الدورة العادية للعام 2011	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل: أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

The weekly work hours, per quarter, in an industrial company in the years 2009 and 2010 are listed in the following table:

Year	2009				2010			
Rank of the quarter: x_i	1	2	3	4	5	6	7	8
Work hours: y_i	38	38.5	39	39.5	40	41	42	42.5

1) Represent, in an orthogonal system, the scatter plot of the points $(x_i ; y_i)$.
2) Calculate the coordinates of the center of gravity $G(\overline{x};\overline{y})$, and plot this point in the preceding system.
3) Calculate the coefficient of correlation of this distribution and interpret the value found.
4) Determine an equation of the regression line $D_{y/x}$ of y in terms of x.

5) Suppose that this model remains valid in the year 2011.

Estimate the number of weekly work hours in this company in the 3rd quarter of the year 2011.

II- (4 points)

In a small town, one supermarket and several small stores share 1200 customers.
A survey conducted during the month of January 2010 revealed that 240 of these customers do their shopping at the supermarket and the others do their shopping in the small stores.
We notice that each month:

- 90% of the supermarket customers continue shopping at the supermarket.
- 15% of the customers of the small stores change their minds and start to do their shopping at the supermarket.

For every natural integer n, let:

a_n be the number of customers of the supermarket after n months, with $a_0 = 240$;
b_n be the number of customers of the small stores after n months, thus $a_n + b_n = 1200$.

1) Calculate a_1 and verify that $b_1 = 840$.
2) Show that, for all n, we have $a_{n+1} = 0.75a_n + 180$.
3) Let (v_n) be the sequence defined by $v_n = a_n - 720$.
a-Show that (v_n) is a geometric sequence whose common ratio and first term are to be determined.
b-Calculate v_n and then a_n in terms of n.

4) a- In how many months would the number of customers of the supermarket exceed 600 for the first time?
b- Show that the sequence (a_n) is increasing and deduce the sense of variation of (b_n).
c- Calculate the limit of b_n **,** and give an interpretation of the result obtained.

III- (4 points)

In 2011, a sports club offers its members three types of activities: volley-ball, basket-ball and tennis. Each member joins only one of these three activities.

- 30 % of the members join the volley-ball activity;
- 20 % of the members join the basket-ball activity;
- The remaining members join the tennis activity.

The club proposes an annual gathering day to all the members.

- 20 % of members who join the volley-ball activity attend this gathering day;
- 25% of members who join the basket-ball activity attend this gathering day;
- 70 % of members who join the tennis activity attend this gathering day.

A member of this club is randomly chosen. Consider the following events:

V : « The chosen member joins the volley-ball activity»
B : « The chosen member joins the basket-ball activity»
T : « The chosen member joins the tennis activity»
R : « The chosen member attends the gathering day».

1) Verify that the probability $p(T \cap R)$ is equal to 0.35 and calculate $p(B \cap R)$ and $p(V \cap R)$.

2) The president of this club affirms that more than half of the members do not attend the gathering day. Justify his statement by calculation.

3) The annual membership fees for this year in the club are as follows:

- 100 000 LL for the tennis activity,
- 60 000 LL for either volley-ball or basket-ball.

Moreover, an additional amount of 15 000 LL is required from each member who wants to attend the annual gathering day.

Let X be the random variable equal to the total sum paid by a member in this club.

a- Find the 4 possible values of X and prove that $p(X = 75000)$ is equal to 0.11.

b- Determine the probability distribution of X.

c - Calculate the expected value E(X).

d- 200 members joined the club. Estimate the revenue of this club for this year.

IV- (8 points)

A- Let f be the function defined on [0 ; +∞[by $f(x) = x - 1 - 4xe^{-2x}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Calculate $\lim_{x \to +\infty} f(x)$ and show that the line (d) with equation y = x–1 is an asymptote to (C).

b- Determine the position of (C) relative to (d).

2) The adjacent table gives the sign of f '(x) on [0 ; +∞[.

x	0		0.28		+∞
f '(x)		–	0	+	

a- Set up the table of variations of f on [0 ; +∞[.

b- Show that the equation f(x) = 0 has a unique solution α and verify that 1.358 < α < 1.359.

3) Draw (d) and (C).

B- A factory manufactures a liquid detergent. The function M_C defined on [0 ;10] by $M_C(x) = 0.8 + 4(1 - 2x)e^{-2x}$ gives the daily marginal cost, in millions LL; x is expressed in thousands of liters.

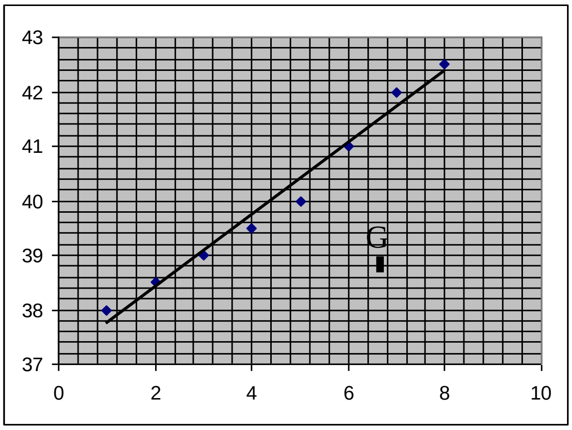
1) Knowing that the fixed costs amount to 1 million LL, prove that the function C representing the total daily cost is given by $C(x) = 1 + 0.8x + 4xe^{-2x}$.

2) The selling price of this liquid is 2000 LL per liter and suppose that 90% of the daily production is sold.

a- Prove that the daily profit, in millions of LL, is represented by f(x).

b- Find the minimal daily production level, in liters, for which this factory starts to achieve a profit.

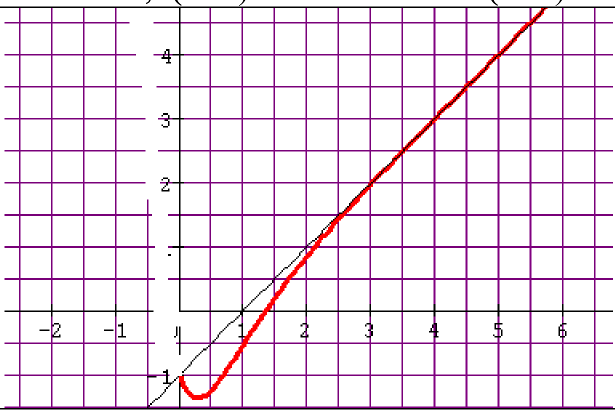
c- What is the maximal daily amount of loss that this factory may suffer?

QI	Answer	M
1		1
2	$\bar{x} = 4.5$ $\bar{y} = 40.0625$. G (4.5 ;40.0625)	1.5
3	$r = \dfrac{\text{cov}(x, y)}{\sigma_x \sigma_y} = 0.99$ (Calculator). r is very close to 1, then there is a strong positive correlation between x and y.	0.5
4	y= 0.6607x+37.089	1
5	For x = 11, y = 0.6607×11 +37.089 = 44.356 h is the weekly work hours during the 11th quarter.	1

QII	Answer	M
1	$a_1 = 0.9\times a_0 + 0.15\times b_0 = 0.9\times 240 + 0.15\times 960 = 360$; $b_1 = 1200 - 360 = 840$	1
2	$a_{n+1} = 0.9\times a_n + 0.15\times b_n = 0.9\times a_n + 0.15\times(1200 - a_n) = 0.9\times a_n + 180 - 0.15a_n = 0.75a_n + 180$	1
3a	$v_{n+1} = a_{n+1} - 720 = 0.75\times a_n + 180 - 720 = 0.75\times a_n - 540 = 0.75(a_n - 720) = 0.75v_n$ Thus, (v_n) is a geometric sequence of common ration 0.75 and of first term $v_0=a_0-720=-480$.	1
3b	$v_n = v_0 \times q^n = (-480)(0.75)^n$ $a_n = 720 + v_n = 720 + (-480)(0.75)^n$	1
4a	$a_n > 500 \Leftrightarrow 720 + (-480)(0.75)^n > 600 \Leftrightarrow (480)(0.75)^n < 120 \Leftrightarrow (0.75)^n < \dfrac{120}{480} \Leftrightarrow n > 4.81$ After 5 months, the number a_n becomes for the first time greater than 600 customers for the first time.	1
4b	$v_{n+1} - v_n = a_{n+1} - a_n$ hence (a_n) has the same sense of variation as (v_n). $v_{n+1} - v_n = 24(0.75)^n$. (v_n) is increasing and (a_n) is increasing $b_{n+1} - b_n = a_n - a_{n+1}$ therefore (b_n) is decreasing.	1
4c	When n tends to $+\infty$ then $(-480)(0,75)^n$ tends to 0 and a_n tends to 720 hence b_n tends to 1200 - 720 = 480. On the long term the number of customers of the small stores decreases but cannot be less than 480.	1

QIII	Answer	M
1	$p(T\cap R)=p(R/T)\cdot p(T)=0.7\times 0.5=0.35.$ $p(B\cap R)=p(R/B)\cdot p(B)=0.25\times 0.2=0.05.$ $p(V\cap R)=p(R/V)\cdot p(V)=0.2\times 0.3=0.06.$	1.5

2	$p(R)=p(T\cap R)+p(B\cap R)+p(V\cap R)=0.35+0.05+0.06=0.46.$ $p(\bar{R})=1-p(R)=0.54.$ So 54% of the members did not attend the gathering day.	0.5
3a	The values of X are: 60000,75000,100000,115000. $p(X = 75\,000) = p(V\cap R)+p(B\cap R) = 0.06 + 0.05 = 0.11.$	
3b	$p(X = 60\,000) = p(V\cap\bar{R})+p(B\cap\bar{R})=p(\bar{R}/V)\cdot p(V)+p(\bar{R}/B)\cdot p(B)$ $= 0.8\times0.3 + 0.75\times0.2 = 0.39.$ $p(X = 100\,000) = p(T\cap\bar{R})=p(\bar{R}/T)\cdot p(T) = 0.3\times0.5 = 0.15.$ $p(X = 115\,000) = p(T\cap R)=0.35.$	1
3c	$E(X) = 60\,000 \times 0.39 + 75\,000 \times 0.11 + 100\,000 \times 0.15 + 115\,000 \times 0.35 = 86\,900.$	0.5
3d	Estimation of annual revenue : $86\,900 \times 200 = 17\,380\,000$ LL.	0.5

QIV	Answer	M
A1a	$\lim\limits_{x\to+\infty} f(x)=+\infty-1-0=+\infty.$ $\lim\limits_{x\to+\infty} f(x)-(x-1)= \lim\limits_{x\to+\infty} -4xe^{-2x}=0$ so (d) : $y=x-1$ is asymptote to (C).	1.5
A1b	$f(x)-(x-1) = -4xe^{-2x}\leq 0$ for all $x\in[0\,;\,+\infty[$. So (C) is below (d) and cuts it at the origin O.	1
A2a	x: 0 ; 0.28 ; $+\infty$ f '(x): − 0 + f (x): −1 ↘ -1.36 ↗ $+\infty$	1.5
A4b	On]0;0.28] f is continuous and strictly decreasing from -1 to -1.36, so $f(x) < 0$. On [0.28 ; +∞[f is continuous and strictly increasing from -1.36 to $+\infty$, so the equation $f(x) = 0$ has a unique solution $\alpha\in[0.28\,;+\infty[$. Moreover, $f(1.35) \approx -0.086 < 0$ and $f(1.36) \approx +0.059 > 0$ thus : $1.35 < \alpha < 1.36$.	2
A5		2
B1	$C(0)=1$ and $C'(x)=0.8+4e^{-2x}-8xe^{-2x}=0.8+4e^{-2x}(1-2x)=M_C(x).$ Thus: $C(x)=1+0.8x+4xe^{-2x}$.	0.5
B2 a	The selling price of x thousands liters is $R(x)=\left(\frac{90}{100}x\right)\frac{2000\times1000}{1000000}=1.8x$; $P(x)=1.8x-1-0.8x-4xe^{-2x}$. $P(x)=1.8x-1-0.8x-4xe^{-2x}=x-1-4xe^{-2x}=f(x)$	1
B2b	$P(x)>0$ for $x>\alpha$, 1359 liters should be produced to realize profit.	1
B2c	The maximal loss occurs when f(x) has a negative minimum, namely $f(x) = -1.36$. The maximal loss is 1.36 million LL.	1