الدورة الإستثنائية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة أربع ساعات	عدد المسائل: ست

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I-(2 points)

In the following table, only one among the answers proposed to each question is correct.
Write down the number of each question and give, **with justification**, the answer that corresponds to it.

N°	Questions	Answers		
		a	b	c
1	If f is the function given by f(x) = lnx , then the domain of definition of $f\circ f$ is:	$]1;+\infty[$	$]0;+\infty[$	$]0;1[\cup]1;+\infty[$
2	The image under the inversion I(O ;1) of the circle (C) with center O and with radius 1 is:	(C)	a straight line	a circle passing through O
3	The nth derivative of the function given by $f(x)=\ln(x+1)$ is:	$\frac{(-1)^{n+1}n!}{(x+1)^n}$	$\frac{(-1)^{n-1}(n-1)!}{(x+1)^n}$	$\frac{(-1)^{n}(n-1)!}{(x+1)^n}$
4	$\int\frac{1}{x^2+4x+8}\,dx=$	$\arctan\frac{x+2}{2}+K$	$\frac{1}{2}\arctan\frac{x+2}{2}+K$	$\frac{1}{4}\arctan\frac{x+2}{2}+K$
5	The function F defined over IR by $F(x)=\int_1^{x^2}\frac{1}{(t^2+1)^2}\,dt$ is:	increasing over IR	decreasing over IR	not monotonous over IR
6	ABC is a triangle such that AB=5, BC=4 and AC=$\sqrt{21}$. Then the median AI is equal to:	2	$\frac{5+\sqrt{21}}{2}$	$\sqrt{19}$

II- (2 points)

In the space referred to a direct orthonormal system $(O\,;\vec{i},\vec{j},\vec{k})$,
consider the points A(1 ; –1 ;1), B(2 ; 0 ; 3), C(–1 ;1 ;1) and G(4 ;2 ;4) , and designate by (P) the plane that is determined by A, B and C.

1) a- Calculate the area of triangle ABC.

b- Calculate the volume of the tetrahedron GABC and deduce the distance from G to plane (P).

2) Prove that $x + y - z + 1 = 0$ is an equation of plane (P).

3) a- Show that the point F(2 ;0 ;6) is symmetric of G with respect to plane (P).

b- Give a system of parametric equations of the line (d) that is the symmetric of the line (AF) with respect to plane (P).

c- Prove that the line (AB) is a bisector of the angle $\widehat{FAG}$.

III- (3 points)

A- An urn U contains: five red balls each carrying the number 2 ,
and three white balls each carrying the number –3.
We draw simultaneously and at random 4 balls from the urn U.

Let X be the random variable that is equal to the sum of the numbers carried by the 4 drawn balls.

1) Determine the 4 possible values of X .

2) Determine the probability distribution of X .

B- In this part suppose that urn U contains 5 red balls and n white balls, (n > 1).

We draw simultaneously and at random two balls from the urn.

1) Calculate the probability of each of the following events :
E: « the two drawn balls are red»
F: « the two drawn balls have the same color».

2) a- Knowing that the two drawn balls have the same color , prove that the probability p that both are red is $p = \dfrac{20}{n^2 - n + 20}$.

b- How many white balls should the urn contain so that $p > \dfrac{10}{13}$?

IV- (3 points)

In the plane referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,)$, consider the hyperbola (H) with focus $F(2\,;\,0)$, directrix the line (d) with equation $x=\frac{1}{2}$ and with eccentricity 2.

1) a- Write an equation of (H) and determine its center .

   b- Determine the vertices and the asymptotes of (H) . Draw (H) .

2) Let (E) be the ellipse with focus F , center O and eccentricity $\frac{1}{2}$ . .

   a- Determine the vertices of (E) and draw (E) in the same system as (H) .

   b- Write an equation of (E) .

3) a- Verify that the point I(2 ; 3) is a point of intersection of (E) and (H) .

   b- Prove that the tangents at I to (E) and (H) are perpendicular.

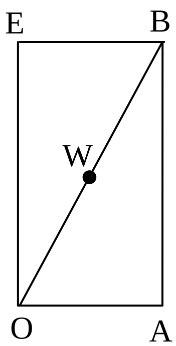
4) Let (D) be the region bounded by (E) , (H) and the two lines with equations $x=1$ and $x=2$.

   Calculate the volume of the solid generated by the rotation of (D) about the axis of abscissas.

V- (3 points)

Consider in an oriented plane the rectangle OABE such that OA = 2 and $\left(\overrightarrow{OA},\overrightarrow{OB}\right)=\frac{\pi}{3}\ (2\pi)$.

Designate by (C) the circle with diameter [OB] and center W.



Let S be the direct plane similitude with center O, ratio $\sqrt{3}$ and angle $\frac{\pi}{3}$.

A-

1) Let A' be a point on the semi-straight line [OB) such that OA' = $2\sqrt{3}$.

   Prove that A' is the image of A under S.

2) a- Verify that the triangle OAW is equilateral.
   b- Determine the image under S of triangle OAW.
   c- Construct then the circle (C'), the image of (C) under S.

B-

The complex plane is referred to a direct orthonormal system $(O\,;\,\vec{u},\,\vec{v})$ such that:

$z_A=2$ and $z_E=2\sqrt{3}i$.

1) Write the complex form of S.

2) Find the affix of W and that of W' the image of W under S.

3) Let f be the plane transformation with complex form $z'=iz+4+2i\sqrt{3}$.
   a- Show that f is a rotation whose angle and center H are to be determined.
   b- Verify that f(W') = W and determine $f\circ S(W)$.
   c- Determine the nature and the characteristic elements of $f\circ S$.

VI- (7 points)

A-

Let f be the function that is defined, on IR, by $f(x) = x^2e^{-2x}$, and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$ and deduce an asymptote of (C).

   b- Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to-\infty} \frac{f(x)}{x}$.

2) Calculate $f'(x)$ and set up the table of variations of f.

3) a- Draw the curve (C).

   b- Determine, according to the values of the real number m, the number of solutions of the equation : $me^{2x} - x^2 = 0$.

B-

Let (I_n) be the sequence defined, for every non-zero natural integer n, by $I_n = \int_0^1 x^n e^{-2x} dx$.

1) Prove that $0 \le I_n \le \frac{1}{n+1}$.
2) Prove that (I_n) is decreasing.
3) Deduce that (I_n) is convergent and specify its limit.
4) Use integration by parts to prove that $I_{n+1} = \frac{1}{2}\left[-\frac{1}{e^2} + (n+1)I_n\right]$.
5) Let h be the function defined on IR by $h(x) = -\frac{1}{4}(2x+1)e^{-2x}$. Calculate h'(x) then calculate I_1.
6) Deduce the area of the region that is bounded by the curve (C), the axis of abscissas and the two lines with equations x = 0 and x = 1.

C-

Let g be the function defined over $]0;+\infty[$ by $g(x) = \ln(f(x))$.

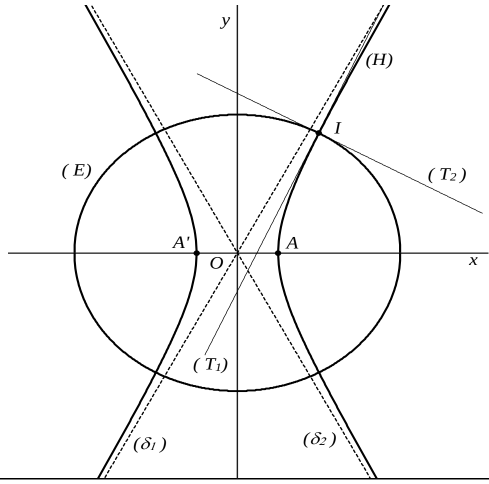
1) Calculate $\lim\limits_{x\to 0} g(x)$ and interpret the result graphically.
2) Calculate $\lim\limits_{x\to+\infty} g(x)$ and show that the representative curve of g has an asymptotic direction .
3) Set up the table of variations of g .
4) Draw the representative curve of g in a new system of axes.

QI	Answer	M
1	$f\circ f(x)=\ln(\ln(x)),\ x>0$ and $\ln x>0 \Rightarrow x>1.$ **(a)**	0.5
2	The circle (C) is the set of invariant points under I therefore the image of (C) is itself. **(a)**	0.5
3	$f'(x)=\dfrac{1}{x+1}$; $f''(x)=\dfrac{-1}{(x+1)^2}=\dfrac{(-1)^1\times 1!}{(x+1)^2}$ **(b)**	0.5
4	$\displaystyle\int\frac{1}{x^2+4x+8}dx=\frac{1}{4}\int\frac{1}{1+\left(\frac{x+2}{2}\right)^2}dx=\frac{1}{2}\arctan\left(\frac{x+2}{2}\right)+K$ **(b)**	1
5	$F'(x)=\dfrac{2x}{(x^2+1)^2}$; F is increasing over $[0;+\infty[$ and decreasing over $]-\infty;0]$ therefore it is not monotonous over IR. **(c)**	0.5
6	$AI^2=AB^2+BI^2-2AB\times BI\cos B$ $AC^2=AB^2+BC^2-2AB\times BC\cos B\Rightarrow\cos B=\dfrac{1}{2}, AI^2=19\Rightarrow AI=\sqrt{19}.$ **(c)**	1

QII	Answers	M
1a	$S=\dfrac{\Vert\vec{AB}\wedge\vec{AC}\Vert}{2}$; $\vec{AB}\wedge\vec{AC}=-4\vec{i}-4\vec{j}+4\vec{k}$; $S=2\sqrt{3}\ u^2.$	0.5
1b	$V=\dfrac{\lvert\vec{AG}.(\vec{AB}\wedge\vec{AC})\rvert}{6}=\dfrac{\lvert-12\rvert}{6}=2\ u^3$; $V=\dfrac{d\times S}{3}$,thus $d=\dfrac{3V}{S}=\dfrac{6}{2\sqrt{3}}=\sqrt{3}$	1
2	$\vec{AM}.(\vec{AB}\wedge\vec{AC})=0$; $-4(x-1)-4(y+1)+4(z-1)=0$; $x+y-z+1=0$. **OR** : The coordinates of A , B and C verify the equation of (P) .	0.5
3a	$\vec{FG}(2\ ;2\ ;-2)$; $\vec{N_P}(1\ ;1\ ;-1)$; $\vec{FG}=2\vec{N_P}$, so $(FG)\perp(P)$. I : midpoint of [FG] ; I(3 ; 1 ; 5) ; 3 + 1 – 5 + 1 = 0 thus I belongs to (P). **OR** : prove that (P) is the mediator plane of [FG] .	0.5
3b	(d) is the line (AG) : x = 3m + 1 ; y = 3m – 1 and z = 3m + 1.	0.5
3c	(AI) is a bisector of $F\hat{A}G$ since AF = AG and I is the midpoint of [FG], moreover, $\vec{AI}(2;2;4)$ and $\vec{AB}(1;1;2)$ hence $\vec{AI}=2\vec{AB}$,thus B belongs to (AI) .	1

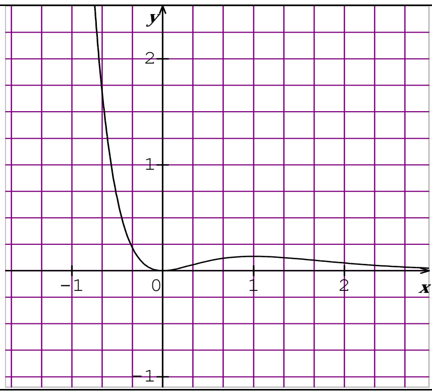
QIII	Answers	M
A1	The four values of X are 8, 3, -2 and -7	**0.5**
A2	x_i : -7 ; -2 ; 3 ; 8 p_i : $\dfrac{C_3^3\times C_5^1}{C_8^4}=\dfrac{5}{70}$; $\dfrac{C_3^2\times C_5^2}{C_8^4}=\dfrac{30}{70}$; $\dfrac{C_3^1\times C_5^3}{C_8^4}=\dfrac{30}{70}$; $\dfrac{C_5^4}{C_8^4}=\dfrac{5}{70}$	**2**
B1	$p(E)=\dfrac{C_5^2}{C_{n+5}^2}=\dfrac{20}{(n+4)(n+5)}.$ $p(F)=\dfrac{20}{(n+4)(n+5)}+\dfrac{n(n-1)}{(n+4)(n+5)}=\dfrac{n^2-n+20}{(n+4)(n+5)}$	1

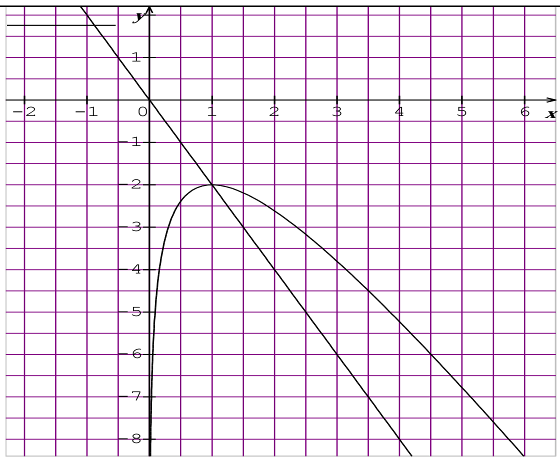
B2a	. $p\left(E/F\right)=\dfrac{p(E\cap F)}{p(F)}=\dfrac{p(E)}{p(F)}=\dfrac{20}{n^2-n+20}$. .	1.5
B2b	$p>\dfrac{10}{13}\Leftrightarrow n^2-n-6<0\Leftrightarrow -2<n<3$. The urn should contain 2 white balls(n>1).	1

QIV	Answers	M
1a	$M(x,y)\in(H)$; $MF^2=4d^2(M\,;(d))$; $x^2-4x+4+y^2=4x^2-4x+1$. (H) : $x^2-\dfrac{y^2}{3}=1$. The center of (H) is the origin $O(0\,;0)$.	1
1b	$a^2=1$, $b^2=3$ and $c^2=4$. Vertices of (H) are $A(1\,;0)$ and $A'(-1\,;0)$. Asymptotes of (H) : $(\delta_1): y=\sqrt{3}x$ and $(\delta_2): y=-\sqrt{3}x$. 	1
2a	$c=OF=2$ and $e=\dfrac{c}{a}=\dfrac{1}{2}$; then $a=4$ and $b^2=a^2-c^2=12$; that is $b=2\sqrt{3}$. The vertices of (E) are : $(4\,;0)$, $(-4\,;0)$, $(0\,;2\sqrt{3})$ and $(0\,;-2\sqrt{3})$.	1
2b	The focal axis of (E) being $x'x$, $a=4$ and $b=2\sqrt{3}$; then (E) : $\dfrac{x^2}{16}+\dfrac{y^2}{12}=1$.	0.5
3a	The coordinates of I verify the equation of (H) and that of (E).	0.5
3b	Tangent at I to (H) is (T_1) : $x x_I-\dfrac{y y_I}{3}=1$, $y=2x-1$. Tangent at I to (E) is (T_2) : $\dfrac{x x_I}{16}+\dfrac{y y_I}{12}=1$; (T_2): $y=-\dfrac{1}{2}(x+1)$;. $a_1\times a_2=-1$; then (T_1) and (T_2) are perpendicular . **Or** (T_1) is the internal bisector of $F\hat{I}F'$ and (T_2) is the external bisector of $F\hat{I}F'$. hence , they are perpendicular .	1
4	$V=\pi\int_1^2\left(Y_E^2-Y_H^2\right)dx=\pi\int_1^2\left[12-\dfrac{3}{4}x^2-3x^2+3\right]dx=\pi\int_1^2(15-\dfrac{15}{4}x^2)dx$ $=\pi\left[15x-\dfrac{15}{4}.\dfrac{x^3}{3}\right]_1^2=\dfrac{25}{4}\pi$ units of volume .	1

QV	Answers	M
A1	$\left(\overrightarrow{OA},\overrightarrow{OA'}\right)=\dfrac{\pi}{3}$ since $\left(\overrightarrow{OA},\overrightarrow{OB}\right)=\dfrac{\pi}{3}$ and $OA'=2\sqrt{3}=OA\sqrt{3}$, so $A'=S(A)$	0.5

A2a	$\left(\overrightarrow{OA},\overrightarrow{OW}\right)=\frac{\pi}{3}$ and $WO=WA$, thus OAW is equilateral .	0.5
A2b	OAW is a direct equilateral triangle, so its image by S is a direct equilateral triangle $OA'W'$ which locates W'.	0.5
A2c	(C') is the circle with center W' and radius $W'O=OA'=2\sqrt{3}$,passing in O	0.5
B1	$z'=\sqrt{3}e^{i\frac{\pi}{3}}z=\left(\frac{\sqrt{3}}{2}+\frac{3}{2}i\right)z$	0.5
B2	$z_W=\frac{z_B}{2}=1+i\sqrt{3}$, $z_{W'}=\sqrt{3}e^{i\frac{\pi}{3}}\left(1+\sqrt{3}i\right)=-\sqrt{3}+3i$	0.5
B3a	$z'=az+b$ where $a=i$, $\lvert a\rvert=1$ and $\arg a=\frac{\pi}{2}(2\pi)$ so f is a rotation with angle $\frac{\pi}{2}$ and center H whose affix is $z_H=\frac{b}{1-a}=2-\sqrt{3}+\left(2+\sqrt{3}\right)i$.	1
B3b	$iz_{W'}+4+2i\sqrt{3}=1+i\sqrt{3}=z_W$ thus $f(W')=W$ $f\circ S(W)=f\left(S(W)\right)=f(W')=W$	1
B3c	f∘S is the composite of two similitudes f(H ; 1 ; $\frac{\pi}{2}$) and S(O ; $\sqrt{3}$; $\frac{\pi}{3}$) thus f∘S is a similitude with ratio $\sqrt{3}$ and angle $\frac{\pi}{2}+\frac{\pi}{3}=\frac{5\pi}{6}$, and since (f∘S)(W) = W then W is the center of f∘S.	1

<table>
<tr><th>QVI</th><th>Answers</th><th>M</th></tr>
<tr><td>A1a</td><td>$\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\frac{x^2}{e^{2x}}=\lim\limits_{x\to+\infty}\frac{x}{e^{2x}}=0$; the x-axis is asymptote to (C)</td><td>0.5</td></tr>
<tr><td>A1b</td><td>$\lim\limits_{x\to-\infty} f(x)=+\infty$; $\lim\limits_{x\to-\infty}\frac{f(x)}{x}=\lim\limits_{x\to-\infty} xe^{-2x}=-\infty$. So vertical asymptotic direction</td><td>0.5</td></tr>
<tr><td>A2</td><td>f’(x) = 2x(1–x) e^{-2x}
<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>0</td><td></td><td>1</td><td></td><td>$+\infty$</td></tr>
<tr><td>f'(x)</td><td></td><td>–</td><td>0</td><td>+</td><td>0</td><td>–</td><td></td></tr>
<tr><td>f(x)</td><td>$+\infty$</td><td>↘</td><td>0</td><td>↗</td><td>e^{-2}</td><td>↘</td><td>0</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>A3a</td><td>
</td><td>1.5</td></tr>
<tr><td>A3b</td><td>$me^{2x}=x^2$; $m=x^2e^{-2x}$
For m < 0 no solution
For m = 0 one solution (double)
For $0<m<e^{-2}$; three solutions
For $m=e^{-2}$ a single solution and a double solution.
For $m>e^{-2}$ one solution</td><td>1</td></tr>
</table>

B1	$0\le x\le 1$, then $-2\le -2x\le 0$, $e^{-2}\le e^{-2x}\le 1$, so $e^{-2}x^n\le x^ne^{-2x}\le x^n$ and $0\le x^ne^{-2x}\le x^n$ hence $0\le I_n\le \int_0^1 x^n dx$ or $0\le I_n\le \left[\frac{x^{n+1}}{n+1}\right]_0^1$, and so $0\le I_n\le \frac{1}{n+1}$.	1
B2	$I_{n+1}-I_n=\int_0^1 x^ne^{-2x}(x-1)dx\le 0$ since $x^n\ge 0$, $e^{-2x}>0$ and $x-1\le 0$.	1
B3	(I_n) has a lower bound 0 and is decreasing, thus it is convergent, and since $\lim_{n\to+\infty}\frac{1}{n+1}=0$, therefore $0\le \lim_{n\to+\infty} I_n\le 0$, so $\lim_{n\to+\infty} I_n=0$	0.5
B4	$I_{n+1}=\int_0^1 x^{n+1}e^{-2x}dx$, $u=x^{n+1}$, $v'=e^{-2x}$, $u'=(n+1)x^n$, $v=-\frac{e^{-2x}}{2}$. $I_{n+1}=-\frac{1}{2}\left[x^{n+1}e^{-2x}\right]_0^1+\frac{1}{2}(n+1)\int_0^1 x^ne^{-2x}dx=\frac{1}{2}\left[-\frac{1}{e^2}+(n+1)I_n\right]$.	1
B5	$h'(x)=-\frac{1}{4}\left[2e^{-2x}-2e^{-2x}(2x+1)\right]=xe^{-2x}$ so $I_1=\int_0^1 xe^{-2x}dx=\left[h(x)\right]_0^1=\frac{1-3e^{-2}}{4}$.	1
B6	$A=\int_0^1 x^2e^{-2x}dx=I_2=\frac{1}{2}\left[-\frac{1}{e^2}+2I_1\right]=\frac{1}{2}\left[-\frac{1}{e^2}+\frac{1-3e^{-2}}{2}\right]=\frac{1-5e^{-2}}{4}u^2$	1
C1	$\lim_{x\to 0} g(x)=-\infty$ so the y-axis is an asymptote to the curve of g .	0.5
C2	$\lim_{x\to+\infty} g(x)=-\infty$, $g(x)=-2x+\ln(x^2)$; $\lim_{x\to+\infty}\frac{g(x)}{x}=\lim_{x\to+\infty}\left(-2+\frac{\ln(x^2)}{x}\right)=-2$ $\lim_{x\to+\infty}[g(x)+2x]=\lim_{x\to+\infty}\ln x^2=+\infty$ thus an asymptotic direction with slope -2.	1
C3	x: 0 || 1 | $+\infty$ $g'(x)$: || + | 0 | − $g(x)$: || $-\infty$ ↗ -2 ↘ $-\infty$	1
C4		1