وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	دورة 2011 العادية
عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the line (d) defined by:

$$(d): \begin{cases} x = t-1 \\ y = t+3 \\ z = t+1 \end{cases} \quad \text{(t is a real parameter).}$$

1) Determine an equation of the plane (Q) determined by the point O and the line (d).
2) a- Calculate the coordinates of point H, the orthogonal projection of O on (d).
   b- Show that the distance from point O to line (d) is equal to $2\sqrt{2}$.
3) (P) is the plane with equation $(2m-1)x - my + (1-m)z + 6m - 2 = 0$, where m is a real parameter.
   a- Verify that H belongs to (P).
   b- Show that (P) contains the line (d).
   c- Calculate, in terms of m, the distance from point O to (P).
4) Determine m so that the line (OH) is perpendicular to plane (P).

II- (4 points)

In a school, each student of the GS and LS sections practices only one sport. The students are distributed as shown in the following table:

	Football	**Basketball**	**Tennis**
LS	1	6	3
GS	4	4	2

The name of each student is written on a separate card, where all the 20 cards used are identical.

A- The cards carrying the names of the LS students are placed in a box B_1 and those carrying the names of the GS students are placed in another box B_2.
The school principal chooses at random a box and then draws, randomly and simultaneously, two cards from the chosen box.
Consider the following events:
E : The chosen box is B_1
S : The two drawn cards carry the names of two students who practice the same sport.

1) a- Show that the probability $p(S/E)$ is equal to $\frac{2}{5}$ and deduce $p(E \cap S)$.
   b- Prove that $p(S) = \frac{31}{90}$.
2) Knowing that the two selected cards carry the names of two students who practice different sports, what is the probability that these two students are in the LS section?

B- Assume, in this part, that the 20 cards carrying the names of the students are placed together in one box B.
Three cards are drawn simultaneously and at random from this box.

1) Prove that the probability that the three drawn cards carry the names of three students, who practice the same sport, is $\frac{7}{57}$.
2) Let X be the random variable equal to the number of sports practiced by the three students whose names are written on the three drawn cards. Determine the probability distribution of X.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points M and M' with respective affixes z and z' such that $z' = (1 + i\sqrt{3})z - 2$.

1) In this part, suppose that $z = 1+i$.

a- Show that the point M' belongs to the line with equation $y = -x$.

b- Show that triangle OMM' is right at O.

2) Let I be the point with affix -2.

a- Verify that $|z'+2| = 2|z|$.

b- Prove that as M describes the circle with center O and radius 2, M' describes a fixed circle whose center and radius are to be determined.

3) Suppose that $z = x+iy$ and $z' = x'+iy'$ where x, y, x' and y' are real numbers.

a- Express x' and y' in terms of x and y.

b- Show that if M describes the line with equation $y = -x\sqrt{3}$, then M' describes a straight line to be determined.

IV- (8 points)

Let f be the function defined, on$]-\infty ; +\infty[$, by $f(x) = x + 2 - \dfrac{3}{1+e^x}$.

(C) is the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Calculate $\lim\limits_{x \to -\infty} f(x)$; Show that the line (d_1) with equation $y = x - 1$ is an asymptote to (C) and specify the position of (d_1) relative to (C).

b- Calculate $\lim\limits_{x \to +\infty} f(x)$; Show that the line (d_2) with equation $y = x + 2$ is an asymptote to (C) and specify the position of (d_2) relative to (C).

2) Prove that the point $I\left(0 ; \dfrac{1}{2}\right)$ is a center of symmetry of (C).

3) Show that f is strictly increasing on $]-\infty ; +\infty[$ and set up its table of variations.

4) Draw (d_1), (d_2) and (C).

5) a - Verify that $f(x) = x + 2 - \dfrac{3e^{-x}}{1+e^{-x}}$.

b - Calculate the area $A(\lambda)$ of the region bounded by the curve (C), the asymptote (d_2) and the two lines with equations $x = 0$ and $x = \lambda$, where $\lambda > 0$, then calculate $\lim\limits_{\lambda \to +\infty} A(\lambda)$.

6) Designate by g the inverse function of f on$]-\infty ; +\infty[$; (G) is the representative curve of g.

a- Verify that E (1+ln2; ln2) is a point on (G).

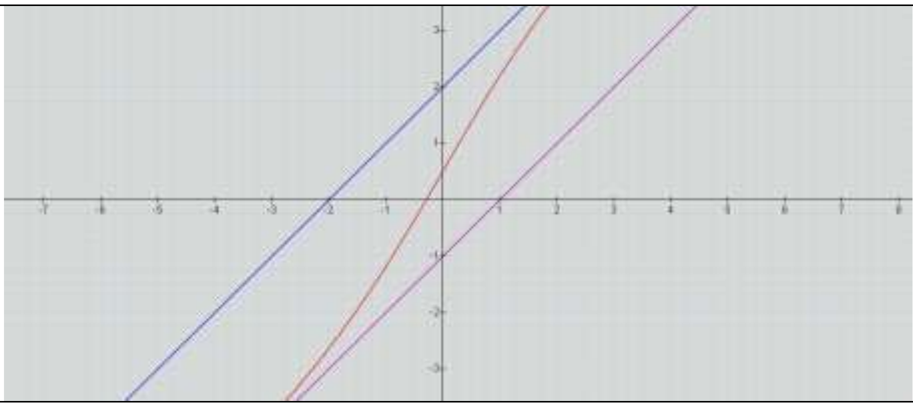
b- Calculate the slope of the tangent to (G) at E.

Q1	Solution	G
1	For t = 0, A (–1, 3, 1) is on (d). Let M(x, y, z) be a point on (Q) ; then $\overrightarrow{OM}\cdot\left(\overrightarrow{OA}\wedge\overrightarrow{v_d}\right)=0 \Leftrightarrow$ (Q) : x + y -2z = 0	0.5
2a	H (x_H, y_H, z_H) is a point on (d) such that (OH) is perpendicular to (d) ; then $\begin{cases} H\in(d)\\ \overrightarrow{OH}\cdot\overrightarrow{v_d}=0\end{cases}$ t–1+ t + 3 + t + 1= 0, therefore t = –1 and H(– 2, 2, 0).	1
2b	$OH=\sqrt{(-2)^2+2^2}=2\sqrt{2}$.	0.5
3a	(2m – 1) (– 2) – 2m + 0 +6m – 2 = 0 The coordinates of H satisfy the equation of (P) ; hence H belongs to (P).	0.5
3b	(2m–1) (t–1) – m(t + 3) + (1– m) (t + 1) + 6m–2 = 2 m t – 2m – t + 1 – m t – 3 m + t +1– m t – m + 6 m – 2 = 0. Thus, (d) lies in (P). **OR** A belongs to (P) and H belongs to (P).	0.5
3c	$d=\dfrac{\lvert 6m-2\rvert}{\sqrt{(2m-1)^2+m^2+(1-m)^2}}=\dfrac{\lvert 6m-2\rvert}{\sqrt{6m^2-6m+2}}$.	0.5
4	(OH) is perpendicular to (P), then d= OH, so $2\sqrt{2}=\dfrac{\lvert 6m-2\rvert}{\sqrt{(2m-1)^2+m^2+(1-m)^2}}$, thus $12(m^2-2m+1)=0$, and consequently m = 1.	0.5

Q2	Solution	G
A1 a	p(S / E) =p(both basketball or both tennis)= $\dfrac{C_6^2+C_3^2}{C_{10}^2}=\dfrac{2}{5}$ $p(E\cap S)=p(E)\times p(S/E)=\dfrac{1}{2}\times\dfrac{2}{5}=\dfrac{1}{5}$.	1
A1 b	$P(S)=P(S\cap E)+P(S\cap\overline{E})=\dfrac{1}{5}+\dfrac{1}{2}\times\dfrac{C_4^2+C_4^2+C_2^2}{C_{10}^2}=\dfrac{1}{5}+\dfrac{13}{90}=\dfrac{31}{90}$.	1
A2	$p(E/\overline{S})=\dfrac{p(E\cap\overline{S})}{p(\overline{S})}=\dfrac{p(E)-P(E\cap S)}{1-p(S)}=\dfrac{\frac{1}{2}-\frac{1}{5}}{1-\frac{31}{90}}=\dfrac{\frac{3}{10}}{\frac{59}{90}}=\dfrac{27}{59}$.	0.5
B1	P(3 students practice the same sport)= $\dfrac{C_5^3+C_{10}^3+C_5^3}{C_{20}^3}=\dfrac{7}{57}$	0.5
B2	X(Ω) = { 1, 2 , 3} since the three students may practice the same sport , two different or three different sports. $p(X=3)=\dfrac{C_5^1\times C_{10}^1\times C_5^1}{C_{20}^3}=\dfrac{25}{114}$; $P(X=1)=\dfrac{7}{57}$; $P(X=2)=1-p(X=1)-p(X=3)=\dfrac{25}{38}$.	1

Q3	Solution	G
1a	$z'=-1-\sqrt{3}+(1+\sqrt{3})i$. Then y' = - x', hence M' belongs to the line with equation y = – x.	0.5
1b	M belongs to the line with equation y = x and M' belongs to the line of equation y = -x, then triangle OMM' is right at O. **OR :** $\overrightarrow{OM}.\overrightarrow{OM'}=0$,so (OM) and (OM') are perpendicular. **OR :** $MM'^2=OM^2+OM'^2$	0.5

2a	$z'+2=\left(1+i\sqrt{3}\right)z$; thus $\lvert z'+2\rvert=\lvert 2z\rvert=2\lvert z\rvert$.	0.5
2b	M belongs to a circle with center O and radius 2, then $\lvert z\rvert=2$; thus $\lvert z'+2\rvert=4$. As a result $\lVert\overrightarrow{IM'}\rVert$= 4, so M' describes the circle of center I and radius 4.	1
3a	x'= x- $\sqrt{3}$ y-2 and y' = y + x $\sqrt{3}$.	0.5
3b	y + x $\sqrt{3}$ = 0, then y' = 0, therefore z' is real, so M' describes the axis of abscissas.	1

Q4	Solution	G
1a	$\lim\limits_{x\to-\infty} f(x)=-\infty$ and $\lim\limits_{x\to-\infty}[f(x)-(x-1)]=\lim\limits_{x\to-\infty}[3-\frac{3}{1+e^{x}}]=\lim\limits_{x\to-\infty}\frac{3e^{x}}{1+e^{x}}=0$ then the straight line (d_1) with equation y = x -1 is an asymptote to (C). $f(x)-(x-1)=\frac{3e^{x}}{1+e^{x}}$ >0 ; then (C) is above (d_1).	1
1b	$\lim\limits_{x\to+\infty} f(x)=+\infty$ and $\lim\limits_{x\to+\infty}[f(x)-(x+2)]=\lim\limits_{x\to+\infty}-\frac{3}{1+e^{x}}=0$. then the straight line (d_2) with equation y = x + 2 is an asymptote to (C). $f(x)-(x+2)=\frac{-3}{1+e^{x}}<0$; then (C) is below ($d_2$).	1
2	0 is the center of the domain of definition of f and f(2a-x) +f(x) = f(–x) + f(x) = 1, then I is a center of symmetry of (C).	1
3	$f'(x)=1+\frac{3e^{x}}{(1+e^{x})^{2}}$ >0 ; for all x in]– ∞ ; +∞[So f is strictly increasing . x: $-\infty$ … $+\infty$ f'(x): + f(x): $-\infty$ ↗ $+\infty$	1
4		1
5a	$f(x)=x+2-\frac{3}{1+e^{x}}=x+2-\frac{3}{1+e^{x}}\times\frac{e^{-x}}{e^{-x}}=x+2-\frac{3e^{-x}}{1+e^{-x}}$.	0.5
5b	$A(\lambda)=\int_0^{\lambda}[(x+2)-(x+2-\frac{3e^{-x}}{1+e^{-x}})]dx=\int_0^{\lambda}\frac{3e^{-x}}{1+e^{-x}}dx=\left[-3\ln(1+e^{-x})\right]_0^{\lambda}$ $=-3\ln(1+e^{-\lambda})+3\ln 2$ thus, $\lim\limits_{\lambda\to+\infty}A(\lambda)=3\ln 2$.	1.5
6a	$f(\ln 2)=\ln 2+2-\frac{3}{1+e^{\ln 2}}=\ln 2+2-1=1+\ln 2$ then, g (1+ln2) = ln 2 and E (1+ ln 2;ln 2) is a point of (G).	0.5
6b	The slope of the tangent to (G) at E is: $g'(1+\ln 2)=\frac{1}{f'(\ln 2)}=\frac{1}{1+\frac{3\times 2}{(2+1)^{2}}}=\frac{3}{5}$.	0.5