

الاسم:  
الرقم:

مسابقة في مادة الرياضيات  
المدّة: ساعة

عدد المسائل: ثلاث

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.  
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

### I- (5 points)

1) Solve the following system : 
$$\begin{cases} 2x + y = 80 \\ 1.7x + 2.7y = 105 \end{cases}$$

2) The price of two shirts and one belt is 80 thousands LL.

After a discount of 15% on the price of one shirt and a discount of 10% on the price of a belt, the price of two shirts and three belts becomes 105 thousands LL.

a- Show that this text is modeled by the system given in the 1<sup>st</sup> question.

b- Find the original price of one shirt and the original price of a belt.

c- Nadim bought four shirts and three belts after the discount.

How much did he pay ?

### II- (5 points)

The following table represents the number of boys and girls, in the LH and SE sections in a certain secondary school:

	LH	SE	Total
Boys	x		
Girls		y	26
Total	20		

The number of boys in SE section is twice that of boys in LH.

The number of girls in LH is 4 more than the number of boys in this section.

1) Determine x and y.

**In what follows suppose that:**  $x = 8$  and  $y = 14$ .

2) Copy and complete the table above.

3) One student is randomly selected from these two sections.

a- Calculate the probability that this student is a boy in LH section.

b- Knowing that the student is a boy, calculate the probability that he is in SE section.

c- Calculate the probability that the student is a girl or in LH section.

### III- (10 points)

Let  $f$  be the function defined on  $]-\infty; 1[ \cup ]1; +\infty[$  as  $f(x) = -x + 2 - \frac{1}{x-1}$ .

(C) is the graph of  $f$  in an orthonormal system.

1) Determine  $\lim_{\substack{x \rightarrow 1 \\ x < 1}} f(x)$  and  $\lim_{\substack{x \rightarrow 1 \\ x > 1}} f(x)$  and deduce an asymptote ( $\Delta$ ) to (C).

2) Determine  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow +\infty} f(x)$ .

3) Show that the line (d) with equation  $y = -x + 2$  is an asymptote to (C).

4) Determine the coordinates of point I the intersection of the asymptotes of (C).

5) Show that  $f'(x) = \frac{x(2-x)}{(x-1)^2}$  and set up the table of variations of  $f$ .

6) Draw ( $\Delta$ ), (d) and (C).

7) a- Prove that  $y = -2x + 3$  is an equation of the line passing through I and joining the two vertices of (C).

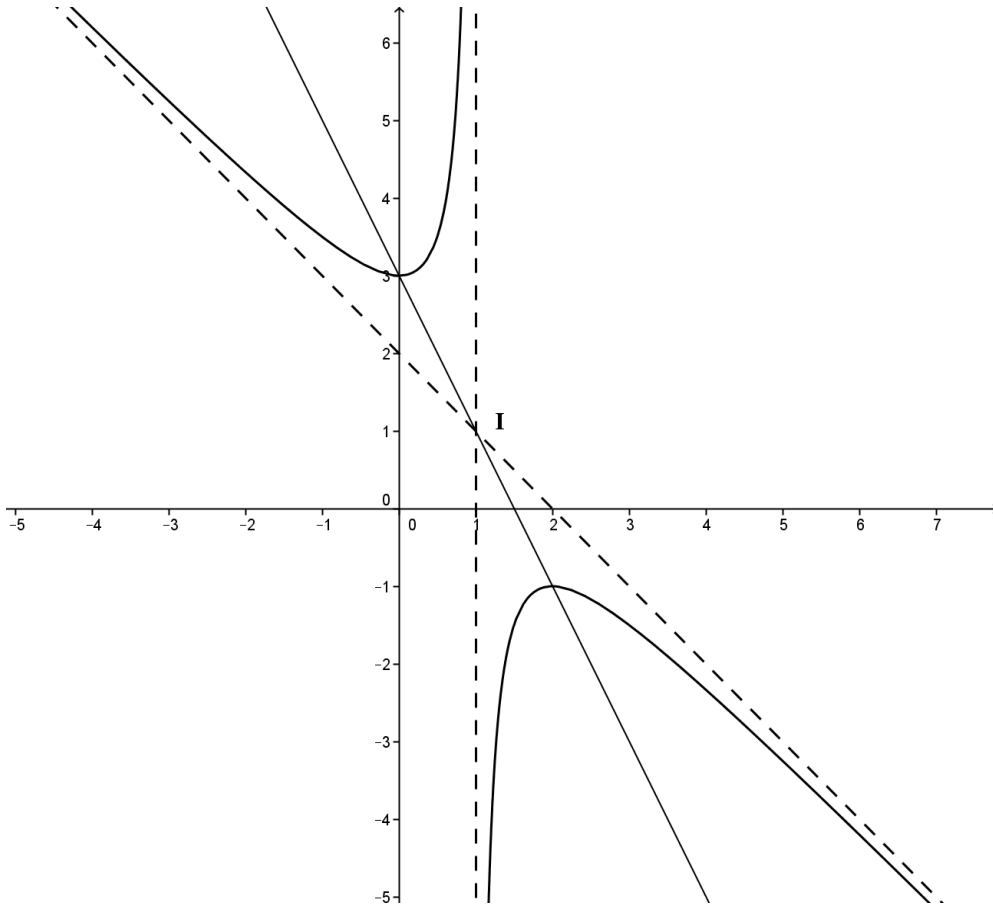
b- Solve the inequality:  $f(x) < -2x + 3$ .

QI	Answers	M
1	$x = 30$ and $y = 20$	1.5
2a	Let $x$ be the price of a shirt and $y$ the price of a belt. $x$ and $y$ verify the system	1.5
2b	The price of a shirt is 30 000 LL and the price of a belt is 20 000 LL	1
2c	$4(30\,000 \times 0.85) + 3(20\,000 \times 0.9) = 156\,000$ Nadim should pay 156 000 LL	1

QII	Answers	M																
1	$x + x + 4 = 20$ and $x + 4 + y = 26$ ; Hence $x = 8$ and $y = 14$ .	1																
2	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Hum</th> <th>Eco</th> <th>Total</th> </tr> </thead> <tbody> <tr> <th>Boys</th> <td>8</td> <td>16</td> <td>24</td> </tr> <tr> <th>Girls</th> <td>12</td> <td>14</td> <td>26</td> </tr> <tr> <th>Total</th> <td>20</td> <td>30</td> <td>50</td> </tr> </tbody> </table>		Hum	Eco	Total	Boys	8	16	24	Girls	12	14	26	Total	20	30	50	1
	Hum	Eco	Total															
Boys	8	16	24															
Girls	12	14	26															
Total	20	30	50															
3a	$P(B \cap H) = \frac{8}{50} = \frac{4}{25}$ .	1																
3b	$P(E/B) = \frac{16}{24} = \frac{2}{3}$ .	1																
3c	$P(G \cup H) = P(G) + P(H) - P(G \cap H) = \frac{26}{50} + \frac{20}{50} - \frac{12}{50} = \frac{17}{25}$ .	1																

QIII	Answers	M																														
1	$\lim_{x \rightarrow 1^+} f(x) = +\infty$ and $\lim_{x \rightarrow 1^-} f(x) = -\infty$ . The line with equation $x = 1$ is an asymptote to (C).	1.5																														
2	$\lim_{x \rightarrow -\infty} f(x) = +\infty$ and $\lim_{x \rightarrow +\infty} f(x) = -\infty$ .	1																														
3	$\lim_{x \rightarrow -\infty} (f(x) - (-x + 2)) = \lim_{x \rightarrow -\infty} \frac{-1}{x-1} = 0$ and $\lim_{x \rightarrow +\infty} (f(x) - (-x + 2)) = 0$ . The line with equation $y = -x + 2$ is an asymptote to (C).	1																														
4	For $x = 1, y = 1$ ; then $I(1; 1)$ .	1																														
5	$f'(x) = -1 + \frac{1}{(x-1)^2} = \frac{-x^2 + 2x - 1 + 1}{(x-1)^2} = \frac{x(2-x)}{(x-1)^2}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <td><math>-\infty</math></td> <td>0</td> <td>1</td> <td>2</td> <td><math>+\infty</math></td> </tr> </thead> <tbody> <tr> <th><math>f'(x)</math></th> <td></td> <td>— 0 +</td> <td style="border-left: 3px double black;"></td> <td>+ 0 —</td> <td></td> </tr> <tr> <th><math>f(x)</math></th> <td><math>+\infty</math></td> <td><math>\searrow</math></td> <td>3</td> <td><math>\nearrow</math></td> <td><math>+\infty</math></td> </tr> <tr> <td></td> <td></td> <td></td> <td style="border-left: 3px double black;"></td> <td><math>\nearrow</math></td> <td><math>-\infty</math></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td><math>\searrow</math></td> <td><math>-\infty</math></td> </tr> </tbody> </table>	x	$-\infty$	0	1	2	$+\infty$	$f'(x)$		— 0 +		+ 0 —		$f(x)$	$+\infty$	$\searrow$	3	$\nearrow$	$+\infty$					$\nearrow$	$-\infty$					$\searrow$	$-\infty$	1.5
x	$-\infty$	0	1	2	$+\infty$																											
$f'(x)$		— 0 +		+ 0 —																												
$f(x)$	$+\infty$	$\searrow$	3	$\nearrow$	$+\infty$																											
				$\nearrow$	$-\infty$																											
				$\searrow$	$-\infty$																											

6



2

a

The points with coordinates  $(1 ; 1)$ ,  $(0 ; 3)$  and  $(2 ; -1)$  belong to line  $y = -2x + 3$ .

1

7

b

$f(x) < -2x + 3$ ; that is (C) is below line  $y = -2x + 3$ :

so  $x \in ]-\infty; 0[ \cup ]1, 2[$

1