المادة: الرياضيات الشهادة: الثانوية العامة الفرع: علوم الحياة نموذج رقم -٣- المدّة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

In the space of an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the point A (2,2,0) and the line (d) with parametric equations : (d) $\begin{cases} x = -t + 1 \\ y = 3 \\ z = -t \end{cases}$

Let (P) be the plane determined by A and (d).

1) Show that x + y – z – 4 = 0 is an equation of (P) .
2) Denote by (Q) the plane containing (d) and perpendicular to (P) .
   Prove that an equation of the plane (Q) is x – 2y – z + 5 = 0.
3) Consider in the plane (Q) the circle (C) with center B(-3,0,2) and radius r = 3$\sqrt{3}$.
   a- Show that (C) is tangent to (d) .
   b- Find the coordinates of E , the tangency point between (d) and (C) .
4) Verify that L(-6,-3,5) is the symmetric point of E with respect to B.
5) Let F (1,3,0) be a given point on (d) and M a variable point on (Δ),the tangent at L to (C) .
   Calculate the area of the triangle MEF.

II- (4 points)

A bag U contains white and black balls.
40 % of those balls are white and the others are black.
20 % of white balls are numbered 0 and 30 % of black balls are numbered 0.
A second bag V contains 5 balls numbered 0 and 5 balls are numbered – 1.

<u>Part A</u>

One ball is randomly selected from U.
Consider the event E:"the selected ball is numbered 0 ".

1) Prove that the probability of E is equal to 0.26.
2) Knowing that the ball selected is not numbered 0, calculate the probability that this ball is white.

<u>Part B</u>

Consider the following game:
One ball is randomly selected from U.

- If the ball drawn from U is numbered 0, then it is placed in V and then two balls are randomly and simultaneously selected from V.
- Otherwise, the ball from U is kept out and then the one ball is selected from V.

Let X be the random variable that is equal to the sum of points obtained at the end of the game.

1) Verify that the possible values of X are -2 , -1 , 0 .
2) Verify that p (x=0) = $\frac{97}{220}$ and determine the probability distribution of X .

III- (4 points)

The complex plane refers to a direct orthonormal system $(O;\ \vec{u},\vec{v})$. Consider the points A and B of respective affixes $a = -4\sqrt{3} - 4i$ and $b = -4\sqrt{3} + 4i$.

1) Find the nature of triangle OAB.
2) Let C be the point of affix $c = \sqrt{3} + i$ and D be the point such that OC = OD and $(\overrightarrow{OC}, \overrightarrow{OD}) = \frac{\pi}{3}\ (2\pi)$. Determine the affix of D.
3) Let G be the point of affix $g = -4\sqrt{3} + 6i$.
   - **a-** Show that OBGD is a parallelogram.
   - **b-** Verify that: $\frac{c-g}{a-g} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$.
   - **c-** Deduce, in radians, the measure of the angle $(\overrightarrow{GA}, \overrightarrow{GC})$ and the value of the ratio $\frac{GC}{GA}$.
   - **d-** What is the nature of triangle AGC?

IV- (8 points)

Part A

Let g be the function defined over R as : $g(x) = e^{-x}(1 - x) + 1$.

1) Find lim g(x) as $x \to -\infty$ and as $x \to +\infty$.

2) a- Set up the table of variations of g .

b- Deduce that $g(x) > 0$ for all x in R .

Part B

Consider the differential equation (E): $y'' + 2y' + y = x + 2$.

1) **a-** Verify that u= x is a particular solution for (E).

   b- Let y = z + u ; Form the differential equation (E')satisfied by z and solve this equation .

   c- Deduce the general solution y = f(x) for the equation (E).

   d-Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{\imath}, \vec{\jmath})$.
   Determine f so that (C) is tangent at O to the line y = 2x .

In what follows, suppose that $f(x) = x\,e^{-x} + x$, and f is defined over R.

2) **a-** Determine lim f(x) as $x \to -\infty$ and as $x \to +\infty$.

   b- Show that the straight line (d) with equation y = x is an asymptote to (C).

   c- Discuss according to x the relative position of (d) and (C).

3) **a-** Verify that $f'(x) = g(x)$ and set up the table of variations of f.

   b- Discuss according to x the concavity of (C).

   c- Determine the point E on (C) where the tangent (T) is parallel to (d).

   d- Plot (d), (T) and (C).

4) Consider the function h defined as: $h(x) = \ln(y_E - f(x))$.

   a- Determine the domain of h.

   b- Set up the table of variations of h.

5) Calculate A, the area of the region bounded by (C), (d) , and the two lines with equations (x = - 1) and (x = 1) .

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أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

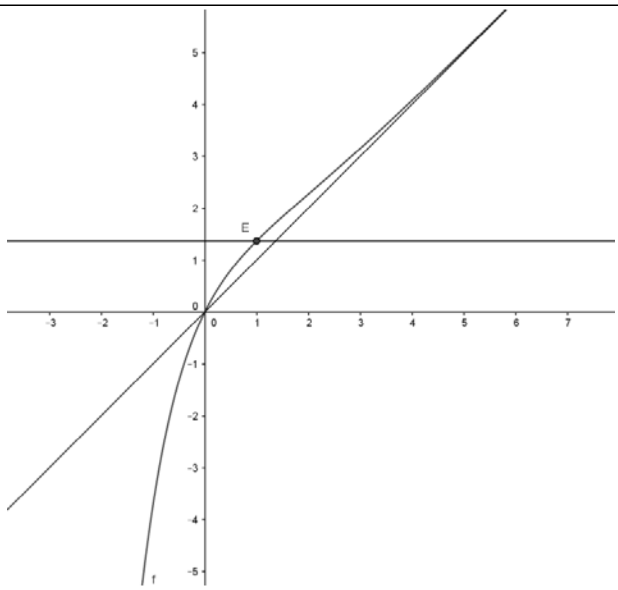
	Question I	Mark
1	(P) = (A ,(d)) : $x + y - z - 4 = 0$.	**0.5**
2	(Q) containing (d) and perpendicular to (P) then (Q) : $x - 2y - z + 5 = 0$.	**0.5**
3.a	d(B,(d)) = $3\sqrt{3}$.= R . (d) is tangent to (C).	**0.75**
3.b	$\overrightarrow{BE}.\overrightarrow{v_d} = 0$ with $\overrightarrow{BE}$ (- t + 4 ,3, - t – 2) and $\overrightarrow{v_d}$ (- 1 , 0,- 1), then t = 1 and E(0,3,- 1).	**0.5**
4.a	B is midpoint of [EL].	**0.5**
5	(Δ)is the parallel through L to (d), then d(M,(d)) = d(L ,(d)) =d(L,(P)) = $6\sqrt{3}$ Area of the triangle MEF = $\frac{EL \times EF}{2} = 3\sqrt{6}\,u^2$	**1.25**

	Question II	Mark
	Part A	
1	P(E) = P(W$\cap$E) + P(B$\cap$E) = 0.08 + 0.18 = 0.26	**1**
2	$P(W/\overline{E}) = \frac{P(W \cap \overline{E})}{P(\overline{E})} = \frac{P(\overline{E}/W) \times P(W)}{P(\overline{E})} = \frac{0.32}{0.74} = 0.432$	**0.75**
	Part B	
1	Then X = $\{0,-1,-2\}$	**0.75**
2	$P(X=0) = 0.26\,\frac{C_6^2}{C_{11}^2} + 0.74\,\frac{C_5^1}{C_{10}^1} = \frac{97}{220}$ $P(X=-1) = 0.26\,\frac{6\times 5}{C_{11}^2} + 0.74\,\frac{5}{10} = \frac{563}{1100}$ $P(X=-2) = 0.26\,\frac{C_5^2}{C_{11}^2} = \frac{13}{275}$ Then sum = 1	**1.5**

	Question III	Mark
1)	OA = | a | = | $-4\sqrt{3}$ – 4i | = 8 OB = | b | = | $-4\sqrt{3}$ + 4i | = 8 AB = | b – a | = | 8i | = 8 Then OAB is an equilateral triangle.	**0.5**
2)	OC = OD, then | z_D | = | z_C | = | $\sqrt{3}$ + i | = 2	**1**

	$(\overrightarrow{OC},\overrightarrow{OD}) = \arg\left(\frac{z_D}{z_C}\right) = \arg(z_D) - \arg(z_C) = \arg(z_D) - \frac{\pi}{6} = \frac{\pi}{3}$, then $\arg(z_D) = \frac{\pi}{2}$ $z_D = \lvert z_D \rvert \times e^{i\arg(z_D)} = 2i$	
3.a	$z_{\overrightarrow{OB}} = z_B = b = -4\sqrt{3} + 4i$ $z_{\overrightarrow{DG}} = z_G - z_D = g - z_D = -4\sqrt{3} + 6i - 2i = -4\sqrt{3} + 4i$ $z_{\overrightarrow{OB}} = z_{\overrightarrow{DG}}$; then OBGD is a parallelogram	**0.5**
3.b	$\frac{c-g}{a-g} = \frac{\sqrt{3}+i-(-4\sqrt{3}+6i)}{-4\sqrt{3}-4i-(-4\sqrt{3}+6i)} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$	**0.5**
3.c	$(\overrightarrow{GA},\overrightarrow{GC}) = \arg\left(\frac{c-g}{a-g}\right) = \arg\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \frac{\pi}{3}\ (2\pi)$ $\frac{GC}{GA} = \left\lvert\frac{c-g}{a-g}\right\rvert = \left\lvert\frac{1}{2} + \frac{\sqrt{3}}{2}i\right\rvert = 1$	**0.5** **0.5**
3.d	AGC is an equilateral triangle being an isosceles triangle with one 60° angle.	**0.5**

	Question IV	**Mark**
	<u>Part A</u>	
1	$x \to -\infty$, $g(x) \to +\infty$. $x \to +\infty$, $g(x) \to 1$.	**1**
2.a	$g'(x) = -e^{-x} - e^{-x}(1-x) = e^{-x}(x-2)$. x: $-\infty$ … 2 … $+\infty$ g'(x): − 0 + g(x): $+\infty$ ↘ $1-\frac{1}{e}$ ↗ 1	**0.5**
2.b	min(g(x)) is positive then g(x) is positive .	**0.25**
	<u>Part B</u>	
1.a	u' = 1 and u''= 0 , u = x solution for (E).	**0.25**
1.b	y = z + u . z'' + u'' + 2z' + 2u' + z + u = x + 2 . z'' +2 z' + z = 0 . $r^2 + 2r + 1 = 0$; r = - 1 then $z = (C_1x + C_2)e^{-x}$ with C_1 and C_2 constants .	**0.5**
1.c	$y = z + u = x + (C_1x + C_2)e^{-x} = f(x)$ with C_1 and C_2 constants .	**0.5**
1.d	f(0)= 0 and f '(0) = 2 so $f(x) = xe^{-x} + x$.	**0.5**
2.a	$x \to -\infty, f(x) \to -\infty$ $x \to +\infty, f(x) \to +\infty$	**0.5**
2.b	$\lim_{x\to+\infty}[f(x) - x] = \lim_{x\to+\infty}\frac{x}{e^x} = 0$ then (d) is an asymptote to (C).	**0.5**
2.c	$f(x) - x = xe^{-x}$. x: … 0 … f(x)- x: − 0 + Position: (C)below (d) ↓ (C) above (d) $(C)\cap(d) = O(0,0)$.	**0.5**

3.a	f ' (x) = $e^{-x}(1-x)+1 = g(x) > 0$. x: $-\infty$ ... $+\infty$ f'(x): + f (x): ↗	**0.25**
3.b	f ''(x) = g'(x) . x: $-\infty$ ... 2 ... $+\infty$ f ''(x): - 0 + concavity: concave down | concave up $I(2\ ;\ 2+\frac{2}{e^2})$ inflection point	**0.5**
3.c	f '(x) = 1 ;g(x) = 1 $e^{-x}(1-x)+1 = 1$. x = 1 $E(1\ ,\ 1+\frac{1}{e})$	**0.25**
3.d	 	**0.75**
4.a	h (x) = ln (y_E – f (x)). y_E - f(x) > 0 ; f (x) < y_E then x < 1 therefore $D_h =]-\infty, 1[$.	**0.5**
4.b	$h'(x) = \frac{-f'(x)}{y_E - f(x)}$ x: $-\infty$ ... 1 ‖ h': – ‖ h: $+\infty$ ↘ $-\infty$ ‖	**0.5**
5	Area $=\int_{-1}^{0}(x-f(x))dx+\int_{0}^{1}(f(x)-x)dx=\int_{-1}^{0}(-xe^{-x})dx+\int_{0}^{1}(xe^{-x})dx$ integration by parts	**0.75**