وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد	الدورة الإستثنائية للعام 2011
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I (4 points)

A factory manufactures sports shoes.
The table below gives the number of pairs of shoes produced and the corresponding cost of production of a pair of shoes:

Number of pairs produced (in hundreds): x_i	1	2	3	4	5
Cost of a pair (in thousands of LL): y_i	60	55	45	25	18

1) Calculate $\overline{X}$ and $\overline{Y}$, the means of the two variables x and y respectively.
2) Represent, in an orthogonal system, the scatter plot of the points $(x_i; y_i)$ as well as the center of gravity $G(\overline{X}; \overline{Y})$.
3) Determine an equation of $(D_{y/x})$, the regression line of y in terms of x and draw this line in the preceding system.

4) In what follows, suppose that the factory decides to produce 350 pairs of shoes.
   a- Knowing that the fixed costs of this factory, during the production period, amount to 2 000 000 LL, estimate the total cost of producing these 350 pairs of shoes.
   b- Each pair of shoes is sold for 75 000 LL.
   Estimate the total profit achieved by this factory upon selling the 350 pairs of shoes.

II (4 points)

Rami inherits an amount of 20 000 000 LL.
He decides to use this amount to pay his monthly rent and to cover his monthly personal expenses.
In the first month, he spends 5% of this amount then he pays 300 000LL for the rent.
In the second month he spends 5% of the amount remaining with him from the previous month and then he pays 300 000LL for the rent, and so on for the following months.
Designate by U_n the amount left with him at the end of the nth month, so $U_0 = 20\,000\,000$.

1) Show that $U_{n+1} = 0.95U_n - 300\,000$.
2) For every natural integer n, let $V_n = U_n + 6\,000\,000$.
   a- Prove that (V_n) is a geometric sequence whose common ratio and first term are to be determined.
   b- Calculate V_n, then U_n, in terms of n.
3) At the end of which month, Rami would not be able, for the first time, to use this amount to pay the rent ?

III-(4 points)
In a game we use:

- A fair die;
- An urn U that contains 4 white and 3 red balls;
- An urn V that contains 17 white and 18 red balls.

A- The die is rolled.
If the six appears then a ball is drawn at random from the urn U, otherwise a ball is drawn at random from the urn V.

1) Prove that the probability that the drawn ball is white and from urn U is equal to $\frac{2}{21}$.
2) Calculate the probability of drawing a white ball.
3) Knowing that the drawn ball is white, calculate the probability that it is drawn from urn V.

B- In this part a new game consists of drawing balls, randomly and successively, from the urn U without replacement. This game ends once a white ball is drawn.

1) Calculate the probability that this game ends at the third draw.
2) Let X be the random variable that is equal to the number of draws needed for this game to end.
   a-Determine the four possible values of X.
   b-Determine the probability distribution of X.

IV (8 points)

A- Given below the table of variations of a function f defined on $]1; +\infty[$ by:
$f(x) = 3\ln(x-1) + ax + b$, where a and b are two real numbers.

x	1		2		$\frac{5}{2}$		$+\infty$
f '(x)	‖		+		0	−	
f(x)	‖	$-\infty$ ↗	3	↗	$2+3\ln\frac{3}{2}$	↘	$-\infty$

Use the information in this table to find the values of a and b.

B- Suppose that f is defined on $]1; +\infty[$ by $f(x) = 3\ln(x-1) - 2x + 7$, and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

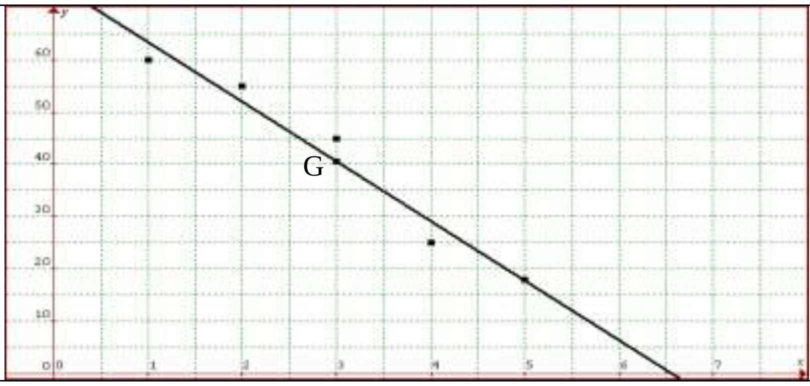
1) Write an equation of the tangent (T) to the curve (C) at the point of abscissa 2.
2) Draw (T) and (C).
3) The line with equation $y = 0.8x$ intersects (C) in two points. Show that the abscissa α of one of them is such that $3.4 < \alpha < 3.5$.

In all what follows, let $\alpha = 3.45$.

C- A factory produces a certain item whose unit price p is expressed in thousands of LL; $(2.5 \le p \le 5.5)$.
The demand D(p) and the supply S(p) of this product, expressed in hundreds of items, are given by:
$D(p) = 3\ln(p-1) - 2p + 7$ and $S(p) = 0.8p$.

1) Calculate the number of items demanded at a unit price of 2000LL.
2) Calculate the unit price for a supply of 320 items.
3) Give an economical interpretation for the value 3.45 of p.
   Calculate in this case the total revenue.
4) a- Calculate E(p), the elasticity of the demand with respect to the price.
   b- Is the demand elastic for p=3? Give an economical interpretation for E(3).

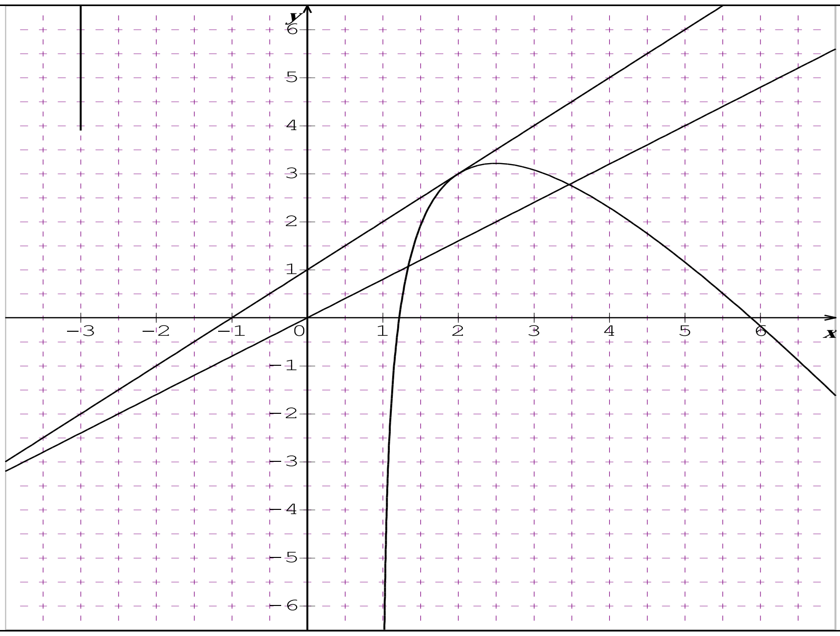
الدورة الإستثنائية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد مشروع معيار التصحيح	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات

QI	Answers	Mark
1	$\overline{X}=3 \quad \overline{Y}=40.6$	0.5
2		1.5
3	$y=-11.4x+74.8$. Graph: see figure.	1,5
4a	For x=3.5, y=34.9. The total cost is then 34900(350)+2 000 000=14 215 000LL.	1.5
4b	Profit = Revenue – Total cost = 75000(350) –14 215 000 = 12 035 000 The company gets a profit of 12035000LL for selling 350 pairs of shoes.	2

QII	Answers	Mark
1	$U_{n+1}=(1-0.5)U_n-300000=0.95U_n-300000.$	1
2a	$V_{n+1}=U_{n+1}+6000000=0.95U_n+5700000=0.95(U_n+6000000)=0.95V_n.$ (V_n) is a geometric sequence of common ratio 0.95 and first term 26000000.	2
2b	$V_n=26000000(0.95)^n$; $U_n=V_n-6000000=2000000\left(13(0.95)^n-3\right)$.	1.5
3	Rami will not be able to pay his rent if $0.95\,S \le 300000$. $2000000\left(13(0.95)^n-3\right)\le\frac{300000}{0.95} \Leftrightarrow 20\times0.95\left(13(0.95)^n-3\right)\le 3$ $\Leftrightarrow 13(0.95)^n \le \frac{3}{19}+3 \Leftrightarrow (0.95)^n \le \frac{60}{247} \Leftrightarrow n \ge 27.58$ so $n=28$ Hence Rami cannot pay his rent for the 29 th month.	2.5

QIII	Answers	Mark
A1	$P(W\cap U)=\frac{1}{6}\times\frac{4}{7}=\frac{2}{21}.$	1
A2	$P(W)=P(U\cap W)+P(V\cap W)=\frac{4}{42}+\frac{5}{6}\times\frac{17}{35}=\frac{4+17}{42}=\frac{1}{2}$	1.5
A3	$P\left(V/W\right)=\frac{P(V\cap W)}{P(W)}=\frac{17/42}{1/2}=\frac{17}{21}$	**1.5**
B1	$P(RRW)=\frac{3}{7}\times\frac{2}{6}\times\frac{4}{5}=\frac{4}{35}$	1
B2a	The values of X are: 1; 2 ; 3 ; 4.	0.5

B2b	$P(X=1)=P(W)=\frac{4}{7}$; $P(X=2)=P(RW)=\frac{3}{7}\times\frac{4}{6}=\frac{2}{7}$ $P(X=3)=\frac{3}{7}\times\frac{2}{6}\times\frac{4}{5}=\frac{4}{35}$; $P(X=4)=P(RRRW)=\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5}\times1=\frac{1}{35}$.	1.5

QIV	Answers	Mark
A	$f(2)=3 \Leftrightarrow 2a+b=3$ $f'(x)=\frac{3}{x-1}+a$ and $f'\left(\frac{5}{2}\right)=\frac{3}{\frac{3}{2}}+a=0 \Leftrightarrow 2+a=0 \Leftrightarrow a=-2$ and b =7. OR: use f(5/2)=2+3ln(3/2) to get $\frac{5}{2}a+b=2$	2
B 1	(T) : $y=f'(a)\times(x-a)+f(a)$ where a =2; f(1)=3 and f '(1) = 1 So, (T) : y = x –2+3 to get y = x+1.	1.5
B2		2
B3	f(3.4)=3ln(2.4)-6.8+7 = 2.82 > 0.8×3.4 = 2.72 f(3.5)=2.74 < 0.8×3.5, then 3.4< α <3.5.	1
C1	D(2)=3 , so the demand is 300 items.	1
C2	For a supply of 320items, S(p)=3.2 thus p =4 and the unit price is 4000LL.	1
C3	For p=3.45 we get D(p)=S(p), so the market is in equilibrium at a unit price of 3450 LL Revenue= $p\times D(p)$ = $3450\times D(3.45)\times 100$ = 961 951 LL.	2.5
C4a	E(p) = $-p\times\frac{D'(p)}{D(p)}=\frac{-p\times\frac{-2p+5}{p-1}}{3\ln(p-1)-2p+7}=\frac{2p^2-5p}{(p-1)(3\ln(p-1)-2p+7)}$	1.5
C4b	E(3) = 0.48<1, so the demand is inelastic. This signifies that at a price of 3000LL, if the price is increased by 1%, then the demand decreases by only 0.48%.	1.5