الدورة الإستثنائية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة:ساعتان	عدد المسائل: اربع

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I-(4 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, with justification, the corresponding answer.

N°	Question	Answers		
		a	b	c
1	The exponential form of $z=-\sin\theta+i\cos\theta$ is	$e^{i\left(\frac{\pi}{2}-\theta\right)}$	$e^{i\left(\theta-\frac{\pi}{2}\right)}$	$e^{i\left(\frac{\pi}{2}+\theta\right)}$
2	If $z=\dfrac{\cos\theta-i\sin\theta}{\cos\theta+i\sin\theta}$, then $\bar{z}=$	$e^{2i\theta}$	$e^{-2i\theta}$	1
3	If $z_A=1-2i$, $z_B=2+3i$ and $z_C=4$ then the triangle ABC is	Right and not isosceles	Isosceles and not right	Right and isosceles
4	$\lim\limits_{x\to0}\dfrac{\int_0^x \ln(t+1)dt}{e^x-1}=$	1	0	$+\infty$
5	$\int\cos^2 x\,dx=$	$\frac{x}{2}-\frac{\sin 2x}{4}+c$	$\frac{\cos^3 x}{3}+c$	$\frac{x}{2}+\frac{\sin 2x}{4}+c$

II- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the points A(4 ; 0 ; 0),

B(0 ; 6 ; 0), C(0 ; 0 ; 4) and E(2 ; 3 ; 0).

1) Show that E belongs to the line (AB).

2) Let (P) be the plane passing through E and parallel to the lines (OB) and (AC).
Show that an equation of (P) is $x + z - 2 = 0$.

3) Write a system of parametric equations of the line (BC).

4) The plane (P) cuts the lines (BC), (OC) and (OA) at F, G and H respectively.
Show that the coordinates of F are (0 ;3 ;2) and specify the respective coordinates of G and H.

5) a- Prove that EFGH is a rectangle.
b- Let Γ be the circle circumscribed about the rectangle EFGH and (T) be the line in plane (P) that is tangent at E to Γ. Determine a system of parametric equations of (T).

III- (4 points)

An urn contains 8 balls:

- 4 white balls each carrying the number 0;
- 3 red balls each carrying the number 5;
- 1 white ball carrying the number 2.

We draw, simultaneously and randomly, 3 balls from the urn.

Consider the following events:

A: « the three drawn balls carry three numbers which could form the number 200».

B: « the three drawn balls carry three identical numbers ».

C: « the three drawn balls are white ».

D: « the three drawn balls are of the same color ».

1) Show that the probability p(A) is equal to $\frac{3}{28}$ and calculate p(B), p(C) and p(D).

2) Determine the probability that among the three drawn balls only one carries the number 0.

3) The three drawn balls are white; calculate the probability that the numbers carried by these balls could form the number 200.

4) let X be the random variable equal to the product of the three numbers carried by the three drawn balls.

a- Give the 3 possible values of X.

b- Determine the probability distribution of X.

IV- (8 points)

A- Let g be the function defined over $]0;+\infty[$ by $g(x) = x + \ln x$.

1) Calculate $\lim\limits_{x \to 0} g(x)$ and $\lim\limits_{x \to +\infty} g(x)$.
2) Set up the table of variations of g.
3) Prove that the equation $g(x) = 0$ has a unique solution α and verify that $0.5 < \alpha < 0.6$.
4) Determine, according to the values of x, the sign of g(x).

B- Consider the function f defined over $]0;+\infty[$ by $f(x) = x(2\ln x + x - 2)$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

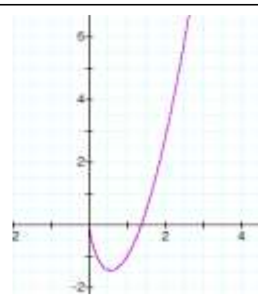
1) Calculate $\lim\limits_{x \to 0} f(x)$, $\lim\limits_{x \to +\infty} f(x)$ and determine f(e).
2) Prove that $f(\alpha) = -\alpha(\alpha + 2)$.
3) Verify that $f'(x) = 2g(x)$ and set up the table of variations of f.
4) Draw (C). (Take $\alpha = 0.55$)
5) Use integration by parts to calculate $\int_{0.5}^{1} x\ln x\,dx$ and deduce the area of the region bounded by the curve (C), the axis of abscissas and the two lines with equations $x = 0.5$ and $x = 1$.
6) The curve (C) cuts the axis of abscissas at a point with abscissa 1.37. Designate by F an antiderivative of f on $]0;+\infty[$; determine, according to the values of x, the variations of F.

QI	Answers	M
1	$z=-\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)=\cos\left(\frac{\pi}{2}+\theta\right)+i\sin\left(\frac{\pi}{2}+\theta\right)=e^{i\left(\frac{\pi}{2}+\theta\right)}$ (c)	0.5
2	$z=\frac{e^{-i\theta}}{e^{i\theta}}=e^{-2i\theta}\Rightarrow\overline{z}=e^{2i\theta}$. (a)	0.5
3	$AB=\lvert z_B-z_A\rvert=\lvert 1+5i\rvert=\sqrt{26},\ AC=\lvert z_C-z_A\rvert=\lvert 3+2i\rvert=\sqrt{13}, BC=\lvert 2+3i\rvert=\sqrt{13}$. ABC is right and isosceles. (c)	1
4	$\lim_{x\to 0}\frac{\int_0^x \ln(t+1)dt}{e^x-1}=\frac{0}{0}=\lim_{x\to 0}\frac{\ln(x+1)}{e^x}=0$ (b)	1
5	$\int\cos^2 x dx=\int\frac{1+\cos 2x}{2}dx=\frac{x}{2}+\frac{1}{4}\sin 2x+c$ (c)	1

QII	Answers	M
1	(AB) : $\begin{cases}x=-2t+4\\ y=3t\\ z=0\end{cases}$ for $t=1: x=2, y=3$ and $z=0$, so E belongs to (AB).	0.5
2	$\overrightarrow{n_P}$ is parallel to $\overrightarrow{OB}\wedge\overrightarrow{AC}$, thus $\overrightarrow{n_P}(1;0;1)$. (P) : x + z + r = 0 ; E (2 ; 3 ; 0) is a point in (P) thus r = – 2 ; (P) : x +y – 2 =0.	0.5
3	$\overrightarrow{BC}(0;-6;4)$ and B is a point on (BC), (BC) : $x=0; y=-3m+6; z=2m$.	0.5
4	$x_F+z_F-2=0+2-2=0$ then F is a point in (P). for m = 1, F(0 ;3 ;2)is a point on (BC). $x_G=y_G=0$ and G is a point in (P), then $z_G=2$ and G (0 ;0 ;2). $y_H=z_H=0$ and H is a point in (P), then $x_H=2$ and H(2 ;0 ;0).	1
5a	**Geometrically** EF is parallel to (AC) which is parallel to (GH), so EF is parallel to (GH). Similarly (EH) is parallel to (OB) and (GF) is parallel to (OB), thus EH is parallel to (GF). Hence EFGH is a parallelogram.. Moreover $(OB)\perp(AC)\Rightarrow(EH)\perp(EF)$ then EFGH is a rectangle. **By calculation** $\overrightarrow{EF}(-2;0;2), \overrightarrow{HG}(-2;0;2)$ so $\overrightarrow{EF}=\overrightarrow{HG}$. $\overrightarrow{FG}(0;-3;0)$ and $\overrightarrow{FG}.\overrightarrow{HG}=0$ thus $FG\perp HG$. } so EFGH is a rectangle.	0.5
5b	$\overrightarrow{EG}\wedge\overrightarrow{n_P}(-3;4;3)$ is a direction vector of (T), thus x= -3λ+2 ; y = 4λ+3 ; z = 3 λ.	1

QIII	Answers	M
1	$p(A)=\frac{C_4^2\times C_1^1}{C_8^3}=\frac{3}{28}$. $p(B)=\frac{C_3^3}{C_8^3}+\frac{C_4^3}{C_8^3}=\frac{5}{56}$, $p(C)=\frac{C_5^3}{C_8^3}=\frac{5}{28}$, $p(D)=\frac{C_5^3}{C_8^3}+\frac{C_3^3}{C_8^3}=\frac{3}{7}$.	1.5
2	$p(\text{only one ball carries } 0)=\frac{C_4^1\times C_4^2}{C_8^3}=\frac{3}{7}$	0.5
3	$p(A/C)=\frac{C_1^1\times C_4^2}{C_5^3}=\frac{3}{5}$.	0.5
4a	X(Ω)={0; 50; 125}	0.5
4b	$p(X=50)=p(\{2,5,5\})=\frac{C_1^1\times C_3^2}{C_8^3}=\frac{3}{56}$. $p(X=125)=p(\{5,5,5\})=\frac{1}{56}$. $p(X=0)=\frac{C_4^1\times C_4^2+C_4^2\times C_4^1+C_4^3}{56}=\frac{52}{56}=\frac{13}{14}$ Or: $1-\frac{C_4^3}{56}=\frac{52}{56}$.	1

<table>
<tr><th>QIV</th><th>Answers</th><th>M</th></tr>
<tr><td>A1</td><td>$\lim_{x\to 0} g(x)=-\infty$, $\lim_{x\to+\infty} g(x)=+\infty$.</td><td>0.5</td></tr>
<tr><td>A2</td><td>$g'(x)=\frac{1}{x}+1$
<table>
<tr><td>x</td><td>0 $+\infty$</td></tr>
<tr><td>g'(x)</td><td>+</td></tr>
<tr><td>g(x)</td><td>$-\infty$ ↗ $+\infty$</td></tr>
</table></td><td>1</td></tr>
<tr><td>A3</td><td>g is continuous, strictly increasing on its domain, changing signs; so it vanishes once, thus g(x)= 0 has a a unique solution α. Moreover g(0.5)=–0.193 and g(0.6)= 0.089 thus 0.5 <α< 0.6.</td><td>1</td></tr>
<tr><td>A4</td><td>g(x) > 0 for x >α , g(x) <0 for x <α and g(x) = 0 for x = α.</td><td>0.5</td></tr>
<tr><td>B1</td><td>$\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(2x\ln x+x^2-2x\right)=0$. $\lim_{x\to+\infty} f(x)=+\infty$ since $\lim_{x\to+\infty}\left(x^2-2x\right)=+\infty$. f(e) = e^2.</td><td>1</td></tr>
<tr><td>B2</td><td>$f(\alpha)=\alpha\left(2\ln\alpha+\alpha-2\right)=\alpha\left(-2\alpha+\alpha-2\right)=-\alpha\left(\alpha+2\right)$</td><td>0.5</td></tr>
<tr><td>B3

B4</td><td>$f'(x)=2\ln x+x-2+x\left(\frac{2}{x}+1\right)=2(\ln x+x)=2g(x)$.
<table>
<tr><td>x</td><td>0   α $+\infty$</td></tr>
<tr><td>f'(x)</td><td>–   0   +</td></tr>
<tr><td>f(x)</td><td>0 ↘ $-\alpha(\alpha+2)$ ↗ $+\infty$</td></tr>
</table>

</td><td>1

1</td></tr>
<tr><td>B5</td><td>$u=\ln x,\ v'=x,\ u'=\frac{1}{x},\ v=\frac{x^2}{2}$. $\int x\ln x\,dx=\frac{x^2}{2}\ln x-\int\frac{x}{2}dx=\frac{x^2}{4}\left(2\ln x-1\right)$.
$\int_{0.5}^{1} x\ln x\,dx=\frac{\ln 2}{8}-\frac{3}{16}$. $A=-\int_{0.5}^{1} f(x)dx=-\frac{\ln 2}{4}+\frac{5}{6}=0.66u^2$.</td><td>1</td></tr>
<tr><td>B6</td><td>$F'(x)\le 0$ on $]0;1.37]$, then F is increasing on this interval.
$F'(x)>0$ on $]1.37;+\infty[$, then F is decreasing on this interval.</td><td>0.5</td></tr>
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