وزارة التربية والتعليم العالي	امتحانات الشهادة الثانوية العامة	دورة العام ٢٠١٦ الإستثنائية
المديرية العامة للتربية	فرع: العلوم العامة	الخميس ٤ آب ٢٠١٦
دائرة الامتحانات الرسميّة		
عدد المسائل: ست	مسابقة في مادة الرياضيات	الاسم:
	المدة: أربع ساعات	الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, with justification, the answer corresponding to it.

Q	Questions	Answers			
		a	b	c	d
1	If $F(x)=\int_1^x e^{t^2}\,dt$, then $F'(2)=$	e^4	$4e^4$	e^{16}	$4e^{16}$
2	If $n\in\mathbb{N}-\{0;1\}$, then $C_n^2+C_n^3+......+C_n^{n-1}+C_n^n=$	$2^{n-1}-n+1$	2^n+n-1	2^n-n-1	2^n+n+1
3	z and z' are two complex numbers such that $z\neq-3i$ and $z'=\dfrac{z+3i}{\overline{z}-3i}$; then $\lvert z'\rvert=$	$\frac{1}{2}$	1	2	3
4	Let $f(x)=\ln\left(e^x-x\right)$ and $g(x)=\arctan(2x)$ where x is a real number, then $(f\circ g)'(0)=$	0	1	2	3

II- (2.5 points)

In an orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the points A(0; −3; 5), B(−2; 0; 1) and the line (D) with parametric equations x = k + 3 ; y = 4k + 1 and z = 2k + 6 where k is a real parameter.

1) Determine a system of parametric equations of line (AB).
2) Show that (AB) and (D) are skew (not coplanar).
3) Verify that $-2x+z-5=0$ is an equation of the plane (Q) containing the line (AB) and parallel to (D).
4) a- Determine a system of parametric equations of the line (D’) passing through A and perpendicular to (Q).
   b- Show that (D) and (D’) inetrsect at a point E whose coordinates are to be determined.
5) Let F be the point in the plane (Q) with zero ordinate and strictly negative abscissa.
   Calculate the coordinates of F so that the volume of the tetrahedron AFBE is equal to 5 cubic units.

III- (2.5 points)

In the plane referred to an orthonormal system $\left(O;\vec{i},\vec{j}\right)$, consider the line (Δ) with equation $x=-\frac{1}{4}$, the points A(4 ; 2), $E(0;1)$ and $F(m;0)$ where m is a real number less than 1.

1) Determine m so that $AF=\frac{17}{4}$.

In what follows, take $m=\frac{1}{4}$**.**

2) Prove that A is on a parabola (P) with focus F and directrix (Δ).

3) a- Write an equation of (P).
b- Draw (P).

4) a- Prove that the line (AE) is tangent to (P).
b- Calculate the area of the region bounded by (P) and the segments [OE] and [AE].

5) The line (AE) intersects (Δ) at point L.
Denote by (d) the line passing through L and perpendicular to line (AL).
Prove that (d) is tangent to (P) at a point K whose coordinates are to be determined.

IV- (3 points)

A box V contains cards such that:

- 20% of the cards are blue and the other cards are red;
- Out of the blue cards, 40% carry odd numbers;
- 32% of the total cards carry odd numbers.

1) A card is randomly selected from box V.
Consider the following events:
    - B : «select a blue card »
    - R : «select a red card »
    - O: «select a card carrying an odd number »

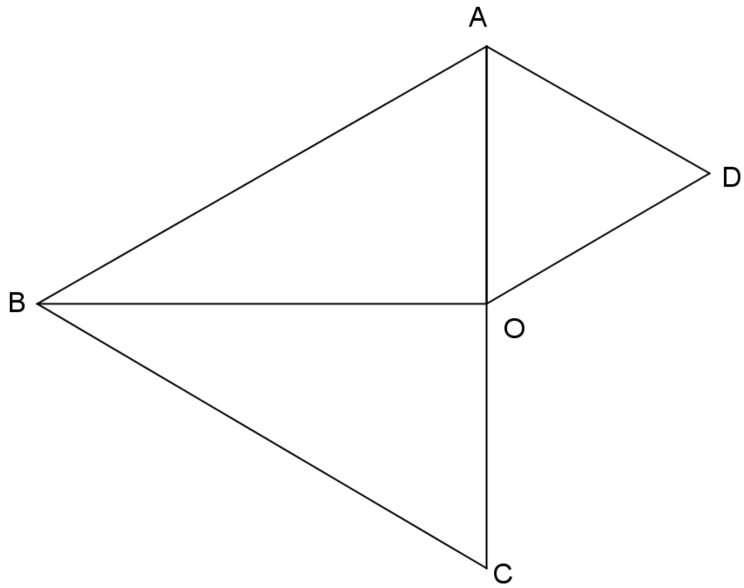
a- Calculate the probabilities $p(O\cap B)$ and verify that $p(O\cap R)=0.24$.
b- Deduce $p(O/R)$.
c- The selected card does not carry an odd number, what is the probability that it is red?

2) In this part, suppose that the number of cards in box V is 50.
Three cards are randomly and simultaneously selected from V.
Consider the following events:
    - M: «among the three selected cards, exactly two carry odd numbers»
    - N: «the three selected cards are blue»
    - L: « among the three selected cards, exactly two carry odd numbers and one is blue».

Calculate the probability p(M), $p(N/M)$ and p(L).

V- (3 points)

In the following figure, ABC and AOD are two direct equilateral triangles where O is the midpoint of [AC].



Let S be the direct plane similitude that transforms B onto O and C onto D.

1)a- Determine the ratio k and an angle α of S.

b- Verify that A is the center of S.

2) Consider the transformation R such that $R(B) = C$ and $R(C) = A$.

a- Prove that R is a rotation and determine an angle of R.

b- Determine the center G of R.

3) Let $h = S \circ R$.

a- Determine h (B) and h(C).

b- Determine the nature, the center and the ratio of h.

4) The plane is referred to the direct orthonormal system $(O;\vec{u},\vec{v})$ such that $\overrightarrow{OA} = 2\vec{v}$.

a- Determine the complex form of S.

b- Consider the ellipse (E) with equation $\frac{x^2}{12} + \frac{y^2}{4} = 1$. Let (E´) be the image of (E) under S.

Determine an equation of the focal axis of (E´).

VI- (7 points)

Consider the function f defined on IR as $f(x)=\dfrac{2e^x}{e^x+1}-x$, and denote by (C) its representative curve in an orthonormal system of axes $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$ and show that line (d) with equation $y=-x$ is an asymptote to (C).

b- Determine $\lim\limits_{x\to+\infty} f(x)$ and show that line (d') with equation $y=-x+2$ is an asymptote to (C).

c- Show that (C) is included between (d) and (d').

2) Show that point $W(0;1)$ is the center of symmetry of curve (C).

3) a- For all real numbers x, prove that $-1<f'(x)<0$. Set up the table of variations of f.

b- Show that the equation $f(x)=0$ has a unique root α, then verify that $1.6<\alpha<1.7$.

c- For all $x\in[0;\alpha]$, prove that $0\le f(x)\le \alpha - x$.

4) Draw (d), (d') and (C) .

5) a- Show that f has an inverse function g whose domain of definition is to be determined .

b- Determine the asymptotes to the representative curve (C') of g.

c- Write an equation of the line (T), the tangent to (C') at its center of symmetry.

d- Draw (C') and (T) in the same system as that of (C).

6) Denote by β the abscissa of the intersection point of (C) and (C') .

Show that the area of the region bounded by (C), (C') and the coordinates axes is equal to $\left[-4\ln(2-2\beta)-2\beta^2\right]$ units of area.

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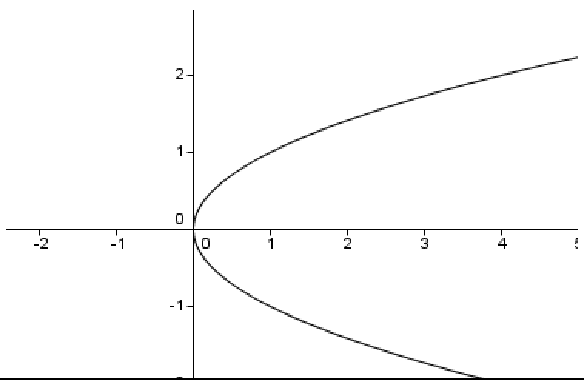
امتحانات الشهادة الثانوية العامة
فرع: العلوم العامة

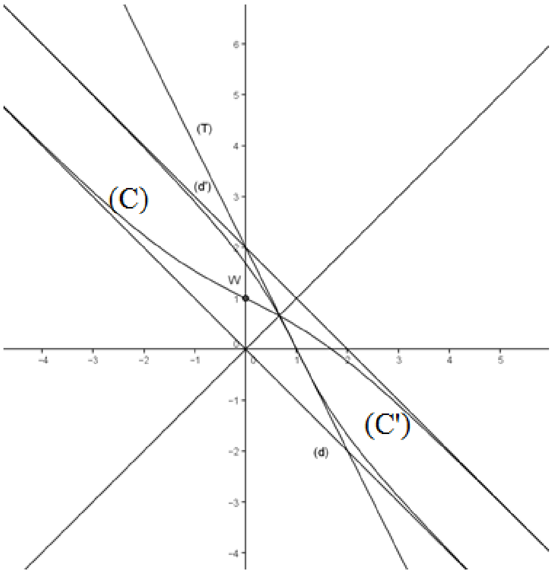
دورة العام ٢٠١٦ الإستثنائية

مشروع معيار التصحيح

مسابقة في مادة الرياضيات
المدة: أربع ساعات

Q-I	Solutions	N
1	$f'(x)=e^{x^2}$; $f'(2)=e^4$. **a**	1
2	$C_n^0+C_n^1+C_n^2+C_n^3+C_n^4+...+C_n^n=(1+1)^n \Rightarrow C_n^2+C_n^3+C_n^4+...+C_n^n=2^n-1-n.$ **c**	1
3	$\|z'\|=\left\|\frac{z+3i}{\overline{z-3i}}\right\|=\left\|\frac{z+3i}{\overline{\overline{z+3i}}}\right\|=1$. **a**	1
4	$f'(x)=\frac{e^x-1}{e^x-x}$, $g'(x)=\frac{2}{1+4x^2}$, $g(0)=0$, $g'(0)=2; f'(g(0))\times g'(0)=0.$ **a**	1
Q-II	**Solutions**	**N**
1	(AB) : $x=-2t$; $y=3t-3$; $z=-4t+5$	0,5
2	Let $C(3;1;6)$ be a point of (D) ; $\overrightarrow{AC}(3;4;1)$; $\overrightarrow{v_{(D)}}(1;4;2)$; $\overrightarrow{AB}(-2;3;-4)$ $\overrightarrow{AC}.(\overrightarrow{V_{(D)}}\wedge\overrightarrow{AB})=-60$ so (AB) and (D) are skew st.lines.	1
3	Let $M(x;y;z)$ be a point of (Q); $\overrightarrow{AM}.(\overrightarrow{V_D}\wedge\overrightarrow{AB})=0$ then $(Q): -2x+z-5=0$	1
4a	(D') : $x=-2m$; $y=-3$; $z=m+5$	0,5
4 b	$4k+1=-3; k=-1; E(2;-3;4)$.	1
5	$F(x;0;2x+5); V=\frac{1}{6}\|(\overrightarrow{AF};\overrightarrow{AB};\overrightarrow{AE})=\frac{1}{6}\|-15x-30\|=5; x=-4$ or $x=0$ (rejected) then $F(-4;0;-3)$ is accepted .	1
Q-III	**Solutions**	**N**
1	$(4-m)^2+4=\frac{289}{16}; m=\frac{1}{4}$ or $m=\frac{-31}{4}$ so $m=\frac{1}{4}$ is accepted	0.5
2	$H\left(-\frac{1}{4};2\right)$ is the orthogonal projection of A on the directrice (Δ). $AF=AH=\frac{17}{4}$	0.5
3a	Vertex S (0 ; 0) and $p=2\times\frac{1}{4}=\frac{1}{2}$. equation of (P) is : $(y-y_S)^2=2\frac{1}{2}(x-x_S)$; $y^2=x$.	0.5
3b		0.75

4a	E is the midpoint of [FH] , AFH is an isoscles triangle and (AE) is the bisector of AFH so it is tangent to (P) at A. verify that : $(AE): y=\frac{1}{4}x+1$	**0.75**
4b	$\int_0^4 (y_{(AE)}-y_{(P)})dx=\frac{8}{3}\ u^2$	**1**
5	$(d): y=-4x+\frac{1}{16}; y_{(P)}{}^2=y_{(d)}{}^2; x=\frac{1}{64}; K\left(\frac{1}{64};-\frac{1}{8}\right)$	**1**
Q-IV	**Solutions**	**N**
1a	$p(O\cap B)=P\left(O/B\right)\times p(B)=0,4\times 0,2=0,08$ $p(O\cap R)=p(O)-p(O\cap B)=0,32-0,8=0,24.$	**1**
1b	$p(O/R)=\frac{p(O\cap R)}{p(R)}=\frac{0,24}{0,8}=0,3$	**1**
1c	$p\left(R/\overline{O}\right)=\frac{p(R\cap\overline{O})}{p(\overline{O})}=\frac{0,7\times 0,8}{0,68}=\frac{14}{17}$	**1**
2	$p(M)=\frac{C_{16}^2\times C_{34}^1}{C_{50}^3}=\frac{51}{245}$; $p(N/M)=\frac{p(N\cap M)}{p(M)}=\frac{\frac{C_4^2\times C_6^1}{C_{50}^3}}{\frac{51}{245}}=\frac{3}{340}$ $p(L)=\frac{C_4^1\times C_{12}^1\times C_{28}^1+C_6^1\times C_{12}^2}{C_{50}^3}=\frac{87}{980}$	**3**
Q-V	**Solutions**	**N**
1a	$k=\frac{OD}{BC}=\frac{1}{2}.\quad \alpha=\left(\overrightarrow{BC};\overrightarrow{OD}\right)=\frac{\pi}{3}(\text{mod}\,2\pi).$	**0,5**
1b	$\frac{OA}{BA}=\frac{1}{2}=K$ and $\left(\overrightarrow{AB};\overrightarrow{AO}\right)=\frac{\pi}{3}.$	**0,5**
2a	$BC=CA$ and $\left(\overrightarrow{BC};\overrightarrow{CA}\right)=2\frac{\pi}{3}(\text{mod}\,2\pi)$ then R is a rotation of angle $\frac{2\pi}{3}$.	**1**
2b	G is the intersection point of the medians of segments [BC] and [CA]..	**0,5**
3a	$h(B)=S\circ R(B)=S(C)=D,\ h(C)=S\circ R(C)=S(A)=A.$	**0,5**
3b	$h=S\circ R=S'(W;\frac{1}{2};\pi)=h\left(W;-\frac{1}{2}\right)$, the center W of h is the intersection point of the two st.lines (BD) and (AC).	**1**
4a	$S:\ z'-z_A=a(z-z_A),\ a=\frac{1}{2}e^{i\frac{\pi}{3}}=\frac{1}{4}+\frac{1}{4}i\sqrt{3},\ z'=\left(\frac{1}{4}+\frac{1}{4}i\sqrt{3}\right)z+\frac{\sqrt{3}}{2}+\frac{3}{2}i.$	**1**
4b	(BO') where O' is the midpoint of the segment [OD]	**1**
Q-VI	**Solutions**	**N**
1a	$\lim_{x\to-\infty} f(x)=+\infty$ and $\lim_{x\to-\infty}\left(f(x)+x\right)=\lim_{x\to-\infty}\frac{2e^x}{e^x+1}=0$, then(d) : $y=-x$ is an asymptote	**1**
1b	$\lim_{x\to+\infty} f(x)=-\infty$ and $\lim_{x\to+\infty}\left(f(x)+x-2\right)=\lim_{x\to+\infty}\frac{-2}{e^x+1}=0$,then (d') : $y=-x+2$ is the asymptote	**1**

<table>
<tr><td>1c</td><td>$\left(f(x)+x\right)=\frac{2e^{x}}{e^{x}+1}>0$ then (C) is above (d).
$\left(f(x)+x-2\right)=\frac{-2}{e^{x}+1}<0$ then (C) is below (d').
So (C) is included between the two st.lines (d) et (d').</td><td>1</td></tr>
<tr><td>2</td><td>$f(x)+f(-x)=\frac{2e^{x}}{e^{x}+1}-x+\frac{2e^{-x}}{e^{-x}+1}+x=2$</td><td>1</td></tr>
<tr><td>3a</td><td>$f'(x)=\frac{-e^{-2x}-1}{(e^{x}+1)^{2}}<0$ et $f'(x)+1=\frac{2e^{x}}{(e^{x}+1)^{2}}>0.$
<table><tr><td>x</td><td>$-\infty$ $+\infty$</td></tr><tr><td>f'(x)</td><td>—</td></tr><tr><td>f(x)</td><td>$+\infty$ ↘ $-\infty$</td></tr></table></td><td>1.5</td></tr>
<tr><td>3b</td><td>f is continuoise and strictly decresing from $+\infty$ to $-\infty$ then f(x)=0 admits a unique solution α and $f(1,6)\times f(1,7)=0.064\times(-0.0089)<0$ then $1,6<\alpha<1,7.$</td><td>1</td></tr>
<tr><td>3c</td><td>$-1<f'(x)<0<1\Rightarrow\left|f'(x)\right|<1$ and f is continuios over $[-1,+1]$ and differentiable over $]-1,+1[$ then $\left|\frac{f(x)-f(\alpha)}{x-\alpha}\right|<1$; $f(\alpha)=0$ and $f(x)\geq 0$ for $x\leq\alpha$; then
$0\leq f\left(x\right)\leq\alpha-x$</td><td>1</td></tr>
<tr><td>4</td><td>
</td><td>1</td></tr>
<tr><td>5a</td><td>f is continuoise and strictly decreasing over $\mathbb{R}$ then it admits an inverse function g defined over $\mathbb{R}$</td><td>1</td></tr>
<tr><td>5b</td><td>(d) and (d') since the two st.lines are perpendicular to y = x</td><td>1</td></tr>
<tr><td>5c</td><td>$g'(2)=\frac{1}{f'(0)}=$-2; $(T): y=-2x+2$</td><td>1</td></tr>
<tr><td>5d</td><td>figure</td><td>1</td></tr>
<tr><td>6</td><td>since y=x is an axis of symmetry, it divides the the region bounded by (C), (C') and the coordinates axes into two rgions of equal areas, then A=double the area of the region limited by y'y , (C) and y=x.
$A=2\int_{0}^{\beta}[f(x)-x]dx=2\left[\ln(1+e^{x})-x^{2}\right]_{0}^{\beta}=2\left[2\ln(1+e^{\beta})-\beta^{2}-2\ln 2\right]$ or $f(\beta)=\beta$;
$\frac{2e^{\beta}}{1+e^{\beta}}=2\beta$; $e^{\beta}=\frac{\beta}{1-\beta}$; $A=-4\ln(1-\beta)-2\beta^{2}-2\ln 2=\left[-4\ln\left(2-2\beta\right)-2\beta^{2}\right]$s.u.</td><td>1.5</td></tr>
</table>