

اسم: الرقم:	• مسابقة في مادة الرياضيات المدة ساعة	عدد المسائل: ثلاث
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ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.  
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

### I- (5 points)

The price of one kg of sugar and one kg of salt together was 1700 LL.

During an economical crisis, the price of sugar increased by 12% while the price of salt decreased by 15% after which the price of one kg of sugar and one kg of salt together becomes 1769LL.

- 1) Calculate the initial price of 1 kg of sugar and of 1 kg of salt.
- 2) A person bought, during the economical crisis, 5 kg of sugar and 2 kg of salt.  
Calculate the sum paid by this person.

### II-(5 points)

The staff of a medical center consists of 100 employees divided into three categories: doctors, nurses and technicians.

20% of the staff are doctors and 60% are nurses.

80% of the doctors are males and 75% of the nurses are females.

60% of the staff are females.

- 1) Copy and complete the following table:

	Doctors	Nurses	Technicians	Total
Males				
Females				60
Total	20	60		100

- 2) A person is chosen randomly from this staff. Calculate the probability of each of the following events:

A : « The chosen person is a female doctor »

B : « The chosen person is a female knowing that she is a doctor »

C : « The chosen person is a female or a doctor »

D : « The chosen person is a male or not a doctor»

- 3) Suppose that all the names of the staff of this center are written each on a card, and the 100 cards are put in a box. We draw at random, successively and without replacement, two cards from this box.

Determine the probability of drawing two cards carrying the names of two male technicians.

### III- (10 points)

Consider the function  $f$  defined over  $]-\infty ; \frac{1}{2}[ \cup ]\frac{1}{2} ; +\infty[$  by  $f(x) = 2x + 1 + \frac{1}{2x - 1}$  and designate by (C) its representative curve in an orthonormal system  $(O; \vec{i}, \vec{j})$ .

1) Determine  $\lim_{x \rightarrow \frac{1}{2}^-} f(x)$ ,  $\lim_{x \rightarrow \frac{1}{2}^+} f(x)$  and deduce an asymptote (d) to (C).

2) Calculate  $\lim_{x \rightarrow +\infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$ . Show that the straight line (D) with equation  $y = 2x + 1$  is an asymptote to (C).

3) Show that the point  $I\left(\frac{1}{2}; 2\right)$  is a center of symmetry for (C).

4) a- Verify that  $f'(x) = \frac{8x(x-1)}{(2x-1)^2}$  and set up the table of variations of  $f$ .

b- Draw (d), (D) and (C).

5) Solve the inequality  $f(x) \geq 0$ .

6) Consider the two points  $A\left(\frac{1}{2}; 1\right)$  and  $M(3; 0)$ .

The straight line (MA) cuts the axis of ordinates at a point N.

a- Verify that an equation of the straight line (MA) is:  $y = -\frac{2}{5}x + \frac{6}{5}$ .

b- Calculate the ordinate of N.

c- Verify that  $OM \times ON = \frac{1}{2}f(3)$ .

QI	Answers	Mark
1	<p>Let <math>x</math> be the price of a kg of sugar and <math>y</math> be the price of a kg of salt.</p> $\begin{cases} x + y = 1700 \\ 1.12x + 0.85y = 1769 \end{cases}$ <p>Using a calculator we get <math>x=1200</math> and <math>y=500</math>. The initial price of a kg of sugar is 1200 LL and that of a kg of salt is 500 LL.</p>	3
2	$5(1.12)(1200) + 2(0.85)(500) = 7570$ <p>The sum paid by the person is 7570 LL.</p>	2

QII	Answers	Mark																				
1	<table border="1"> <thead> <tr> <th></th> <th>Doctors</th> <th>Nurses</th> <th>Technicians</th> <th>Total</th> </tr> </thead> <tbody> <tr> <th>Males</th> <td>16</td> <td>15</td> <td>9</td> <td>40</td> </tr> <tr> <th>Females</th> <td>4</td> <td>45</td> <td>11</td> <td>60</td> </tr> <tr> <th>Total</th> <td>20</td> <td>60</td> <td>20</td> <td>100</td> </tr> </tbody> </table>		Doctors	Nurses	Technicians	Total	Males	16	15	9	40	Females	4	45	11	60	Total	20	60	20	100	1.5
	Doctors	Nurses	Technicians	Total																		
Males	16	15	9	40																		
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Total	20	60	20	100																		
2	$P(A) = \frac{4}{100} = 0.04, \quad P(B) = \frac{4}{20} = 0.2$ $P(C) = P(\text{Female or a doctor})$ $= P(\text{Female}) + P(\text{doctor}) - P(\text{Female and doctor}).$ $= \frac{60}{100} + \frac{20}{100} - \frac{4}{100} = 0.76$ $p(D) = p(\text{Male or not a doctor})$ $= p(\text{Male}) + p(\text{not a doctor}) - p(\text{Male and not a doctor})$ $= \frac{40}{100} + \frac{80}{100} - \frac{24}{100} = \frac{96}{100} = 0.96$	2																				
3	<p>Probability of drawing the names of two male technicians</p> $= \frac{9}{100} \times \frac{8}{99} = \frac{2}{275} = 0.0072$	1.5																				

QIII	Answers	Mark
1	$\lim_{x \rightarrow 1/2^+} f(x) = +\infty$ ; $\lim_{x \rightarrow 1/2^-} f(x) = -\infty$ . The of equation $x = \frac{1}{2}$ is an asymptote to (C).	1
2	$\lim_{x \rightarrow +\infty} f(x) = +\infty$ ; $\lim_{x \rightarrow -\infty} f(x) = +\infty$ $\lim_{x \rightarrow -\infty} [f(x) - (2x + 1)] = \lim_{x \rightarrow -\infty} \frac{1}{2x - 1} = 0$ and $\lim_{x \rightarrow +\infty} [f(x) - (2x + 1)] = \lim_{x \rightarrow +\infty} \frac{1}{2x - 1} = 0$ . The straight line of equation $y = 2x + 1$ is an asymptote to (C).	1
3	$f(1-x) + f(x) = 2(1-x) + 1 + \frac{1}{2(1-x)-1} + 2x + 1 + \frac{1}{2x-1} = 4 + \frac{1}{-2x+1} + \frac{1}{2x-1} = 4 = 2 \times 2.$ <p>Then, the point <math>I(\frac{1}{2}; 2)</math> is a center of symmetry for (C).</p>	1
4a	$f'(x) = 2 - \frac{2}{(2x-1)^2} = \frac{2(4x^2 - 4x + 1) - 2}{(2x-1)^2} = \frac{8x^2 - 8x}{(2x-1)^2} = \frac{8x(x-1)}{(2x-1)^2}$ <p><math>f'(x) = 0</math> for <math>x = 0</math> or <math>x = 1</math>.</p>	2
4b		2
5	$f(x) \geq 0$ when (C) is above or on the axis of abscissas which is true for $x > \frac{1}{2}$ or $x=0$ .	0.5
6a	The coordinates of A and of M satisfy the given equation.	1
6b	For $x = 0$ , $y = \frac{6}{5}$ .	1
6b	$OM \times ON = 3 \times \frac{6}{5} = \frac{18}{5}$ and $\frac{1}{2} f(3) = \frac{1}{2} (7 + \frac{1}{5}) = \frac{36}{10} = \frac{18}{5}$ ; then $OM \times ON = \frac{1}{2} f(3)$ .	0.5

