وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	الدورة العادية الاستكمالية للعام 2011
عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points A(1; –1 ; 1), B(–2 ; 2 ; 1), $I\left(\frac{1}{2};-\frac{1}{2};1\right)$ and the line (d) defined by: $\begin{cases} x=-t+1 \\ y=-t \\ z=2t \end{cases}$ (t is a real number).

1) Write a system of parametric equations of line (AB).
2) Prove that (AB) and (d) intersect at I.
3) Show that an equation of the plane (P) determined by (AB) and (d) is $x+y+z-1=0$.
4) Consider the point $H\left(2;1;\frac{5}{2}\right)$.
   a- Prove that I is the orthogonal projection of H on (P).
   b- Verify that (AB) and (d) are perpendicular.
   c- K is a point on (d) such that IK = IA. Calculate the volume of the tetrahedron HABK.

II- (4 points)

An urn contains 4 black balls, 3 white balls and n red balls; ($n \geq 2$).

A-

In this part take **n = 2**.

We draw randomly and simultaneously 3 balls from the urn.

1) Calculate the probability of drawing three balls having the same color.
2) Designate by E the event:
   « Among the three drawn balls there are exactly two balls of the same color ».
   Prove that the probability P(E) is equal to $\frac{55}{84}$.

B-

In this part we draw randomly and simultaneously 2 balls from the **n+7** balls in the urn.

Designate by X the random variable equal to the number of red balls obtained among the three drawn.

1) Prove that $P(X=2)=\frac{n(n-1)}{(n+6)(n+7)}$.
2) Determine the probability distribution of X.
3) Calculate n so that the mathematical expectation E(X) is equal to 1.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B with affixes $z_A = 1$ and $z_B = e^{i\frac{\pi}{4}}$. Designate by E the midpoint of segment [AB].

1) Verify that $z_E = \dfrac{2+\sqrt{2}}{4} + i\dfrac{\sqrt{2}}{4}$.

2) a- Verify, for every real number θ, that $1+e^{i\theta} = e^{i\frac{\theta}{2}}\left(e^{i\frac{\theta}{2}} + e^{-i\frac{\theta}{2}}\right)$.

b- Show that $z_E = \left(\cos\dfrac{\pi}{8}\right)e^{i\frac{\pi}{8}}$.

c- Deduce from the preceding results the exact value of $\cos\dfrac{\pi}{8}$.

3) Let M be a variable point with affix z such that $\left|2z-\sqrt{2}-i\sqrt{2}\right| = 2$.

Prove that M describes a circle (C) and verify that O belongs to (C).

IV- (8 points)

Consider the function f defined on $\mathbb{R}$ by $f(x) = \dfrac{e^x}{e^x+1}$. Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Deduce the asymptotes of the curve (C).

2) Calculate $f'(x)$ and set up the table of variations of f.

3) Show that $f''(x) = \dfrac{e^x(1-e^x)}{(1+e^x)^3}$. Prove that (C) has a point of inflection I to be determined.

4) Write an equation of the tangent (T) to (C) at the point I.

5) Draw (T) and (C).

6) The function f has on $\mathbb{R}$ an inverse function g.

a- Draw the representative curve (G) of g in the given system.

b- Verify that $g(x) = \ln\left(\dfrac{x}{1-x}\right)$.

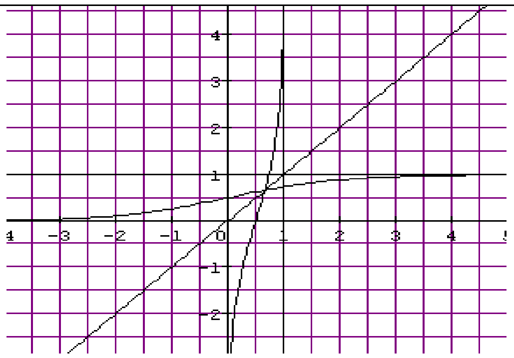
c- (G) and (C) intersect at a point with abscissa α. Calculate, in terms of α, the area of the region bounded by (C), (G) and the two axes of coordinates.

مشروع معيار التصحيح	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	الدورة العادية الاستكمالية للعام 2011 مسابقة في مادة الرياضيات

QI	Answers	M
1	$\overrightarrow{AB}(-3,3,0)$ so $\vec{V}(1,-1,0)$ is a direction vector of (AB) then : $x=m+1, y=-m-1, z=1$.	0.5
2	For $t=\frac{1}{2}$, I belongs to (d). $\vec{AI}\left(-\frac{1}{2};\frac{1}{2};0\right)$; $\vec{BI}\left(\frac{5}{2};-\frac{1}{2};0\right)$; hence $\vec{BI}=-5\,\vec{AI}$ and B , A and I are collinear. I belongs to (AB) and (d) with $A \not\in (d)$ therefore (AB) and (d) intersect at I. OR: For $m=-\frac{1}{2}$; I belongs to (AB) where (d) and (AB) are distinct.	0.5
3	The coordinates of A and B verify the given equation since: $x_A+y_A+z_A-1=1-1+1-1=0$ and $x_B+y_B+z_B-1=0$. Moreover $(d)\subset(P)$ since the coordinates of the point $(-t+1;-t;2t)$ verify the given equation for every t.	1
4a	$\vec{IH}\left(\frac{3}{2};\frac{3}{2};\frac{3}{2}\right)$ and $\vec{n}_P(1;1;1)$ are collinear. And I belongs to plane (P), hence I is the orthogonal projection of H on (P).	0.5
4b	$\vec{AB}.\vec{V_d}=3-3+0=0$, then (d) and (AB) are perpendicular at I.	0.5
4c	Volume = $\frac{Area(ABK)\times IH}{3}$. Area of KAB $=\frac{IK\times AB}{2}=\frac{IA\times AB}{2}=\frac{\frac{\sqrt{2}}{2}\times3\sqrt{2}}{2}=\frac{3}{2}u^2$. Therefore V = $\frac{3\times3\sqrt{3}}{2\times3\times3}=\frac{3\sqrt{3}}{4}u^3$. (**Or**: Find the coordinates of point K (two possibilities) then use $V=\frac{\left\lvert\overrightarrow{AB}\cdot\left(\overrightarrow{AH}\wedge\overrightarrow{AK}\right)\right\rvert}{6}$	1

<table>
<tr><th>QII</th><th>Answers</th><th>M</th></tr>
<tr><td>A1</td><td>P(3 balls of same color) = P(3B) + P(3W) = $\frac{C_4^3+C_3^3}{C_9^3}=\frac{5}{84}$</td><td>0.5</td></tr>
<tr><td>A2</td><td>P(E) = P(2 balls of same color)=
$P(2R,1\overline{R})+P(2B,1\overline{B})+P(2W,1\overline{W})=\frac{C_2^2\times C_7^1+C_4^2\times C_5^1+C_3^2\times C_6^1}{C_9^3}=\frac{55}{84}$</td><td>1</td></tr>
<tr><td>B1</td><td>p(X=2) =p(2 red) = $\frac{C_n^2}{C_{n+7}^2}=\frac{n!}{2!(n-2)!}\times\frac{2!(n+5)!}{(n+7)!}=\frac{n(n-1)}{(n+7)(n+6)}$</td><td>1</td></tr>
<tr><td>B2</td><td>p(X= 0) = p(0 red)
$=\frac{C_7^2}{C_{n+7}^2}=\frac{7\times6}{(n+7)(n+6)}$;
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>p</td><td>$\frac{42}{(n+7)(n+6)}$</td><td>$\frac{14n}{(n+7)(n+6)}$</td><td>$\frac{n^2-n}{(n+7)(n+6)}$</td></tr>
</table>
p(X=1)= $P(1R,1\overline{R})=\frac{C_n^1\times C_7^1}{C_{n+7}^2}=\frac{7n\times2}{(n+7)(n+6)}$;</td><td>1</td></tr>
<tr><td>B3</td><td>E(X)= $\frac{14n+2n^2-2n}{n^2+13n+42}=1$ then $n^2-n-42=0$, so n = 7 or n = –6. Therefore n = 7</td><td>0.5</td></tr>
</table>

QIII	Answers	M
1	$z_E = \frac{z_A + z_B}{2} = \frac{1}{2}\left(1+\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) = \frac{1}{2}\left(1+\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = \frac{2+\sqrt{2}}{4}+i\frac{\sqrt{2}}{4}$.	0.5
2a	$e^{i\frac{\theta}{2}}\left(e^{i\frac{\theta}{2}}+e^{-i\frac{\theta}{2}}\right) = e^{i\left(\frac{\theta}{2}+\frac{\theta}{2}\right)} + e^{i\left(\frac{\theta}{2}-\frac{\theta}{2}\right)} = e^{i\theta} + e^{i(0)} = 1 + e^{i\theta}$.	0.5
2b	$z_E = \frac{1}{2}\left(1+e^{i\frac{\pi}{4}}\right) = \frac{1}{2}e^{i\frac{\pi}{8}}(e^{i\frac{\pi}{8}}+e^{-i\frac{\pi}{8}}) = e^{i\frac{\pi}{8}}\frac{1}{2}\left(2\cos\frac{\pi}{8}\right) = \left(\cos\frac{\pi}{8}\right)e^{i\frac{\pi}{8}}$.	1
2c	$\cos\frac{\pi}{8}e^{i\frac{\pi}{8}} = \frac{2+\sqrt{2}}{4}+\frac{\sqrt{2}}{4}i$, $\cos^2\frac{\pi}{8}+i\cos\frac{\pi}{8}\sin\frac{\pi}{8} = \frac{2+\sqrt{2}}{4}+\frac{\sqrt{2}}{4}i$ hence $\cos\frac{\pi}{8} = +\sqrt{\frac{2+\sqrt{2}}{4}}$ $\left(\cos\frac{\pi}{8} > 0\right)$	1
3	**$\lvert 2Z - 2Z_B \rvert = 2$; $\lvert Z - Z_B \rvert = 1$** hence BM=1 , and M describes the circle with center B and radius 1 . Since BO=1 then O belongs to (C).	1

<table>
<tr><th>QIV</th><th colspan="3">Answers</th><th>M</th></tr>
<tr><td>1</td><td colspan="3">$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\frac{e^x}{e^x+1} = \frac{0}{0+1} = 0$. the line with equation y = 0 is an asymptote to (C).
$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}\frac{e^x}{e^x+1} = \frac{+\infty}{+\infty} = \lim_{x\to+\infty}\frac{e^x}{e^x} = 1$. the line with equation y = 1 is an asymptote to (C).</td><td>1</td></tr>
<tr><td>2</td><td colspan="3">$f'(x) = \frac{e^x(e^x+1)-e^x(e^x)}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2} > 0$
$\begin{array}{c|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & 0 & \nearrow & 1 \end{array}$</td><td>1</td></tr>
<tr><td>3</td><td colspan="3">$f''(x) = \frac{e^x(e^x+1)^2 - 2e^x(e^x+1)e^x}{(1+e^x)^4} = \frac{e^x(1-e^x)}{(1+e^x)^3}$, so f "(x) vanishes while changing sign at x = 0. Hence the point I(0,1/2) is a point of inflection.</td><td>1.5</td></tr>
<tr><td>4</td><td colspan="3">$f'(0) = \frac{1}{4}$; $y - \frac{1}{2} = \frac{x}{4}$; $y = \frac{x}{4} + \frac{1}{2}$</td><td>0.5</td></tr>
<tr><td rowspan="2">5</td><td rowspan="2"> </td><td rowspan="2">1</td><td>6a | (G) is symmetric of (C) with respect to the line with equation y = x.</td><td>1</td></tr>
<tr><td>6b | $e^x = \frac{y}{1-y}$ hence $x = \ln\left(\frac{y}{1-y}\right)$;
so $g(x) = \ln\left(\frac{x}{1-x}\right)$</td><td>1</td></tr>
<tr><td>6c</td><td colspan="3">Using the symmetry with respect to the first bisector, the area of the region is twice the area of the region bounded by (C) and the first bisector,
$A = 2\int_0^{\alpha}\left(\frac{e^x}{e^x+1} - x\right)dx = 2\left[\ln\left(e^x+1\right) - \frac{1}{2}x^2\right]_0^{\alpha} = \left(2\ln\left(e^{\alpha}+1\right) - 2\ln 2 - \alpha^2\right)u^2$</td><td>1</td></tr>
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