المادة: الرياضيات الشهادة: الثانوية العامة الفرع: علوم الحياة نموذج رقم -٤- المدّة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

In the space of an orthonormal system $(O\,;\,\vec{i},\vec{j},\vec{k})$, consider the plane (P) with equation :

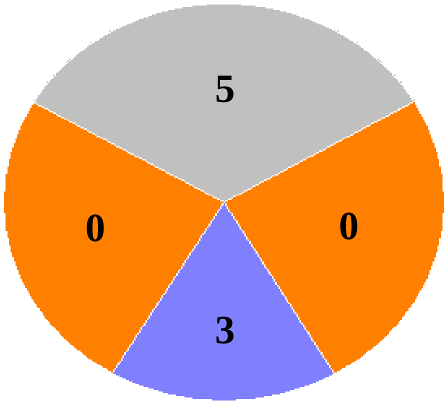
$x + y + z - 1 = 0$, and the line (d) with parametric equations $\begin{cases} x = -t-1 \\ y = t+5 \\ z = 3t+9 \end{cases}$ $(t \in \mathbb{R})$,

Let H (1, 1, -1) be a point on (P).

1) Determine A, the common point between (d) and (P).
2) Let (Δ) be the line passing through H and perpendicular to the plane (P).
   a- Write a system of parametric equations of (Δ).
   b- Verify that E (2,2,0) is the intersection point between (Δ) and (d).
   c- Calculate the angle formed by (d) and (P).
3) Let (Q) be the plane passing through O and the point F (2,1,0) and perpendicular to(P).
   a- Write an equation of the plane (Q).
   b- Let M(x,y,z) be a variable point on (Q).
   Prove that the volume of the tetrahedron MEAH is constant.
   c- Deduce that the two planes (Q) and (EAH) are parallel

II- (4 points)

A game consists of throwing a dart at a target .The target is divided into four sectors as shown in the figure at right.



Denote by P_0 the probability of obtaining 0 point, P_3 the probability of obtaining 3 points and P_5 the probability of obtaining 5 points.

1) We know that the dart touches the target on every throw,
   $P_5 = \frac{1}{2} P_3$ and $P_5 = \frac{1}{3} P_0$.
   Verify that $P_5 = \frac{1}{6}$.
2) In this part ,the game consists of throwing a maximum of two darts. Suppose that the two throws are independent. A player wins a round if s/he obtains a total greater than or equal to 5,but the game stops if she obtains 5 at the first throw.
   Consider the following events :
   - G_1 : « The player wins a round in 1 throw».
   - G_2 :«The player wins a round in 2 throws ».
   - G_0 :« The player loses a round ».

   Show that $P(G_2) = \frac{1}{4}$,then deduce $P(G_0)$.
3) To participate in the game, a player should pay 2000 L.L.
   If the player wins a round in one throw, she gains 5 000 L.L.
   If the player wins a round in two throws, she gains 3 000 L.L.
   If the player loses a round, s/he gains nothing.
   Denote by X the random variable that corresponds to the algebraic gain of a player in one round
   a- Verify that the possible values of X are : -2000, 1000 and 3000.

b- Determine the probability distribution of X.
c- A game is fair if E(x) > 0. Is this game fair?

III- (4points)
The complex plane refers to a direct orthonormal system $(O;\vec{u},\vec{v})$.
Consider the points E, A,B,M and M' with affixes i, 2, 2i, z et z'.
Let z' be the complex number defined as: z' = $\frac{2-z}{2+iz}$.

1) If z = − 2i, write z' in exponential form.
2) **a)** Prove that (z'-i)(2+iz) =2 – 2i .
   b) Verify that 2 + iz = i (z – 2i).
   c) Deduce the value of (z'- i)(z – 2i).
   d) Calculate BM$\times$EM' and ($\vec{u}$, $\overrightarrow{BM}$) + ($\vec{u}$, $\overrightarrow{EM'}$).
3) Given z = x+iy and z' = x'+iy'.
   a) Find x' and y' in terms of x and y.
   b) If z' is pure imaginary, prove that M moves on a line whose equation should be determined.
   c) Calculate, in this case, the angle ($\vec{u}$, $\overrightarrow{BM}$).

IV- (8points).
Part A
Let g be the function defined over]0; + ∞[as g(x) = ax^2 −2 ln x + b.
(C_g) is its graph and A is the point on(C_g) so that x_A = 1.
1) Find a and b so that (C_g) is tangent at A to the line (d) with equation : y = 2x + 2 .
2) In what follows let a = b = 2.
a) Find $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.
b) Set up the table of variations of g, deduce that g (x) > 0.
3) h is the function defined over]0; + ∞[as h(x) = x^2 - ln^2 x + 2 lnx – 1 .
a) Find $\lim\limits_{x\to 0} h(x)$ and $\lim\limits_{x\to+\infty} h(x)$.
b) Prove that h'(x) = $\frac{g(x)}{x}$. Deduce that h is increasing.
c) Calculate h (1), then discuss according to x the sign of h(x).

Part B
f is the function defined over]0;+∞[as f(x) = x – 1 + $\frac{1+\ln^2 x}{x}$; (C) is the graph of f.
1) a-Find $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.
b-Prove that the line (Δ) with equation y = x – 1 is an asymptote to (C).
c- Show that (C) is above (Δ).
2) a-Prove that f '(x) = $\frac{h(x)}{x^2}$.
b-Set up the table of variations of f.
c- Find the point B on (C) where the tangent (T) is parallel to the line (Δ).
d- Calculate f ($\frac{1}{2}$), f (2), then plot (Δ), (T) and (C).
3) a- For x ≥ 1, prove that f has an inverse function P, Find D_P.
b- Plot the graph (C') of P in the same system as that of (C).
4) Let P (2) = α .
a-Prove that 2.2 < α < 2.3.
b- Prove that P'(2) = $\frac{\alpha^2}{2\alpha^2-3\alpha+2\ln\alpha}$.

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أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

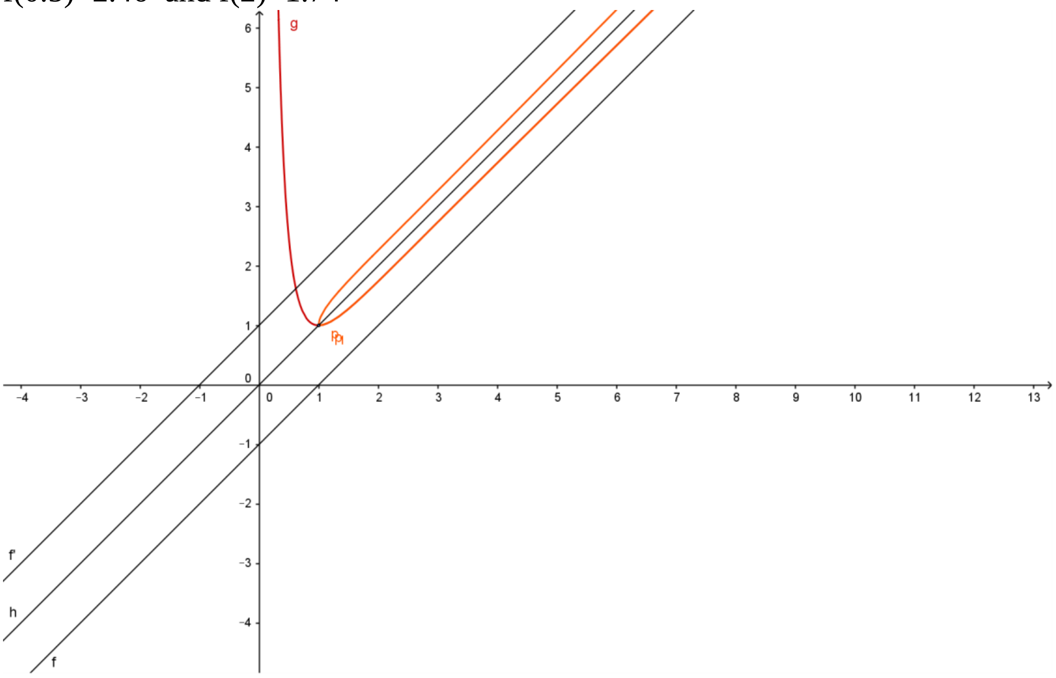
	Question I	Note
1	A(3;1;-3) for t=-4	0.5
2.a	$\begin{cases} x = k+1 \\ y = k+1 \\ z = k-1 \end{cases}$	0.5
2.b	$E \in (\Delta)$ for t=-3 and $E \in (d)$ for k=1 $\Rightarrow E = (\Delta) \cap (d)$.	0.5
2.c	The angle is $H\hat{A}E$ and $\cos H\hat{A}E = \frac{2\sqrt{2}}{\sqrt{11}} \simeq 0.85$ then $H\hat{A}E \simeq 32^\circ$.	0.5
3.a	$\overrightarrow{OM}.\left(\overrightarrow{OF} \wedge \overrightarrow{N_P}\right) = 0 \Rightarrow (Q): x - 2y + z = 0$.	0.75
3.b	$V = \frac{1}{6}\left\|\overrightarrow{EM}.\left(\overrightarrow{EA} \wedge \overrightarrow{EH}\right)\right\| = \frac{2}{3}U^3$	0.75
3.c	The volume is independent of M then the distance from (Q) to (EAH) is constant then (Q)//(EAH).	0.5

	Question II	Note
1	$P_0 + P_3 + P_5 = 1$, then $P_5 = \frac{1}{6}$	0.5
2	$\frac{1}{2} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3} = \frac{1}{4}$	1
	$1 - \frac{1}{6} - \frac{1}{4} = \frac{7}{12}$	0.5
3.a	$-2000 \rightarrow P(G_0)$ $1000 \rightarrow P(G_2)$ $3000 \rightarrow P(G_1)$	0.5
3.b	$\begin{array}{\|c\|c\|c\|c\|} \hline X = x_i & -2000 & 1000 & 3000 \\ \hline p(X = x_i) & \frac{7}{12} & \frac{1}{4} & \frac{1}{6} \\ \hline \end{array}$	1
3.c	$E(X) = \frac{-1000}{3} < 0$, then this game is not favorable.	0.5

	Question III	Note
1	$z' = \frac{\sqrt{2}}{2} e^{\frac{i\pi}{4}}$	0.5
2.a	$\left(\frac{2-z}{2+iz} - i\right)(2+iz) = 2-2i$	0.5
2.b	$2 + iz = i\left(z + \frac{2}{i}\right) = i(z-2i)$	0.5

2.c	$\left(z'-i\right)\left(z-2i\right)=\dfrac{2-2i}{i\left(z-2i\right)}\left(z-2i\right)=\dfrac{2-2i}{i}=-2-2i$	**0.5**
2.d	$EM'\times BM=\lvert -2-2i\rvert=2\sqrt{2}$; $\left(\vec{U},\overrightarrow{BM}\right)+\left(\vec{U},\overrightarrow{EM'}\right)=\arg\left(-2-2i\right)=\dfrac{5\pi}{4}+2k\pi$	**0.5**
3.a	$x'=\dfrac{4-2x-2y}{x^2+\left(2-y\right)^2}$; $y'=\dfrac{x^2+y^2-2x-2y}{x^2+\left(2-y\right)^2}$	**0.5**
3.b	z'is pure imaginary ,then x'=0 and y'≠0 then M moves on the line with equation 2−x−y=0, without points A and B.	**0.5**
3.c	$(\vec{U};\overrightarrow{EM'})=\pm\dfrac{\pi}{2}$ then $\left(\vec{U},\overrightarrow{BM}\right)=\dfrac{3\pi}{4}$ or $\dfrac{-\pi}{4}$.	**0.5**

	Question IV	**Note**
Part A		
1	g(1)=4 and g'(1)=2 then a=b=2.	**0.5**
2.a	$\lim\limits_{x\to 0} g(x)=+\infty$ $\lim\limits_{x\to+\infty} g(x)=+\infty$	**0.5**
2.b	g'(x)= $4x-\dfrac{2}{x}=\dfrac{4x^2-2}{x}$ **x**: 0 ; $\dfrac{\sqrt{2}}{2}$; $+\infty$ **g'(x)**: - ; 0 ; + **g(x)**: $+\infty$ ↘ $3-2\ln\dfrac{\sqrt{2}}{2}$ ↗ $+\infty$ since min(g(x)>0) then g(x)>0 for all x in its domain.	**0.5**
3.a	$\lim\limits_{x\to 0} h(x)=-\infty$ $\lim\limits_{x\to+\infty} h(x)=+\infty$	**0.5**
3.b	$h'(x)=2x-\dfrac{2\ln x}{x}+\dfrac{2}{x}=\dfrac{g(x)}{x}$ and h'(x)>0 then h is increasing.	**0.5**
3.c	h(1)=0 then h(x) > 0 for x > 1 and h(x)<0 for 0<x<1.	**0.25**
Part B		
1.a	$\lim\limits_{x\to 0} f(x)=+\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty$	**0.5**
1.b	$\lim\limits_{x\to+\infty}(f(x)-(x-1))=\lim\limits_{x\to+\infty}\dfrac{1+\ln^2 x}{x}=0$ then $\left(\Delta\right)$ is asymptote to (C).	**0.25**
1.c	$f(x)-(x-1)=\dfrac{1+\ln^2 x}{x}\succ 0$ then (Δ) below (C).	**0.25**
2.a	$f'(x)=1+\dfrac{2\ln x-1-\ln^2 x}{x^2}=\dfrac{h(x)}{x^2}$	**0.5**
2.b	**x**: 0 ; 1 ; $+\infty$ **f'(x)**: - ; 0 ; + **f(x)**: $+\infty$ ↘ 1 ↗ $+\infty$	**0.5**

2.d	f’(x)=1 then $h(x)=x^2 \;\Rightarrow\; (\ln x-1)^2=0 \;\Rightarrow\; B(e, f(e))$.	**0.5**
2.c	f(0.5)=2.46 and f(2)=1.74  	**1**
3.a	for $x\in[1;+\infty[$, f is continuously and strictly increasing then it has an inverse P= f^{-1}et $D_P=[1;+\infty[$	**0.25**
3.b	(C’) graph of P is the symmetric of (C) with respect to y=x.	**0.5**
4.a	$(2,\alpha)$ $on(C'), then$ $(\alpha,2)$ is on (C) $with$ $\alpha\geq 1$ $f(\alpha)=2$, $f(2.2)\prec 2$ and $f(2.3)\succ 2$ Since f is increasing for $x\geq 1$ then $2.2\prec\alpha\prec 2.3$	**0.5**
4.b	$P'(2)=\frac{1}{f'(\alpha)}=\frac{\alpha^2}{h(\alpha)}=\frac{\alpha^2}{\alpha^2-\ln^2\alpha+2\ln\alpha-1}$.Hence $f(\alpha)=2\Rightarrow \alpha^2-\alpha+1+\ln^2\alpha=2\alpha\Rightarrow P'(2)=\frac{\alpha^2}{2\alpha^2-3\alpha+2\ln\alpha}$	**0.5**