ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

The original price of a car is 15 000 000 LL.
The table below shows the selling price y_i , expressed in millions LL, after x_i years in use.

x_i	0	1	2	3	4	5	6
y_i	15	14	13.5	11	10	8.5	7

1) Represent graphically the scatter plot of the points (x_i, y_i) as well as the center of gravity G $(\overline{X}, \overline{Y})$ in a rectangular system.

2) Determine an equation of the regression line $D_{y/x}$ of y in terms of x; draw this line in the preceding system.

3) Calculate the linear correlation coefficient r and give an interpretation of the value thus obtained.

4) Determine the percentage decrease of the original price after four years in use.

5) Suppose that the above model remains valid until 10 years in use.
Estimate the number of years in use so that the price becomes, for the first time, less than or equal to 5 000 000 LL .

II- (4 points)

A touristic resort offers its customers two optional sports activities: swimming and tennis.
A tourist has the choice to practice only one activity or none.
Among the tourists spending a week of vacation at this resort,

- 40% are men
- Out of the men, 50% practice tennis and 10% do not practice any activity
- Out of the women, 60% practice swimming and 20% do not practice any activity.

A tourist is randomly chosen and interviewed. Consider the following events:
M: “The interviewed tourist is a man” ; W: “The interviewed tourist is a woman”
S: “The interviewed tourist practices swimming” ; T: “The interviewed tourist practices tennis”
R: “The interviewed tourist does not practice any activity”.

1) Calculate the probability $p(M \cap S)$ and verify that $p(S) = 0.52$.

2) The interviewed tourist does not practice swimming, calculate the probability that this tourist is a woman.

3) A group of tourists spend a week at this resort. Each tourist must pay 750 000 LL with an additional amount of 100 000 LL for tennis, and an additional amount of 50 000 LL for swimming.
Let X be the random variable equal to the sum paid by a tourist to spend a week at this resort.

a- Determine the probability distribution of X.
b- Calculate the average amount paid by a tourist for a week.
c- Knowing that the sum paid by a tourist for one weekis less than 830 000 LL, calculate the probability that the chosen tourist is a man who does not practice any activity.

III- (4 points)

Two cinemas **A** and **B** schedule two different movies each Saturday evening from 8 till 10.
Each spectator watches one of the two movies.
The same group of spectators visits the two cinemas all weeks.
85% of spectators who watch movie in **A** and 10% of those who watch movie in **B** one week, will watch a movie in cinema **A** the week after.
The first week, 70% of these spectators watch a movie in cinema **A**.
Denote by a_n and b_n the respective ratios of spectators at cinemas **A** and **B** during the n^{th} week.
Thus $a_1 = 0.7$, $b_1 = 0.3$ and for all nonzero narural numbers n: $a_n + b_n = 1$.

1) Verify that $a_2 = 0.625$.

2) For all nonzero natural numbers n, show that $a_{n+1} = 0.75a_n + 0.1$.

3) Let (u_n) be the sequence defined as $u_n = a_n - 0.4$.

a- Show that (u_n) is a geometric sequence whose common ratio and first term are to be determined.

b- Deduce that $a_n = 0.4 + 0.3 \times (0.75)^{n-1}$.

4) a- Prove that the sequence (a_n) is decreasing.

b- Will the ratio of spectators in cinema **A** be less than 0.3? Justify your answer.

IV- (8 points)

A-

Consider the function f defined on $[0;+\infty[$ as $f(x) = 1 + (8x-4)e^{-2x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim\limits_{x \to +\infty} f(x)$ and deduce an asymptote to (C).

2) Verify that $f'(x) = 16(1-x)e^{-2x}$ and set up the table of variations of f.

3) The curve (C) intersects the x- axis at the point with abscissa 0.28.

a- Draw the curve (C).

b- Deduce, according to the values of x, the sign of f(x).

B-

A factory produces daily a quantity x of a certain product.

The total cost C of production is modeled as: $C(x) = 0.8x + 1 + 4xe^{-2x}$ where $0 \le x \le 6$.

(x is expressed in hundreds of kg and C(x) in millions LL).

The whole production is sold at a price of 18 000 LL per kg.

1) Show that the profit function is given as $P(x) = x - 1 - 4xe^{-2x}$.

2) Verify that $P'(x) = f(x)$ and set up the table of variations of P.

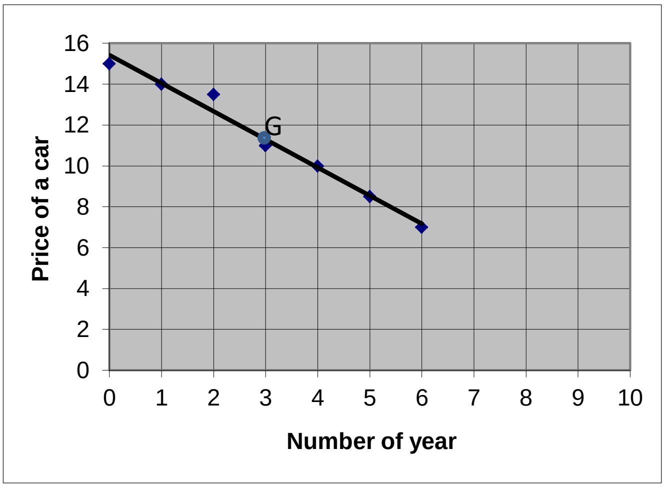
3) Prove that the equation P(x) = 0 has a unique solution α. Verify that α is in the interval [1.35 ; 1.37].

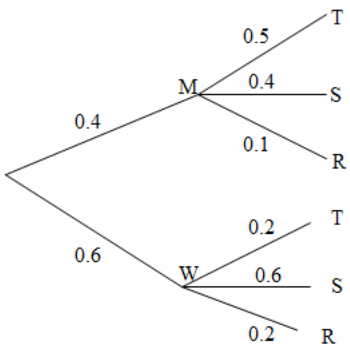
4) Take $\alpha = 1.36$.
Determine the break-even quantity of the factory.

5) Calculate, in kg, the quantity of production so that the marginal cost is equal to 1.8 million LL.

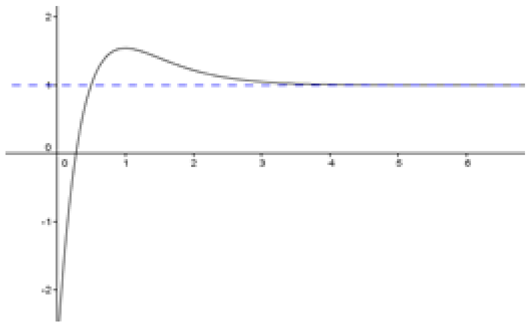
مشروع معيار التصحيح

مسابقة في مادة الرياضيات
المدة: ساعتان

<table>
<tr><th>I</th><th></th><th>Mark</th></tr>
<tr><td>1</td><td>$\overline{X} = 3$ and $\overline{Y} = 11.286$.

</td><td>1.5</td></tr>
<tr><td>2</td><td>$a = -1.375$ and $b = 15.4107$.
$D_{y/x}$: $y = -1.375x + 15.41$.</td><td>1.5</td></tr>
<tr><td>3</td><td>$r = -0.9907$ there is a strong negative correlation between x and y.</td><td>1.5</td></tr>
<tr><td>4</td><td>$\dfrac{10-15}{15} \times 100 = -33.34\%$. Hence the percentage decrease is 33.34%</td><td>1</td></tr>
<tr><td>5</td><td>$y \le 5$ gives $-1.375x + 15.41 \le 5$.
We get $x \ge 7.5$.
Hence after 8 years the selling price becomes less than or equal to 5 000 000 LL for the first time.</td><td>1.5</td></tr>
</table>

<table>
<tr><th>II</th><th>Answer</th><th>Mark</th></tr>
<tr><td>1</td><td>$P(M \cap S) = P(S / M) \times P(M) = \quad 0.4 \times 0.4 = 0.16$.
$P(S) = P(S \cap M) + P(S \cap W) = \quad 0.16 + 0.6 \times 0.6 = 0.52$

</td><td>1.5</td></tr>
<tr><td>2</td><td>$P(W/\overline{S}) = \dfrac{P(W \cap \overline{S})}{P(\overline{S})} = \dfrac{P(W) - P(W \cap S)}{P(\overline{S})} = \dfrac{0.6-0.36}{1-0.52} = 0.5$</td><td>1.5</td></tr>
<tr><td>3a</td><td>$P(X = 750\ 000) = P(R) = P(R \cap M) + P(R \cap W) = 0.4 \times 0.1 + 0.2 \times 0.6 = 0.16$
$P(X = 800\ 000) = P(S) = 0.52$ R
$P(X = 850\ 000) = P(T) = P(T \cap M) + P(T \cap W) = 0.5 \times 0.4 + 0.2 \times 0.6 = 0.32$
<table>
<tr><td>$X = x_i$</td><td>750 000</td><td>800 000</td><td>850 000</td><td>Total</td></tr>
<tr><td>$P(X = x_i)$</td><td>0.16</td><td>0.52</td><td>0.32</td><td>1</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>3b</td><td>$E(X) = 750\ 000 \times 0.16 + 800\ 000 \times 0.52 + 850\ 000 \times 0.32 = 808\ 000$.
The average amount paid by a tourist during one week is 808 000 LL</td><td>1</td></tr>
<tr><td>3c</td><td>$P\big((M \cap R)/(\le 830\ 000)\big) = \dfrac{P(M \cap R)}{P(S)+P(R)} = \dfrac{0.1 \times 0.4}{0.52+0.16} = \dfrac{1}{17}$</td><td>1.5</td></tr>
</table>

III	Answers	Mark
1	$a_2 = 0.85a_1 + 0.1b_1 = 0.85\times 0.7 + 0.1\times 0.3 = 0.625$	1
2	$a_{n+1} = 0.85a_n + 0.1b_n = 0.85a_n + 0.1(1-a_n) = 0.75a_n + 0.1.$	1
3a	$u_{n+1} = a_{n+1} - 0.4 = 0.75a_n + 0.1 - 0.4 = 0.75\times a_n - 0.3 = 0.75(a_n - 0.4) = 0.75u_n.$ The sequence (u_n) is geometric with common ratio r = 0.75 and first term $u_1 = 0.7 - 0.4 = 0.3.$	2
3b	$u_n = u_1 \times r^{n-1} = 0.3\times(0.75)^{n-1}$ and $a_n = u_n + 0.4 = 0.3\times(0.75)^{n-1} + 0.4$	1
4a	$a_{n+1} - a_n = (0.4 + 0.3\times 0.75^n) - (0.4 + 0.3\times 0.75^{n-1}) = -0.075\times 0.75^{n-1} < 0$ hence (a_n) is decreasing.	1
4b	$a_n < 0.3;\ 0.3\times(0.75)^{n-1} + 0.4 < 0.3;\ 0.3\times(0.75)^{n-1} < -0.1$ which is impossible; thus the ratio of spectators in cinema A will not be less than 0.3 **OR** The sequence (a_n) is decreasing but its limit is 0.4, hence the minimum is 0.4 .	1

IV	Answer	Mark
A1	$\lim\limits_{x\to+\infty} f(x) = 1$, y = 1 is the equation of a horizontal asymptote .	1
A2	f ´(x) = $8e^{-2x} - 2(8x-4)\,e^{-2x}$ = $16(1-x)\,e^{-2x}$. $\begin{array}{c\|ccccc} x & 0 & & 1 & & +\infty \\ \hline f'(x) & & + & 0 & - & \\ \hline f(x & -3 & \nearrow & 1.54 & \searrow & 1 \end{array}$	2
A3a		2
A3b	f(x) < 0 for 0 £ x < 0.28. f(x) > 0 for x > 0.28. f(x) = 0 for x = 0.28.	1
B1	R(x) = $\dfrac{18000\times x\times 100}{1000000}$ = 1.8x P(x) = R(x) – C(x) = 1.8x – 0.8x – 1 – $4xe^{-2x}$ = x – 1 – $4xe^{-2x}$	2
B2	P´(x) = 1 – $4e^{-2x}$ + $8xe^{-2x}$ = 1 + (8x – 4) e^{-2x} = f(x). $\begin{array}{c\|ccccc} x & 0 & & 0.28 & & 6 \\ \hline P'(x) & & - & 0 & + & \\ \hline P(x) & -1 & \searrow & -1.36 & \nearrow & 4.9 \end{array}$	1.5
B3	On the interval [0 ; 0.28]: P(x) = 0 has no solution since the maximum of P(x) is –1 < 0 On the interval [0.28 ; 6]: P is continuous, P is strictly increasing from –1.36 to +4.99. Thus the equation P(x) = 0 has a unique root α. P(1.35) × P(1.37) = –0.01 × 0.016 < 0, α ∈ [1.35 ; 1.37].	2
B4	The break even is attained when P(x) = 0; x = α = 1.36 The break even quantity of the factory is 136 kg.	1
B5b	$M_C(x)$ = 1.8 P(x) = R(x) – C(x), then P '(x) = 1.8 – C '(x) = 1.8 – $M_C(x)$ = 1.8 – 1.8 = 0; x = 0.28 The quantity produced so that the marginal cost is 1.8 million LL is 28 kg	1.5