الدورة العادية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة أربع ساعات	عدد المسائل:ست.

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات

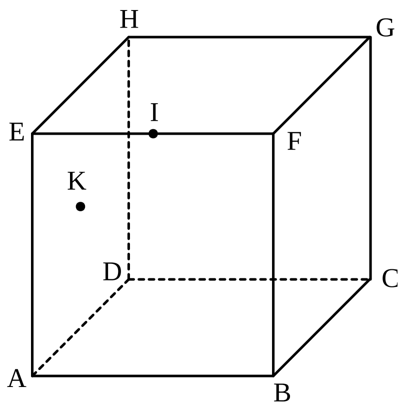
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I- (2 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of each question and give, ***with justification***, the answer corresponding to it.

		Answers		
N°	**Questions**	**a**	**b**	**c**
1	$\int_{-a}^{a}\left(x^5-\sin x\right)dx=$	$\frac{a^6}{6}$	$\frac{a^6}{24}$	0
2	$\arg\left(\frac{e^{i\pi}}{i}\right)=$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π
3	The roots of the equation $z+\lvert z\rvert^2=3+i$ are:	$1+i$ and i	$1+i$ and $-2+i$	$-2+i$ and $-i$
4	If $u=z-2\bar{z}+i$, then $i\bar{u}=$	$i\bar{z}+2iz+1$	$i\bar{z}-2iz+1$	$i\bar{z}-2iz-1$
5	$\lim\limits_{x\to-\infty}\left(x+e^{-x}\right)=$	$+\infty$	0	$-\infty$
6	If $\alpha=\arcsin\left(\sin\frac{7\pi}{5}\right)$, then $\alpha=$	$\alpha=\frac{7\pi}{5}$	$\alpha=-\frac{3\pi}{5}$	$\alpha=-\frac{2\pi}{5}$

II- (2 points)



Consider the cube ABCDEFGH represented in the adjacent figure .

The space is referred to a direct orthonormal system $(A\,;\,\overrightarrow{AB}\,,\,\overrightarrow{AD}\,,\overrightarrow{AE}\,)$.

Designate by I the midpoint of $[EF]$ and by K the center of the square ADHE .

1) a- Calculate the area of triangle IGA .

b- Calculate the volume of the tetrahedron ABIG.

c- Deduce that the distance from point B to the plane (AIG) is $\frac{\sqrt{6}}{3}$.

2) a- Write an equation of the plane (AFH) .

b- The line (CE) cuts the plane (AFH) at a point L. Calculate the coordinates of L .

c- Prove that L belongs to the line (FK). What does the point L represent for the triangle AFH?

III-(3 points)

Consider two urns U_1 and U_2.
U_1 contains four red balls and three green balls.
U_2 contains two red balls and one green ball.

A-

We draw at random a ball from U_1 and we put it in U_2, then we draw at random a ball from U_2.
Designate by X the random variable that is equal to the number of red balls remaining in the urn U_2 after the two preceding draws.

1) Prove that the probability $P(X = 2)$ is equal to $\frac{9}{14}$.

2) Find the three values of X and determine the probability distribution of X.

B-

In this part, each red ball carries the number 1 and each green ball carries the number – 1.
We choose at random an urn then we draw simultaneously and at random two balls from the chosen urn.
Consider the following events:

E: « The chosen urn is U_1 »
F: « The sum of the numbers carried by the two drawn balls is equal to 0 ».

1) a- Calculate the probabilities $P(F/E)$ and $P(F/\overline{E})$.

b- Deduce that $P(F) = \frac{13}{21}$.

2) Designate by G the event « The sum of the numbers carried by the two drawn balls is equal to – 2».
Calculate P(G).

IV- (3points)

In the plane referred to an orthonormal system $(O ; \vec{i}, \vec{j})$, consider the line (d) with equation $x = -4$ and the parabola (P) with focus O and directrix (d).

1) a-Show that an equation of (P) is $y^2 = 8x + 16$. Determine the vertex S of (P).

b- Draw (P).

c- Let D be the region bounded by (P) and the axis of ordinates. Calculate the area of D.

d- Calculate the volume of the solid generated by the rotation of D about the axis of abscissas.

2) Let A(6 ; 8) be a point on (P) .

a- Write an equation of the tangent (T_A) at A to (P).

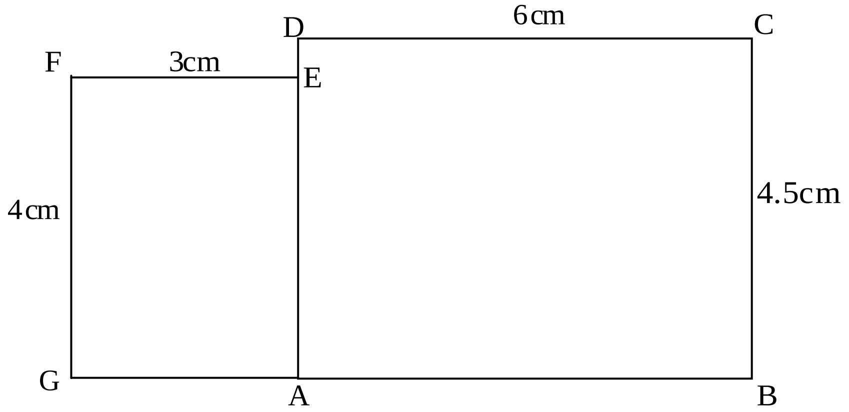
b-The line (OA) intersects (P) again at a point B.

Calculate the coordinates of B and write an equation of the tangent (T_B) at B to (P).

c-Verify that (T_A) and (T_B) are perpendicular and that they intersect on the directrix of (P).

3) Let $M(x_o ; y_o)$ be a point on (P), distinct from S.
N is the orthogonal projection of M on the tangent through S to (P).
The perpendicular through N to the line (MS) intersects the axis of abscissas at I.
Show that the abscissa of I is independent of x_o and y_o.

V- (3 points)



In the figure above, ABCD and AEFG are two direct rectangles so that $(\vec{AB}, \vec{AD}) = \frac{\pi}{2} \pmod{2\pi}$.

S is the direct plane similitude that transforms B onto E and C onto F;

T is the translation with vector $\vec{EF}$;
f is the similitude defined by T o S.

1) a- Determine the ratio k and an angle α of S.

b- Determine the image of D by S.

c- Prove that A is the center of S.

2) a- Find f(B) and f(A).

b- Specify the ratio and an angle of the similitude f.

c- Construct the center W of f.

3) The complex plane is referred to a direct orthonormal system $(A\ ;\ \frac{1}{6}\overrightarrow{AB}, \frac{1}{4}\overrightarrow{AE})$.

a- Write the complex form of f.

b-Deduce the affix of point W.

4) Let F_1 be the image of F by S, and for any nonzero natural integer n, let F_{n+1} be the image of F_n by S.

Determine the values of n so that A, F_1 and F_n are collinear.

VI- (7 points)

Consider the function f defined over $]-\infty\,;5[$ by $f(x) = \ln(5 - x)$.

Designate by (C) the representative curve of f in an orthonormal system $(O\,;\vec{i}\,,\vec{j})$.

1) a- Calculate $\lim\limits_{x\to5} f(x)$, $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to-\infty} \frac{f(x)}{x}$. Interpret, graphically, the results thus obtained.

b- Set up the table of variations of f over $]-\infty\,;5[$.

2) a- Determine an equation of the tangent (T) to (C) at the point with abscissa 4 .

b- Draw (T) and (C).

c- The curve (C) intersects the line with equation $y = x$ at a point with abscissa α.
Verify that $1< \alpha <2$.

3) f has an inverse function f^{-1}. Designate by (C') the representative curve of f^{-1} in the same system of (C) .

a- Prove that the tangent (T) to (C) is also tangent to (C') .

b- Draw (C').

4) Let h be the function defined on $]-\infty\,;5[$ by $h(x) = (5 - x)\ln(5 - x)$.

a- Verify that $h'(x)+f(x)=-1$ and deduce an antiderivative of the function f.

b- Designate by $A(\alpha)$ the area of the region bounded by (C), the axis of abscissas and the two lines with equations $x = \alpha$ and $x = 4$. Prove that $A(\alpha)=-\alpha^2+6\alpha-4$.

5) Let I be the interval $[0\,;3]$.

a- Prove that $f(I)$ is included in I .

b- Prove, for all x in I, that $|f'(x)|\le\frac{1}{2}$.

c- Deduce that, for all x in I, $|f(x)-\alpha|\le\frac{1}{2}|x-\alpha|$.

6) Consider the sequence (U_n) defined by: $U_0=1$ and , for all $n\ge0$, $U_{n+1}=f(U_n)$.

a- Prove by mathematical induction that , for all $n\ge0$, U_n belongs to I .

b- Show that, for all $n\ge0$, $|U_{n+1}-\alpha|\le\frac{1}{2}|U_n-\alpha|$.

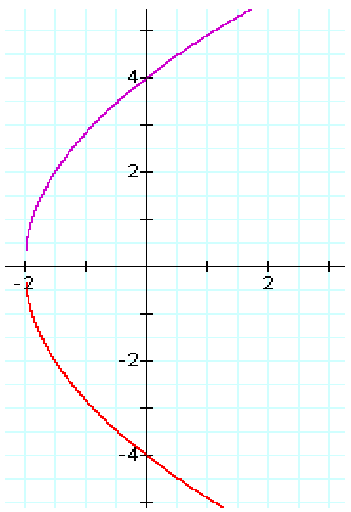
c- Prove, for all $n\ge0$, that $|U_n-\alpha|\le\frac{1}{2^n}$ and deduce that the sequence (U_n) is convergent.

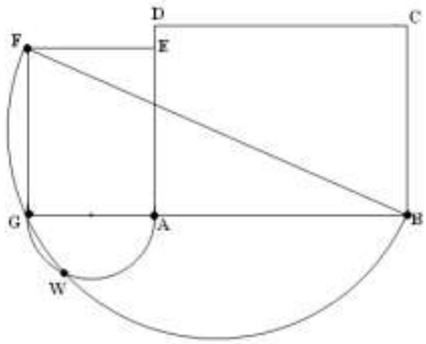
Q1	Solution		G
1	The integral of an odd function on $[-a,a]$ is zero.	c	1
2	$\arg\left(\frac{e^{i\pi}}{i}\right)=\arg\left(\frac{-1}{i}\right)=\arg(i)=\frac{\pi}{2}$.	b	0.5
3	$x + iy + x^2 + y^2 = 3 + i$, then $y = 1$ and $x^2 + y^2 + x= 3$; $x^2 + x-2= 0$. Thus, $x = 1$ or $x= -2$. Therefore, $z = 1 + i$ or $z = -2 +i$. or by verification.	b	1
4	$\bar{u}=\left(\overline{z-2\bar{z}+i}\right)=\bar{z}-2z-i$; $i\bar{u}=i\bar{z}-2iz+1$	b	0.5
5	$\lim\limits_{x\to-\infty}\left(x+e^{-x}\right)=\lim\limits_{t\to+\infty}\left(-t+e^{t}\right)=-\lim\limits_{t\to+\infty}e^{t}\left(\frac{t}{e^{t}}-1\right)=+\infty$	a	0.5
6	$\alpha=\arcsin\left(\sin\frac{7\pi}{5}\right)=-\frac{2\pi}{5}$ since $-\frac{2\pi}{5}\in\left[-\frac{\pi}{2}\ ;\ \frac{\pi}{2}\right]$ and $\sin\left(-\frac{2\pi}{5}\right)=\sin\frac{7\pi}{5}$.	c	0.5

Q2	Solution	G
1a	$I(\frac{1}{2};0;1)$, $G(1;1;1)$; $\overrightarrow{IG}(\frac{1}{2};1;0)$ and $\overrightarrow{IA}$ (-1/2 ; 0 ; -1) $\overrightarrow{IG}\wedge\overrightarrow{IA}=-\vec{i}+\frac{1}{2}\vec{j}+\frac{1}{2}\vec{k}$. Area of $(IGA)=\frac{1}{2}\sqrt{1+1/4+1/4}=\frac{\sqrt{6}}{4}$.	0.5
1b	$\overrightarrow{AB}(1;0;0)$; $\overrightarrow{AB}.(\overrightarrow{IG}\wedge\overrightarrow{IA})=-1$ The volume of the tetrahedron ABIG is $V=\frac{1}{6}\left\|\overrightarrow{AB}.(\overrightarrow{IG}\wedge\overrightarrow{IA})\right\|=\frac{1}{6}$.	0.5
1c	Let d be the distance of B to plane (AIG) . $V=\frac{1}{3}$Area of $(IGA)\times d=\frac{1}{3}\cdot\frac{\sqrt{6}}{4}\cdot d$. Thus, $d=\frac{\sqrt{6}}{3}$.	0.5
2a	$\overrightarrow{AF}\wedge\overrightarrow{AH}=-\vec{i}-\vec{j}+\vec{k}$. (AFH) : $x+y-z=0$. OR $\overrightarrow{AM}.\left(\overrightarrow{AF}\wedge\overrightarrow{AH}\right)=0$	0.5
2b	(C E) : $x=t; y=t; z=-t+1$. $(CE)\cap(AFH)$: $t + t + t - 1 = 0$. So, $t=\frac{1}{3}$ and $L(\frac{1}{3};\frac{1}{3};\frac{2}{3})$	1
2c	$\overrightarrow{FL}\left(-\frac{2}{3};\frac{1}{3};-\frac{1}{3}\right)$ and $\overrightarrow{FK}\left(-1;\frac{1}{2};-\frac{1}{2}\right)$, then $\overrightarrow{FL}=\frac{2}{3}\overrightarrow{FK}$. Thus, L belongs to [FK] , the median in triangle AFH. Therefore, L is the center of gravity of triangle AFH.	1

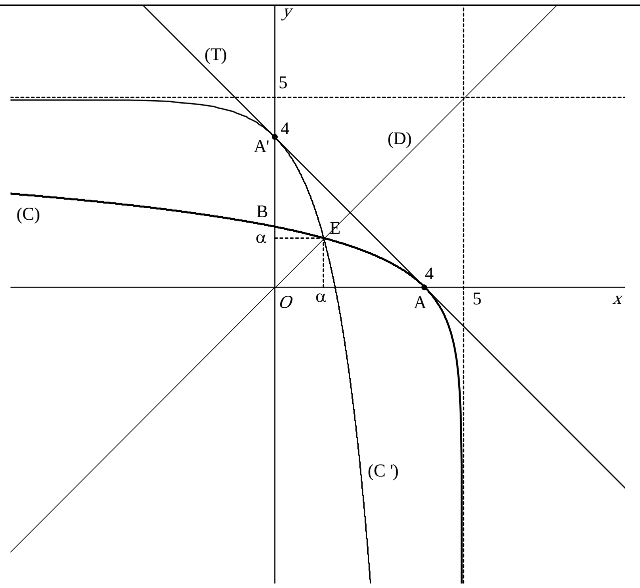
Q3	Solution	G
A 1	X = 2 occurs when we draw one red ball from U_1 then one red ball from U_2 <u>or</u> one green ball from U_1 then one green ball of U_2 . Thus: $P(X=2)=\frac{4}{7}\times\frac{3}{4}+\frac{3}{7}\times\frac{2}{4}=\frac{9}{14}$.	1.5
A 2	The values of X are1, 2 and 3. X = 1 occurs when one red ball remains in the urn U_2 . That is drawing a green ball from U_1 then a red ball from U_2 . Thus, $P(X=1)=\frac{3}{7}\times\frac{2}{4}=\frac{3}{14}$ X = 3 occurs when we draw a red ball from U_1 and a green ball from U_2 .	1.5

	$P(X=3)=\frac{4}{7}\times\frac{1}{4}=\frac{1}{7}$. Or : $P(X=3)=1-\left(\frac{9}{14}+\frac{3}{14}\right)=\frac{1}{7}$.	
B 1a	To get a sum 0 , we should draw a red ball and a green ball. $P(F/E)=\frac{4\times3}{C_7^2}=\frac{12}{21}=\frac{4}{7}$; $P(F/\bar{E})=\frac{2\times1}{C_3^2}=\frac{2}{3}$.	1
B 1b	$P(F)=P\left(F\cap E\right)+P\left(F\cap\bar{E}\right)=P(E)\times P(F/E)+P(\bar{E})\times P(F/\bar{E})=\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{2}{3}=\frac{13}{21}$.	1
B2	G occurs when we draw two green balls which is only possible from the urn U_1 since the urn U_2 contains only one green ball, thus $P(G)=\frac{1}{2}\times\frac{C_3^2}{C_7^2}=\frac{1}{14}$.	1

Q4	Solution	G
1a	$MO=d(M\to(d))$; $MO^2=d^2(M\to(d))$; $x^2+y^2=(x+4)^2$; $y^2=8x+16$. $y^2=8x+16$; $(y-0)^2=8(x+2)$ The vertex is S (– 2; 0).	1
1b		0.5
1c	$A=2\int_{-2}^{0}\sqrt{8x+16}\,dx=\frac{1}{6}\left[\sqrt{(8x+16)^3}\right]_{-2}^{0}=\frac{32}{6}u^2$.	1
1d	$V=\pi\int_{-2}^{0}y^2dx=\pi\int_{-2}^{0}\left(8x+16\right)dx=16\pi u^3$.	0.5
2a	$2yy'=8$; $y'=\frac{4}{y}$; $y'_A=\frac{1}{2}$. The equation of (T_A) is $y=\frac{1}{2}x+5$	0.5
2b	(OA) : $y=\frac{4}{3}x$. The abscissas of the points of intersection of (OA) and (P) verifies the equation : $\frac{16}{9}x^2=8x+16$, $2x^2-9x-18=0$; $x'=6$ and $x''=-\frac{3}{2}=x_B$. $B(-\frac{3}{2};-2)$. The equation of (T_B) is $y+2=y'_B(x+\frac{3}{2})$; $y=-2x-5$	1
2c	The product of the slopes of the tangents (T_A) and (T_B) is equal to – 1 thus (T_A) and (T_B) are perpendicular. Moreover, $\frac{1}{2}x+5=-2x-5$; $x=-4$ and $y=3$, thus (T_A) and (T_B) intersect on the directrix (d).	0.5
3	Let I(a ; 0) . We have N(– 2 ; $y_\circ$) ; $\vec{MS}(-2-x_\circ;-y_\circ)$; $\vec{MI}(a+2;-y_\circ)$ $\vec{MS}\bullet\vec{MI}=0$; $(-2-x_\circ)(a+2)+{y_\circ}^2=0$; $(-2-x_\circ)(a+2)+8(x_\circ+2)=0$; $(x+2_\circ)(6-a)=0$; $a=6$ ($x_\circ\neq-2$) Therefore, the abscissa of I is independent of x_0 and y_0.	1

Q5	Solution	G
1a	$S = sim(k\,;\,\alpha)$; $B \xrightarrow{S} E$; $C \xrightarrow{S} F$ $EF = k\,BC$; $k = \frac{3}{4{,}5} = \frac{2}{3}$; $\alpha = (\vec{BC},\vec{EF}) = (\vec{BC},\vec{AD}) + (\vec{AD},\vec{EF}) = \frac{\pi}{2}\ (2\pi)$	1
1b	Triangle EFG is similar to triangle BCD and in the same sense . Thus, S(D)= G.	
1c	S(BCDA) is the direct rectangle EFGA, S(A) = A, then A is the center of S	1
2a	f(B) = T(S(B)) = T(E) = F ; f(A) = T(S(A)) = T(A) = G	0.5
2b	f = similitude of ratio $\frac{2}{3}$ and angle $\frac{\pi}{2}$.	0.5
2c	$(\vec{WB},\vec{WF}) = \frac{\pi}{2}$ and $(\vec{WA},\vec{WG}) = \frac{\pi}{2}$; W is the point of intersection of the two circles of diameters [BF] and [AG] other than G. 	1
3a	f : M(z) → M'(z') ; $z' = \frac{2}{3}iz + b$; $z_G = = \frac{2}{3}iz_A + b$; $b = -3$. The complex form of f is $z' = \frac{2}{3}iz - 3$.	0.5
3b	$z_W = \frac{2}{3}iz_W - 3$; $3z_W - 2iz_W = -9$; $z_W = \frac{-9}{3-2i} = -\frac{27}{13} - \frac{18}{13}i$.	0.5
4	$(\vec{AF_1},\vec{AF_n}) = (\vec{AF_1},\vec{AF_2}) + (\vec{AF_2},\vec{AF_3}) + \ldots + (\vec{AF_{n-1}},\vec{AF_n}) = (n-1)\frac{\pi}{2}$ A, F_1 and F_n are collinear for $(n-1)\frac{\pi}{2} = k\pi$; then n = 2k+1 where k is a natural integer.(n is odd).	1

Q6	Solution	G
1a	$\lim\limits_{x\to-\infty} f(x) = +\infty$, $\lim\limits_{x\to5} f(x) = -\infty$ and $\lim\limits_{x\to-\infty}\frac{f(x)}{x} = 0$. The straight line of equation x = 5 is an asymptote to (C) and the curve (C) has a horizontal asymptotic direction at $-\infty$.	1.5
1b	$f'(x) = \frac{1}{x-5}$ with $x - 5 < 0$ over $]-\infty\,;\,5[$. x: $-\infty$ … 5 f'(x): − f(x): $+\infty$ ↘ $-\infty$	1
2a	(C) cuts $x'x$ at the point $A(4\,;\,0)$ and $y'y$ at the point $B(0\,;\,\ell n5)$. (T) is tangent at A to (C) ; (T) : $y = -x + 4$	0.5

2b		1.5
2c	f(1) = ln4 > 1 and f(2) = ln3 < 2 then $1<\alpha<2$.	**0.5**
3a	(C') is symmetric to (C) with respect to the straight line (D) of equation y = x . (C) cuts $x'x$ at $A(4\,;0)$ and admits (T) as a tangent at A . By symmetry with respect to (D), (C') cuts $y'y$ at the point $A'(0\,;4)$ and admits the symmetric of (T) with respect to (D), as a tangent at A' . But $(T)\perp(D)$, thus (T) is the symmetric of itself with respect to (D) . As a result , (T) is the tangent at A' to (C'). **Check the figure part 2b**.	1
3b	(C) and (C') are symmetric with respect to the straight line of equation y = x.	1
4a	$h'(x)=-1-\ell n(5-x)$; so $h'(x)+f(x)=-1$, therefore, F(x) = – h(x) – x.	1
4b	$A(\alpha)=\int_{\alpha}^{4} f(x)\,dx=\left[-x-h(x)\right]_{\alpha}^{4}=\alpha-4-(5-\alpha)\,\ell n(5-\alpha)$ But, $\ell n(5-\alpha)=\alpha$; thus $A(\alpha)=-4+\alpha+5\alpha-\alpha^2=-\alpha^2+6\alpha-4$ u^2.	1
5a	f is continuous and strictly decreasing ; then f(I)= [f(3),f(0)]=[ln2,ln5]$\subset$ I.	0.5
5b	$f'(x)=\frac{1}{x-5}$ with $x-5<0$; then $\lvert f'(x)\rvert=\frac{1}{5-x}$. but $0\le x\le 3$, so $2\le 5-x\le 5$ and $\frac{1}{5}\le\frac{1}{5-x}\le\frac{1}{2}$. Consequently, $\lvert f'(x)\rvert\le\frac{1}{2}$.	1
5c	Using the mean value inequality, we can write : $\lvert f(x)-f(\alpha)\rvert\le\frac{1}{2}\lvert x-\alpha\rvert$ with $f(\alpha)=\alpha$. therefore, $\lvert f(x)-\alpha\rvert\le\frac{1}{2}\lvert x-\alpha\rvert$.	0.5
6a	$U_0=1$; then $U_0\in I$. If $U_n\in I$, then $f(U_n)\in f(I)$. Hence, $U_{n+1}\in I$.	1
6b	$U_n\in I$, then $\lvert f(U_n)-\alpha\rvert\le\frac{1}{2}\lvert U_n-\alpha\rvert$. Consequently, $\lvert U_{n+1}-\alpha\rvert\le\frac{1}{2}\lvert U_n-\alpha\rvert$	0.5
6c	$U_0=1$ and $1<\alpha<2$; then, $-1<U_0-\alpha<0$ and $\lvert U_0-\alpha\rvert\le\frac{1}{2^0}$. If $\lvert U_n-\alpha\rvert\le\frac{1}{2^n}$, then $\lvert U_{n+1}-\alpha\rvert\le\frac{1}{2}\lvert U_n-\alpha\rvert\le\frac{1}{2^{n+1}}$. Or by using multiplication and simplification. $\lim_{x\to+\infty}\left(\frac{1}{2}\right)^n=0$; then $\lim_{x\to+\infty}\lvert U_n-\alpha\rvert=0$ and $\lim_{x\to+\infty}U_n=\alpha$	1.5