دورة سنة ٢٠١٢ العادية	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة أربع ساعات	عدد المسائل : ست

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (2 points)

In the following table, only one among the answers proposed to each question is correct. Write down the number of each question and give, **with justification**, the answer that corresponds to it.

№	Questions	Answers			
		a	b	c	d
1	If $z = r e^{i\frac{5\pi}{6}}$ (r >0) and $z' = z(z - \bar{z})$ then the exponential form of z' is :	$r^2 e^{-i\frac{2\pi}{3}}$	$r^2 e^{i\frac{2\pi}{3}}$	$r\ e^{i\frac{2\pi}{3}}$	$r\ e^{-i\frac{2\pi}{3}}$
2	If f is the function defined over R by $f(x) = xe^x$, then the n^{th} derivative of f is :	$(x+2n-1)e^x$	$(x+n+1)e^x$	$(x-n)e^x$	$(x+n)e^x$
3	If $g(x) = \arcsin(2x^2-1)$, then the domain of definition of g is:	$\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$	[–1 ; 1]	[0 ; 1]	$[1;+\infty[$
4	n is a strictly positive integer. If $U_n = \int_0^1 \frac{e^{nx}}{e^x+1}dx$, then $U_{n+1} + U_n =$	$\frac{e^n-1}{n}$	$ne^n - 1$	$e^n - 1$	$\frac{e^n}{n}$

II - (2 points)

The space is referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$.

Consider the points A(3 ; 1 ; 1) , B(2 ; 0 ; 3) and C(1 ; 2 ; 2) .

1) Write an equation of the plane (P) determined by the points A, B and C.

2) a- Show that triangle ABC is equilateral.

b- G (2 ; 1 ; 2) is the center of the circle (γ) circumscribed about triangle ABC.

Determine a system of parametric equations of the tangent (T) at A to (γ).

2) a- Calculate the area of triangle ABC.

b- Consider the point M(1;7; α) . In the case where the volume of the tetrahedron ABCM is equal to 3, calculate α.

III- (3 points)

In a given population, 15% of the individuals have a disease D_a.

Out of the individuals having disease D_a, 20% have another disease D_b.

Out of the individuals not having disease D_a, 90% don't have disease D_b.

An individual is randomly chosen from this population. Consider the following events:

A: «The chosen individual has disease D_a»

B: «The chosen individual has disease D_b»

1) Calculate the probability $P(A \cap B)$ and prove that P(B) = 0.115.

2) An individual of the population declares that he doesn't have disease D_b, calculate the probability that he has disease D_a .

3) An individual is randomly chosen from this population . Denote by X the random variable equal to the number of diseases, mentioned before, and this person may have.

Determine the probability distribution of X.

4) In this question, suppose that this population counts 200 individuals. A group of 4 individuals is randomly chosen from this population. Calculate the probability that at most 2 individuals among the chosen 4 have the disease D_a.

IV- (3 points)

In the plane referred to an orthonormal system $(O ; \vec{i}, \vec{j})$, consider the parabola (P) with vertex $V(-2 ; 0)$ and focus $F\left(-\frac{7}{4};0\right)$.

1) a- Prove that $y^2 = x + 2$ is an equation of (P).

   b- Draw (P).

2) a- E(2 ; 2) is a point on (P).Write an equation of (T), the tangent to (P) at E.

   b- Denote by (D) the region bounded by the axis of abscissas , the parabola (P) and the line (T).

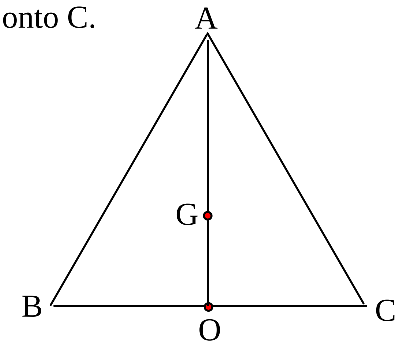
   Show that the area of (D) is less than 8.

3) (L) is the line with equation y = mx where m is a non-zero real number. (L) intersects (P) in two points A and B. Show that the midpoint I of [AB] moves on a parabola (P′) whose focus and directrix are to be determined.

V- (3 points)

Consider a direct equilateral triangle ABC with center G. Let O be the midpoint of [BC] and F that of [AC].

Let *S* be the direct plane similitude that transforms A onto B and G onto C.



A-

1) a-Determine the ratio and an angle of *S* .
   b-Prove that F is the center of *S* .

2) a-Determine the line (d) , image of (AB) under *S* .

   b-Construct the point D image of B under *S* .

B- The complex plane is referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$ such that: $z_A = i\sqrt{3}$ and $z_C = 1$.

a- Find the complex form of *S* .

b- Determine the nature and characteristic elements of $S \circ S$.

c- Express $\overrightarrow{FD}$ in terms of $\overrightarrow{FA}$.

4) Denote by (E) the ellipse with center O and vertices B,C and G. (E') is the image of (E) under *S*.

   a- Determine the center and the focal axis of the ellipse (E').

   b- Calculate the area of the region bounded by (E') .

VI- (7 points)

A- Consider the differential equation $(E)\ :\ y + xy' = e^x \quad (x \neq 0)$.

Let z = xy.

1) Form the differential equation (E') satisfied by z.

2) Solve the equation (E') and deduce the general solution of (E).

3) Determine the particular solution of (E) whose representative curve in an orthonormal system has at the point with abscissa 1 a tangent parallel to the line with equation y = x.

B- Let h be the function defined over $]0;+\infty[$ by $h(x) = \dfrac{e^x - 1}{x}$.

Denote by (C) the representative curve of h in an orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j})$.

1) a- Verify that $h'(x) = \dfrac{(x-1)e^x + 1}{x^2}$.

b- Let g be the function defined over $]0;+\infty[$ by $g(x) = (x-1)e^x$.

Set up the table of variations of g and deduce that h'(x) > 0.

2) a- Calculate $\lim\limits_{x\to 0} h(x)$, $\lim\limits_{x\to+\infty} h(x)$ and $\lim\limits_{x\to+\infty} \dfrac{h(x)}{x}$.

. b- Set up the table of variations of h .

3) a- Write an equation of (Δ), the tangent to (C) at the point with abscissa 1.

b- Draw (Δ) and (C).

4) a- Show that h has an inverse function h^{-1} whose domain of definition is to be determined.

b- Calculate $(h^{-1})'(e-1)$.

C- Consider the function f defined over $]0\ ;\ +\infty[$ by $f(x) = h(x) + \ln x$ and denote by (Γ) its representative curve in the same system as (C).

1) a- Calculate $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

b- Set up the table of variations of the function f .

2) a- Prove that the equation f(x) = 0 has a unique solution α and that $0.3 < \alpha < 0.4$.

b- Compare $h(\alpha)$ and $h(1)$. Deduce that $\ell n\,\alpha > 1 - e$.

3) a- Discuss, according to the values of x, the relative positions of (C) and (Γ).

b- Draw (Γ) .

4) A is a point on (C) and B is a point on (Γ) such that A and B have the same abscissa x .

m is any real number such that m > 0. If AB = m, prove that there exist two values of x whose product is independent of m .

5) For 0 < t < 1, calculate the area S(t) of the region bounded by (C), (Γ) and the two lines with equations x = t and x = e .Calculate $\lim\limits_{t\to 0} S(t)$.

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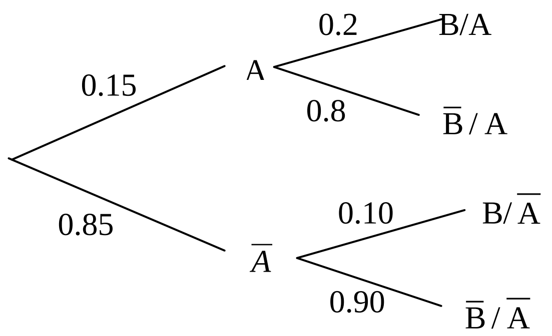
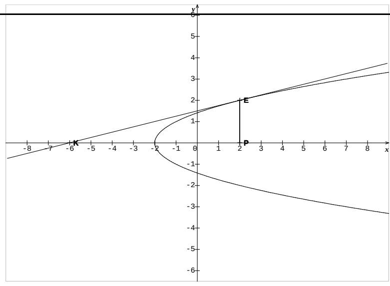
مشروع معيار التصحيح	مسابقة في مادة الرياضيات المدة أربع ساعات	الاسم: الرقم:

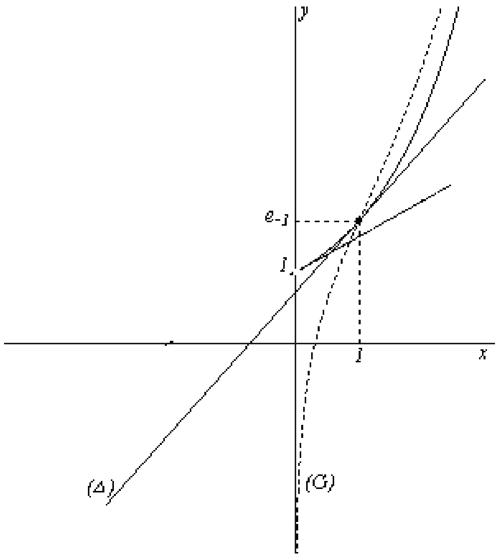
I	Solution	Grade
1	$z' = r\,e^{i\frac{5\pi}{6}}(2i\,I_m(z)) = r^2\,e^{i(\frac{5\pi}{6}+\frac{\pi}{2})} = r^2\,e^{i\frac{8\pi}{6}} = r^2 e^{-i\frac{2\pi}{3}}$ **a**	**1**
2	$f(x) = xe^x$; $f'(x) = (x+1)e^x$ and $f''(x) = (x+2)e^x$ **d**	**1**
3	g is defined for $-1 \le 2x^2 - 1 \le 1$; $0 \le x^2 \le 1$; $-1 \le x \le 1$ **b**	**1**
4	$U_{n+1} + U_n = \int_0^1 e^{nx}dx = \frac{1}{n}\left[e^{nx}\right]_0^1 = \frac{e^n - 1}{n}$ **a**	**1**

II	Solution	Grade
1	An equation of (P) is given by $\begin{vmatrix} x-3 & y-1 & z-1 \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix} = 0$; (P) : $x + y + z - 5 = 0$.	0.5
2a	$AB = BC = CA = \sqrt{6}$. ABC is equilateral.	0.5
2b	$\vec{V}$ is a directing vector of (T). (T) lies completely in (P) and (T) is perpendicular to (GA) ; $\vec{V}$ is collinear to vector $\vec{N}_P \wedge \overrightarrow{GA}$. $\vec{N}_P \wedge \overrightarrow{GA} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \vec{i} - 2\vec{j} + \vec{k}$ (T) : $x = t + 3$, $y = -2t + 1$, $z = t + 1$ with t being a real parameter.	1.5
3a	$A_{ABC} = \frac{1}{2}\lVert \overrightarrow{AB} \wedge \overrightarrow{AC} \rVert = \frac{1}{2}\sqrt{9+9+9} = \frac{3\sqrt{3}}{2}$. OR $A_{ABC} = \frac{1}{2} AB \times AC \times \sin A = \frac{1}{2}\sqrt{6} \times \sqrt{6} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$.	0.5
3b	$d(M \longrightarrow (P)) = \frac{\lvert 1+7+\alpha-5 \rvert}{\sqrt{1+1+1}} = \frac{\lvert \alpha+3 \rvert}{\sqrt{3}}$ Volume $= \frac{1}{3} \times d(M \longrightarrow (P)) \times A_{ABC} = \frac{\lvert \alpha+3 \rvert}{2}$ $\lvert \alpha + 3 \rvert = 6$ for $\alpha + 3 = 6$ or $\alpha + 3 = -6$; $\alpha = 3$ or $\alpha = -9$.	1

<table>
<tr><th>III</th><th>Solution</th><th>Grade</th></tr>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>1</td><td>$P(A\cap B) = P(A) \times P(B/A) = 0.15 \times 0.2 = 0.03.$
$P(B) = P(A\cap B) + P(\overline{A} \cap B) = 0.03 + P(\overline{A}) \times P(B/\overline{A}) = 0.03 + 0.85\times0.1 = 0.115$</td><td>1</td></tr>
<tr><td>2</td><td>$P\left(A/\overline{B}\right)=\dfrac{P\left(A\cap\overline{B}\right)}{P\left(\overline{B}\right)}=\dfrac{P(A).P(\overline{B}/A)}{1-p(B)}=\dfrac{0.12}{0.885}=\dfrac{120}{885}=\dfrac{24}{177}=\dfrac{8}{59}$</td><td>1</td></tr>
<tr><td>3</td><td>The possible values of x are : 0;1;2
$P\left(X=0\right)=P\left(\overline{A}\cap\overline{B}\right)=P\left(\overline{A}\right)\times P\left(\overline{B}/\overline{A}\right)=0.85\times0.90=0.765$
$P\left(X=1\right)=P\left(A\cap\overline{B}\right)+P\left(\overline{A}\cap B\right)=0.15\times0.8+0.85\times0.1=0.12+0.085=0.205$
$P\left(X=2\right)=P\left(A\cap B\right)=0.03$
<table>
<tr><td>x_i</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>P_i</td><td>0.765</td><td>0.205</td><td>0.030</td></tr>
</table></td><td>2</td></tr>
<tr><td>4a</td><td>P(at most 2) = P(0) + P(1) + P(2) = $\dfrac{C_{30}^{0}\times C_{170}^{4}+C_{30}^{1}\times C_{170}^{3}+C_{30}^{2}\times C_{170}^{2}}{C_{200}^{4}} = 0.988$</td><td>2</td></tr>
</table>

IV	Solution	Grade
1-a	$\overrightarrow{SH} = -\overrightarrow{SF}$; $x_H = -\frac{9}{4}$; the directrix (d) has an equation $x + \frac{9}{4} = 0$ M(x ; y) is a point of (P) if and only if $d(M \longrightarrow F) = d(M \longrightarrow (d))$; $(x + \frac{9}{4})^2 = (x + \frac{7}{4})^2 + y^2$Hence, $y^2 = x + 2$.	1
1-b		0.5
2-a	2yy’ =1 ; $y' = \frac{1}{2y}$; $(y_E)' = \frac{1}{4}$ (T) : $y - 2 = \frac{1}{4}(x - 2)$; $y = \frac{x}{4} + \frac{3}{2}$	1
2-b	T intersects the axis of abscissas at the point K(– 6 ; 0). Area of (D) < Area of (triangle EKP) with P = proj(E/x’x) ; Thus, Area (D) < 8.	1.5
3	y=mx x_A and x_B are the roots of the equation : $m^2x^2 - x - 2 = 0$ $x_I = ½ (x_A + x_B) = \frac{1}{2m^2}$ and $y_I = mx_I$; $(y_I)^2 = m^2(x_I)^2$; $(y_I)^2 = \frac{1}{2} x_I$. I moves on the parabola (P') of equation $y^2 = \frac{1}{2}x$ .F'$(\frac{1}{8};0)$ and (d'): $x=\frac{-1}{8}$	2

V	Solution	Grade
A-1-a	$\frac{BC}{AG}=\frac{BC}{\frac{2}{3}AO}=\frac{BC}{\frac{2}{3}AB\frac{\sqrt{3}}{2}}=\sqrt{3}$, $\left(\overrightarrow{AG};\overrightarrow{BC}\right)=\frac{\pi}{2}(2\pi)$.	0.5
A-1-b	S(G) = C and S(A) = B, then $\left(\overrightarrow{\Omega G};\overrightarrow{\Omega C}\right)=\frac{\pi}{2}(2\pi)$, $\left(\overrightarrow{\Omega A};\overrightarrow{\Omega B}\right)=\frac{\pi}{2}(2\pi)$ So, Ω is the point of intersection, other than O, of the two circles with diameters [AB] and [GC] respectively since $\left(\overrightarrow{OG};\overrightarrow{OC}\right)=-\frac{\pi}{2}(2\pi)$, hence it is the point F.	1
A-2-a	S(A) = B , (d) = S(AB) passing through B and perpendicular to (AB).	0.5
A-2-b	S(B) = D and S(A) = B ; then (BD) is perpendicular to (AB). S(F) = F and S(B) = D ; then (FD) is perpendicular to (FB) = (AC). F is the point of intersection of (d) and (AC).	1
B	B(– 1) ; A($i\sqrt{3}$)	
B-1-a	S is of the form z' = $\sqrt{3}$ iz + b but S(A) = B ; – 1 = $\sqrt{3}$ i(i $\sqrt{3}$) + b so b = 2 z' = $\sqrt{3}$ iz + 2	0.5
B-1-b	S o S is the similtude of center F , ratio 3 and angle $\frac{\pi}{2}+\frac{\pi}{2}=\pi$ Thus, S o S is the negative dilation of center F and ratio – 3.	0.5
B-1-c	$S\circ S(A)=S(B)=D$ then, $\overrightarrow{FD}=-3\overrightarrow{FA}$	0.5
B-2-a	S(O)= O' . $z_O = 0$; then $z_{O'} = 2$ Focal axis of (E) is (BC). Focal axis of (E') is the perpendicular to (BC) through O'.	1
B-2-b	$A_{(E)} = \pi$ ab ; a = OB = OC = 1 and b = OG = $\frac{\sqrt{3}}{3}$ $A_{(E)}=\pi\frac{\sqrt{3}}{3}$ and $A_{(E')} = k^2\times A_{(E)}=\pi\sqrt{3}$.	0.5

VI	**Solution**	**Grade**
A1	$z' = e^x$	0.5
A2	The general solution of (E') is $z = e^x + C$; $C \in IR$. The general solution of (E) is $y = \dfrac{e^x + C}{x}$.	0.5
A3	$y'(1) = 1$. $y' = \dfrac{e^x - y}{x}$; $1 = \dfrac{e - (e + c)}{1}$. Then, c=-1 A particular solution of (E) is $y = \dfrac{e^x - 1}{x}$	0.5
B1a	$h'(x) = \dfrac{e^x x - (e^x - 1)}{x^2} = \dfrac{e^x (x - 1) + 1}{x^2}$	0.5
B1b	$g(x) = (x-1)e^x$. $\lim\limits_{x \to -\infty} g(x) = 0$; $\lim\limits_{x \to +\infty} g(x) = +\infty$ $g'(x) = x e^x$. x: 0 … $+\infty$; $g'(x)$: 0 + ; $g(x)$: -1 ↗ $+\infty$ For all $x \neq 0$, $g(x) > -1$. so $(x-1)e^x + 1 > 0$. Thus h'(x)>0	1
B2a	. $\lim\limits_{x \to 0} h(x) = 1$ and $\lim\limits_{x \to +\infty} h(x) = +\infty$. $\lim\limits_{x \to +\infty} \dfrac{h(x)}{x} = +\infty$	1
B2b	For all $x \neq 0$, $g(x) > -1$. Therefore, $(x-1)e^x + 1 > 0$ x: 0 … $+\infty$; $h'(x)$: + ; $h(x)$: 1 ↗ $+\infty$	0.5
B3a	y-(e-1) = 1(x-1) : y = x+e-2	0.5
B3b	(C) admits at $+\infty$ an asymptotic direction parallel to $y'y$.  	0.5
B4a	h is continuous and strictly increasing over $]0 ; +\infty[$; then h has an inverse function h^{-1} defined over $]1 ; +\infty[$.	0.5

B4b	$(h^{-1})'(e-1)=(h^{-1})'(h(1))=\dfrac{1}{h'(1)}=1$.	0.5
C1a	$\lim\limits_{x\to 0} f(x)=-\infty$ and $\lim\limits_{x\to+\infty} f(x)=+\infty$	0.5
C1b	For $x\in]0\,;+\infty[$, $f'(x)=h'(x)+\dfrac{1}{x}>0$. x: 0 ... $+\infty$ $f'(x)$: $+$ $f(x)$: $-\infty$ ↗ $+\infty$	0.5
C2a	f is continuous and strictly increasing over $]0\,;+\infty[$ from $-\infty$ to $+\infty$. thus, the equation $f(x)=0$ has a unique solution α . Moreover, $f(0.3)\times f(0.4)\approx -0.038\times 0.313<0$, then $0.3<\alpha<0.4$	1
C2b	h is strictly increasing over $]0\,;+\infty[$ and $\alpha<1$ then $h(\alpha)<h(1)$; $\dfrac{e^{\alpha}-1}{\alpha}<e-1$; but $\dfrac{e^{\alpha}-1}{\alpha}=-\ln\alpha$; therefore $\ln\alpha>1-e$.	1
C3a	$f(x)-h(x)=\ell n\,x$. If $0<x<1$,$\ln x<0$ then (Γ) is below(C). If $x>1$, $\ln x>0$ then (Γ) is above (C). (C) and (Γ) intersect at $(1\,;e-1)$	1
C3b	$\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=+\infty$; (Γ) has an asymptotic direction prallel to y'y at $+\infty$. $\lim\limits_{x\to 0^+} f(x)=-\infty$; $y'y$ is an asymptote to (Γ). Drawn in 3b	0.5
C4a	$AB=\lvert f(x)-g(x)\rvert=\lvert \ell n\,x\rvert$; $AB=m$ is equivalent to $\lvert \ln x\rvert=m$; $\ln x=m$ or $\ln x=-m$. thus $x=e^{m}$ or $x=e^{-m}$. $e^{m}\times e^{-m}=1$	1
C5	$S(t)=-\int_t^1 \ln x\,dx+\int_1^e \ln x\,dx=-\left[x\ln x-x\right]_t^1+\left[x\ln x-x\right]_1^e=t\ln t-t+2$ square units . $\lim\limits_{t\to 0} S(t)=2$.	1.5