 10^{-6}\times 10^{11} = 10^{5}$ Bq.	¼
B.2	The liberated energy is : $E = 10^{11}\times 1.56\times 10^{-13} = 1.56\times 10^{-2}$ J	¼
B.3	The absorbed dose is: $D = \frac{E}{m} = \frac{1.56\times 10^{-2}}{25\times 10^{-3}} = 0.624$ J/kg	½
C.1.a	After the duration t_1=1 day, according to the law of radioactive decay the number of remaining nuclei is $N_1 = N_0 e^{-\lambda t} = N_0 e^{-\lambda}$ (λ in day^{-1}) An additional number N_0 is taken when drinks the second quantity next morning. Thus the number of non-decay nuclei is then: $N_1 = N_0 + N_0 e^{-\lambda} = N_0 (1 + e^{-\lambda})$	¾
C.1.b	On the 2nd day, an additional N_0 from the third quantity and the number remaining from the previous milk is $N_1 e^{-\lambda}$: $N_2 = N_1 e^{-\lambda} + N_0 = N_0(1 + e^{-\lambda}) e^{-\lambda} + N_0 = N_0(1 + e^{-\lambda} + e^{-2\lambda})$	¼
C.2	On the 3rd day, an additional N_0 from the fourth quantity and the number remaining from the previous milk is $N_2 e^{-\lambda}$. $N_3 = N_2 e^{-\lambda} + N_0 = N_0(1 + e^{-\lambda} + e^{-2\lambda}) e^{-\lambda} + N_0 = N_0(1 + e^{-\lambda} + e^{-2\lambda} + e^{-3\lambda})$	¼
C.3	À t=3 days, the number N_3 of nuclei is : $N_2 = N_0 (1 + e^{-0.087} + e^{-2\times 0.087} + e^{-3\times 0.087}) = 9.17\times 10^{16}$ nuclei. The corresponding activity becomes : $A_3 = \lambda N_3 = 10^{-6}\times 9.17\times 10^{16} = 91.7$ GBq $>$ 75G Bq At the instant t_3 , the person is thus in danger.	¾