ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المُرشّح الإجابة بالترتيب الذي يناسبه (دون الإلتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

The results of a survey conducted by a company on the evolution of the price x_i of an article and the number of customers who purchased this article y_i, are given by the following table:

Unit price of the article: x_i in thousands of LL	12	15	18	22	24
Number of purchasers: y_i in hundreds	16	13	11	9	6

1) Construct, in an orthogonal system, the scatter plot of the points (x_i, y_i) .
2) Calculate the coordinates of the center of gravity G and plot this point in the preceding system.
3) Determine the coefficient of correlation and interpret the value thus found.
4) Write an equation of the regression line $(D_{y/x})$, of y in terms of x, and plot this line in the preceding system.
5) Suppose that the evolution of the price follows the given pattern and that the price of this article reaches 25 000LL.
   a- Show that an estimation of the number of purchasers, at this price, is 581.
   b- In this case, suppose that each of these customers bought the article. The production cost of this article is 8000 LL . Estimate, thus, the total profit.

II- (4 points)

The third secondary class of a school consists of 3 sections: ES, LS and GS.
40 % of the students of this class are in the ES section and 40% are in the LS section.
All the students of this class sit for the first term exam.

- $\dfrac{5}{8}$ of the students of the ES section pass this exam;

- 50% of the students of the LS section pass this exam;

- 60 % of all the students pass this exam;

A student is randomly chosen from the third secondary class of this school.

Consider the following events :

E : «The chosen student is in the ES section»

L : «The chosen student is in the LS section»

G : «The chosen student is in the GS section»

R : «The chosen student pass the exam».

1) Calculate the probabilities $P(E \cap R)$, $P(L \cap R)$ and deduce that $P(G \cap R)$ is equal to 0.15.

2) The chosen student pass the exam. Calculate the probability that this student is from the GS section?

3) In this question, suppose that third secondary class counts 60 students.

A group of 2 students is chosen randomly and simultaneously from these 60 students.

Denote by X the random variable equal to the number of students who pass the exam in this group.

   a- Verify that $P(X = 1) = \dfrac{144}{295}$.

   b- Determine the probability distribution of X.

III- (4 points)

A bank proposes to its customers, who are younger than 25 years, the following offer:

Depositing an amount of 2 000 000 LL in an account at an annual interest rate of 9% compounded monthly to which the bank adds directly and each month an amount of 9000 LL.

Imad decides to take advantage of this offer.

Denote by S_n the amount in Imad's account after n months. Thus, S_0=2 000 000.

1) Prove that $S_{n+1}=1.0075 \, S_n+9000$.

2) (V_n) is the sequence defined by $V_n=S_n+1\,200\,000$ for all natural numbers n.

    a- Show that (V_n) is a geometric sequence whose common ratio and first term V_0 are to be determined.

    b- Express V_n in terms of n. Deduce S_n in terms of n.

3) After how many months will the amount in Imad's account exceed 4 000 000 LL for the first time?

IV- (8 points)

Consider the function f defined over $[0,+\infty[$ by $f(x) = x+e^{-x+2}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

A-

1) a- Determine $\lim\limits_{x \rightarrow +\infty} f(x)$.

    b- (d) is the line with equation $y = x$. Prove that (C) is above (d).

    c- Verify that (d) is an asymptote to (C).

2) a- Calculate f '(x) and set up the table of variations of f.

    b- Draw (d) and (C).

3) Determine over $[0;+\infty[$ an antiderivative F of f and deduce the area of the region bounded by (C), the axis of abscissas (x-axis) and the two lines with equations $x = 0$ and $x = 2$.

B-

A company wants to launch a new product to the market. The average cost function is given by $f(x) = x+e^{-x+2}$ where x represents the daily quantity produced in tens of units $(1 \le x \le 8)$ and f(x) represents the average cost in millions of LL.

1) Give an economical interpretation to the minimum of f(x).

2) Determine the total cost C_T (x) of producing x tens of units.
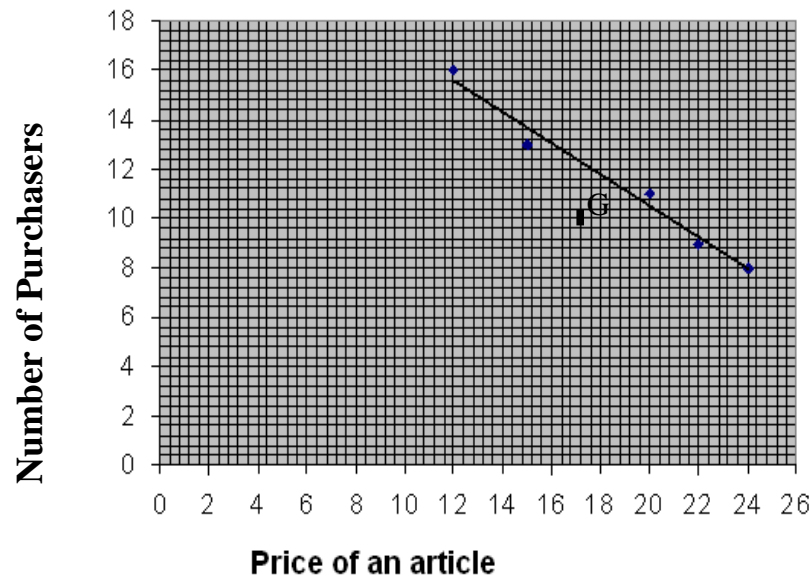
3) a- A unit is sold for a fixed price of 200 000 LL. Show that the profit obtained by the sale of x tens of units, is given by $P(x) = 2x - x^2 - xe^{-x+2}$.

    b- If the company sells 60 units of the product, does it make a profit?

    c- Copy and complete the table of variations of P given below:

x	1		2 - ln2		8
P'(x)	0	+	0	−	
P(x)					

    d- Is the company interested in launching this product to the market? Justify.

I	**Answers**	**Marks**
1		1
2	$\overline{x} = 18.2$; $\overline{y} = 11$ (by calculator) G(18.2; 11).	1
3	r = – 0.9885 (by calculator) There is a strong negative linear correlation between the two variables.	1
4	y = – 0.76x + 24.913(by calculator)	1
5a	The price of an article is 25000LL so x = 25 Then y = – 0.76 × 25 + 24.913; y = 5.81. Thus, the number of purchasers is 581.	1.5
5b	The total profit is $(25000 – 8000) \times 581 = 9877000$ LL	1.5

II	**Answers**	**Marks**
1	$P(E \cap R) = P(E) \times P(R/E) = (40/100) \times (5/8) = 0.25$. $P(V \cap R) = P(V) \times P(R/V) = (40/100) \times (1/2) = 0.2$. $P(R) = P(G \cap R) + P(V \cap R) + P(E \cap R)$. Thus, $P(G \cap R) = 0.6 – 0.25 – 0.2 = 0.15$.	2.5
2	$P(G/R) = \dfrac{P(G \cap R)}{P(R)} = \dfrac{0.15}{0.6} = 0.25$.	1.5
3a	The students that pass the exam are $0.6 \times 60 = 36$. $P(X=1) = \dfrac{C_{36}^1 \times C_{24}^1}{C_{60}^2} = \dfrac{864}{1770} = \dfrac{144}{295}$	1
3b	The possible values of X are 0,1and 2. $P(X=0) = \dfrac{C_{24}^2}{C_{60}^2} = \dfrac{46}{295}$; $P(X=1) = \dfrac{144}{295}$; $P(X=2) = \dfrac{C_{36}^2}{C_{60}^2} = \dfrac{21}{59}$.	1

III	**Answers**	**Marks**
1	$S_{n+1} = S_n(1 + \dfrac{0.09}{12}) + 9000 = 1.0075 S_n + 9000$.	1
3a	$V_{n+1} = S_{n+1} + 75 = 1.0075 S_n + 9000 + 1200000 = 1.0075(S_n + \dfrac{1209000}{1.0075})$ $\quad = 1.0075(S_n + 1200000) = 1.0075 V_n$ Thus, (V_n) is a geometric sequence with first term $V_0 = S_0 + 1200000 = 3200000$ and with common ratio r = 1.0075.	2

3b	$V_n = V_0 r^n = 3200000(1.0075)^n$ $\qquad$ $S_n = V_n - 1200000 = 3200000(1.0075)^n - 1200000$.	2
4	$S_n > 4000000.$; $(3200000)(1.0075)^n > 5200000$ then $n \ln(1.0075) > \ln(1.625)$; $n > 64.97$ After 65 months the amount in Imad's account exceeds 4000000 for the first time.	2

IV	Answers	Marks
A1a	$\lim\limits_{x \to +\infty} f(x) = +\infty$ since $\lim\limits_{x \to +\infty} e^{-x+2} = 0$	0.5
A1b	$f(x) - x = e^{-x+2} > 0$ for all values of $x \in [0; +\infty[$.	1
A1c	$\lim\limits_{x \to +\infty} [f(x) - x] = \lim\limits_{x \to +\infty} e^{-x+2} = 0$; (d) of equation $y = x$ is an asymptote to (C).	1
A2a	$f'(x) = 1 - e^{-x+2}$ over $[0, +\infty[$. 	1.5
A2b		1.5
A3	$F(x) = \dfrac{x^2}{2} - e^{-x+2}$; Area of the region is $\displaystyle\int_0^2 f(x)dx = \left[\dfrac{x^2}{2} - e^{-x+2}\right]_0^2 = \left(1 + e^2\right)$.	1.5
B1	Economical interpretation: For the production of 20 units, the average minimum cost is 3 000 000LL.	1
B2	$C_T(x) = x f(x)$ then $C_T(x) = x^2 + x e^{-x+2}$.	1
B3a	The revenue $R(x) = (x \times 10) \times (200\,000) \times \dfrac{1}{1000000} = 2x$ The profit $P(x)$ for the sale of x ten units is: $P(x) = 2x - x^2 - x e^{-x+2}$.	1.5
B3b	$P(6) = 12 - 36 - 6e^{-4} = -24 - 6e^{-4}$. The company does not make profit.	1
B3c		1
B3d	From the table of variations of P, we notice that P(x) <0 so the company has no interest in launching the new product.	1.5