

عدد المسائل: خمسة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:
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ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختران المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (2 points)

1) Calculate the greatest common divisor (GCD) of 154 and 112.

2) Write the fraction $\frac{154}{112}$ in its simplest form.

3) Let $m = \frac{154}{112} + \frac{1}{8}$.

a. Write m as a fraction in the simplest form .

b. Is the number m decimal? Justify.

II- (3 points)

The two parts **A** and **B** of this exercise are independent.

Part A

Given $P(x) = (3x - 2)(x + 2) - (3x - 2)^2$.

1) a. Develop and reduce P(x).

b. Calculate $P(\sqrt{5})$.

2) a. Factorize P(x).

b. Solve the equation $P(x) = 0$.

Part B

Given two real numbers x and y such that $xy = 2\sqrt{3}$ and $x + y = 2 + 2\sqrt{3}$.

1) Calculate $x^2y + xy^2$. Give the result in the form $a + b\sqrt{3}$ where a and b are two integers.

2) Calculate $x^2 + y^2$.

III- (3 points)

To buy **three** copybooks and **two** pens we must pay 4 500 LL. To buy **six** copybooks and **three** pens we must pay 7 500 LL. This information is translated into the following system:

$$\begin{cases} 3x + 2y = 4\ 500 \\ 6x + 3y = 7\ 500 \end{cases}$$

1) What does x and y represent in this system?

2) Solve the previous system, showing the details of the steps you follow, to find the price of a copybook and that of a pen.

3) A student bought a pack which contains copybooks and pens, and he paid 11 000 LL. Calculate the number of copybooks and the number of pens in this pack knowing that the sum of these two numbers is 12.

IV- (6 points)

In an orthonormal system of axes x'Ox and y'Oy, consider the points A(3; 4) , B(3;- 1), C(1; 3) and the line (d) with equation $y = -2x + 5$.

1) Plot the points A, B and C.

2) Prove that B and C are two points on (d), then draw (d).

3) a. Find the equation of the line (CA).

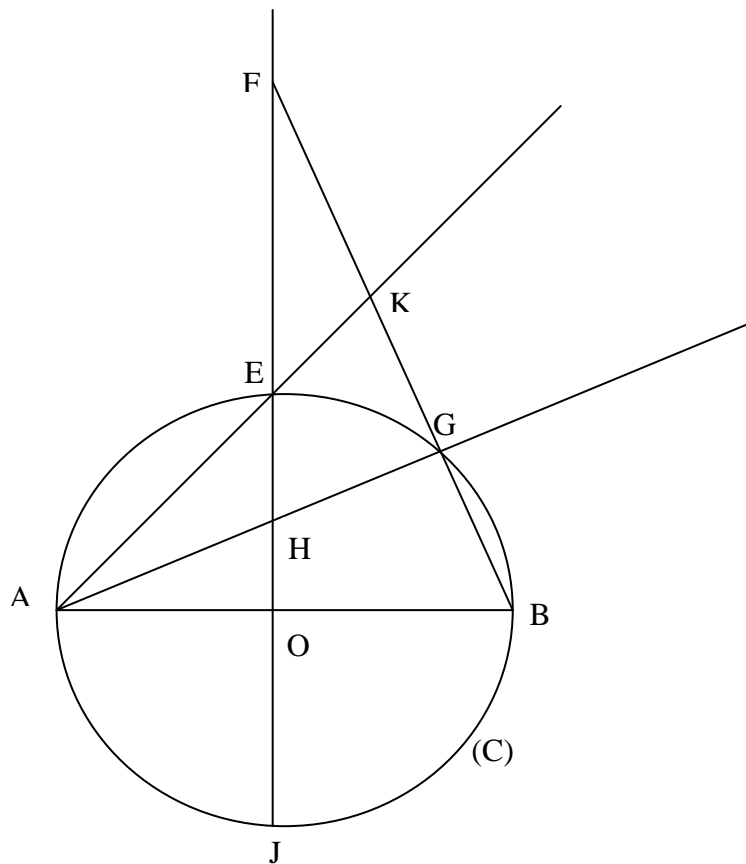
b. prove that the triangle ABC a right at C .

- 4) D is the point defined by $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{CB}$.
- Prove that CADB is a rectangle.
 - Calculate the coordinates of D.
- 5) E is the symmetric of C with respect to A.
- What is the nature of the quadrilateral ABDE? Justify.
 - Prove that CDE is an isosceles triangle.
 - Prove that (DE) is parallel to $y'Oy$ and write the equation of (DE).

V- (6 points)

In the following figure:

- (C) is a circle with diameter [AB], center O and radius 3 cm
- The perpendicular at O to (AB) intersects (C) at E and J
- The bisector of the angle \widehat{EAB} intersects [OE] at H and intersects (C) at another point G
- The line (BG) intersects (AE) at K and (OE) at F.

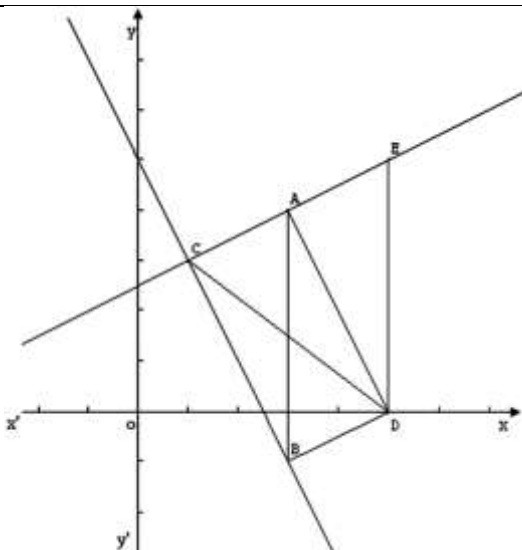


- Reproduce the figure.
- Verify that $\widehat{BAG} = \frac{45^\circ}{2}$.
- Prove that the triangle ABK is isosceles with vertex A.
- Calculate AE and EK.
- Prove that the two triangles AOH and AGB are similar. Deduce the value of the product $AH \times AG$.
- Using $\cos\left(\frac{45^\circ}{2}\right)$ in the triangle AOH, calculate AH to the nearest hundredth.
 - Deduce an approximate value of the similarity ratio of the triangles AOH and AGB.
- (BH) and (AF) intersect at I, prove that I is a point on (C).

الاسم:
الرقم:

مسابقة في مادة الرياضيات
المدة ساعتان

مشروع معيار التصحيح

I(2points)	Answer	Mark	
1	$154 = 2 \times 11 \times 7$ $112 = 2^4 \times 7$; $GCD(154; 112) = 2 \times 7 = 14$.	0.75	
2	$\frac{154}{112} = \frac{11}{8}$.	0.50	
3.a	$m = \frac{154}{112} + \frac{1}{8} = \frac{12}{8} = \frac{3}{2}$	0.75	
3.b	$m = 1,5$; m is a decimal number .		
II(3points)	Answer	Note	
A	1.a	$P(x) = -6x^2 + 16x - 8$.	0.5
	1.b	$P(\sqrt{5}) = 16\sqrt{5} - 38$	0.5
	2.a	$P(x) = (3x-2)(-2x+4) = -2(3x-2)(x-2)$	0.5
	2.b	$x = \frac{2}{3}$ or $x = 2$	0.5
B	1	$x^2y + xy^2 = xy(x+y) = 2\sqrt{3}(2+2\sqrt{3}) = 4\sqrt{3} + 12$.	0.5
	2	$x^2 + y^2 = (x+y)^2 - 2xy = (2\sqrt{3}+2)^2 - 4\sqrt{3} = 16 + 4\sqrt{3}$.	0.5
III.(3points)	Answer	Mark	
1	x is the price of a copybook ; y is the price of a pen .	0.50	
2	$\begin{cases} -6x - 4y = -9000 \\ 6x + 3y = 7500 \end{cases}$ $y = 1500$ and $x = 500$. The price of a copybook is 500LL and of a pen is 1500LL.	1.25	
3	Let a be the number of pens ; $(12 - a)$ the number of copybooks . $1500a + 500(12 - a) = 11000$. Hence $a = 5$ and $12 - a = 7$	1.25	
IV(6points)	Answer	Mark	
1		0.50	
2	$Y_B = -2x_B + 5$; then B is on (d). $Y_C = -2x_C + 5$; then C is on (d), hence (d) is (BC),	0,75	

3.a	(CA), $y = ax + b$ $\begin{cases} 3a + b = 4 \\ a + b = 3 \end{cases} \quad a = \frac{1}{2}, \quad b = \frac{5}{2} \quad y = \frac{x}{2} + \frac{5}{2}$	1
3.b	$(-2) \times \frac{1}{2} = -1$ then $(d) \perp (AC)$. ABC is right at C.	0.50
4.a	Given $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{CB}$ then CADB is a parallelogram . Since $\hat{C} = 90^\circ$, , then CADB is a rectangle .	0.50
4.b	$\overrightarrow{BD} = \overrightarrow{CA}$; then $x_D - x_B = x_A - x_C$ and $y_D - y_B = y_A - y_C$, thus D(5; 0)	0.75
5.a	Since AE = BD and (AE) parallel to (BD). Then ABDE is a parallelogram	0.75
5.b	A is the midpoint of [CE] and (DA) is perpendicular to (CE), then CDE is an isosceles triangle with vertex D.	0.50
5.c	ABDE is a parallelogram, then (DE) // (AB). Since $x_A = x_B$, then (AB) // y'Oy. Therefore (DE) is parallel to y'Oy. The equation of (DE) $x = 5$.	0.75
V(6points)	Answer	Mark
1		0.50
2	$BAC = \frac{45^\circ}{2} \dots$	0.50
3	$\widehat{AGB} = 90^\circ$; [AG] is at the same time bisector and altitude in the triangle BAK. Hence BAK is isosceles with vertex A.	0.75
4	$AE = 3\sqrt{2}$ cm, $EK = AK - AE = (6 - 3\sqrt{2})$ cm.	0.75
5	$\hat{O} = \hat{G} = 90^\circ$ and $\widehat{OAH} = \widehat{BAG}$. $\frac{AO}{AG} = \frac{AH}{AB} = \frac{OH}{GB}$, then $AH \times AG = AO \times AB$; $AH \times AG = 18$.	1.25
6.a	AOH right at O and $\widehat{OAH} = \frac{45^\circ}{2}$. $\cos \widehat{OAH} = \frac{OA}{AH}$, $AH \approx 3.25$	0.75
6.b	$\frac{AH}{AB} \approx \frac{3.25}{6}$, hence 0.54 is an approximate value of the ratio..	0.50
7	[AG] and [OF] are two altitudes of the triangle ABF; Hence [BI] is the third altitude. Therefore $\widehat{AIB} = 90^\circ$ and I is a point on (C).	1