وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة العام ٢٠١٦ العاديّة
الاربعاء ١٥ حزيران ٢٠١٦

عدد المسائل: أربع	مسابقة في مادة الرياضيات	الاسم:
	المدة: ساعتان	الرقم:

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المُرشّح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider the points

A (3,1 , 1) and F (2 ,2 , −2). Denote by (d) the line defined as: $\begin{cases} x = -t \\ y = t + 2 \\ z = t \end{cases}$ (t is a real parameter).

1) Let (P) be the plane through the point F and containing the line (d).
   Verify that: $x + z = 0$ is an equation of the plane (P).
2) Let $E(1,1,-1)$ be a point on (d).
   Verify that E is the orthogonal projection of A on the plane (P).
3) Denote by L the point on the line (d) so that $x_L \neq 0$ and the triangle EFL is isosceles with principle vertex E. Calculate the coordinates of L.
4) Calculate the volume of the tetrahedron AEFL.

II- (4 points)

Consider a box V containing six cards numbered 1 ; 2 ; 3 ; 4 ; 7 ; 9, and two urns U_1 and U_2 such that:

- U_1 contains 3 red balls and 5 black balls
- U_2 contains 4 red balls and 4 black balls.

One card is randomly selected from the box V.
If this card shows an even number, then two balls are randomly and simultaneously selected from U_1. If the card shows an odd number then two balls are randomly and simultaneously selected from U_2.

Consider the following events:
E: "The card selected shows an even number"
O: "The card selected shows an odd number"
R: "The two selected balls are red"
B: "The two selected balls are black".

1) a- Calculate the probability $P\left(R/E\right)$ and deduce that $P(E \cap R) = \frac{1}{28}$.
   b- Calculate $P(O \cap R)$ and P(R).
2) Show that $P(B) = \frac{11}{42}$.
3) Knowing that the two selected balls are black, calculate the probability that these two balls come from urn U_1 .

III- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points E(2i), A(–i), M(z) and $M'(z')$ where z and z' are two complex numbers such that: $z' = 2i - \dfrac{2}{z}$. $(z \neq 0)$.

1) a- Show that $z\left(z' - 2i\right) = -2$.

   b- Calculate $\arg(z) + \arg(z' - 2i)$.

2) a- Verify that: $z' = \dfrac{2i(z+i)}{z}$.

   b- Show that $OM' = \dfrac{2AM}{OM}$.

   c- As M moves on the perpendicular bisector of $[OA]$, prove that M' moves on a circle (C) whose center and radius are to be determined.

3) Suppose that $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

   a- Show that $x' = \dfrac{-2x}{x^2+y^2}$ and $y' = 2 + \dfrac{2y}{x^2+y^2}$.

   b- If x = y, show that the lines (OM) and (EM′) are perpendicular.

IV- (8 points)

Consider the function f defined over $]-1;+\infty[$ as: $f(x) = e^x - \dfrac{2e^x}{x+1}$.

Denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) a- Determine $\lim\limits_{\substack{x\to -1\\ x>-1}} f(x)$. Deduce an asymptote (D) to (C).

   b- Determine $\lim\limits_{x\to+\infty} f(x)$ and calculate f(2.5).

2) Prove that $f'(x) = \dfrac{(x^2+1)e^x}{(x+1)^2}$ and set up the table of variations of the function f.

3) Let (d) be the line with equation y = x. The curve (C) intersects (d) at a unique point A with abscissa α. Verify that $1.8 < \alpha < 1.9$.

4) a- Specify the coordinates of the points of intersection of (C) with the coordinates axes.

   b- Draw (D), (d) and (C).

5) a- Prove that, over $]-1;+\infty[$, f has an inverse function f^{-1}.

   b- Draw (C'), the representative curve of f^{-1}, in the same system as that of (C).

6) Suppose that the area of the region bounded by (C), the x-axis and the lines with equations x = 0 and x = 1 is 0.53 units of area.
   Calculate the area of the region bounded by (C'), the line (d), the y-axis and the line with equation $x = -1$.

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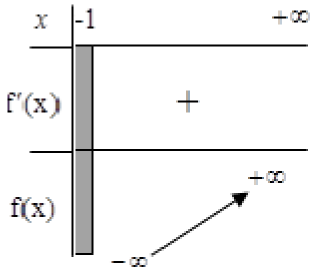
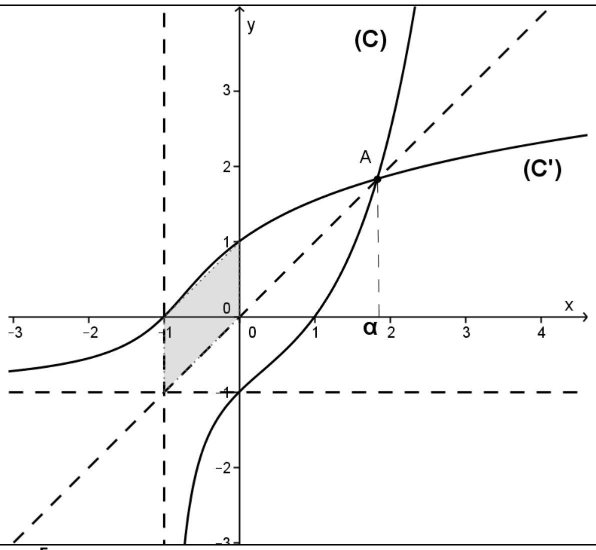
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	مشروع معيار التصحيح

I	Answer	M
1	Verify that F belongs to (P) and (d) lies in it.	1
2	E is on (d) then E belongs to (P) and $\overrightarrow{AE}(-1,0,-2)$ so $\overrightarrow{AE}=-2\overrightarrow{N}_{(P)}$	1
3	$EF^2=3$ and $EL^2=3(t+1)^2$ then $3(t+1)^2=3\Rightarrow$ $t=-2$ or $t=0$ So $L(2,0,-2)$	1
4	$V=\frac{1}{6}\left\lvert\overrightarrow{AL}.(\overrightarrow{AE}\wedge\overrightarrow{AF})\right\rvert=\frac{\lvert-8\rvert}{6}=\frac{4}{3}u^3$	1

II		Answer	M
1	a	$P\left(R/E\right)=\frac{C_3^2}{C_8^2}=\frac{3}{28}$; $p(E\cap R)=p(E).\ P\left(R/E\right)=\frac{2}{6}\times\frac{C_3^2}{C_8^2}=\frac{1}{28}$	1
	b	$p(O\cap R)=p(O).\ P\left(R/O\right)=\frac{4}{6}\times\frac{C_4^2}{C_8^2}=\frac{1}{7}$ $P(R)=p(E\cap R)+p(O\cap R)=\frac{5}{28}$.	1
2		$P(B)=p(E\cap B)+p(O\cap B)=p(E).p(B/E)+p(O).p(B/O)=$ $\frac{1}{3}\times\frac{C_5^2}{C_8^2}+\frac{2}{3}\times\frac{C_4^2}{C_8^2}=\frac{11}{42}$.	1
3		$p(E/B)=\frac{p(E\cap B)}{p(B)}=\frac{5/42}{11/42}=\frac{5}{11}$	1

III		Answer	M
1	a	$z'-2i=\frac{-2}{z}$ then $z(z'-2i)=-2$	0.5
	b	$\arg(z(z'-2i))=\arg z+\arg(z'-2i)=\arg(-2)=\pi[2\pi]$	0.5
2	a	$z'=\frac{2i(z+i)}{z}$	0.5
	b	$OM'=\left\lvert\frac{2i(z+i)}{z}\right\rvert=\frac{\lvert 2i(z+i)\rvert}{\lvert z\rvert}=\frac{\lvert 2i\rvert\lvert z+i\rvert}{\lvert z\rvert}=\frac{2AM}{OM}$	0.5
	c	M belongs to perp bis of $[OA]$ then MA=MO so $OM'=2$, therefore M' Moves on circle center O and radius 2.	0.5
3	a	$x'=\frac{-2x}{x^2+y^2}$ and $y'=2+\frac{2y}{x^2+y^2}$	1
	b	x= y then $M'(\frac{-1}{x},2+\frac{1}{x})$ so $\overrightarrow{EM'}(\frac{-1}{x},\frac{1}{x})$ and $\overrightarrow{OM}(x,y)$ $\overrightarrow{EM'}.\ \overrightarrow{OM}=0$ so $(EM')\perp(OM)$.	0.5

IV		Answer	M
1	a	$\lim\limits_{\substack{x\to -1\\ x>-1}} f(x) = -\infty$ then the line (D) : x = -1 is an asymptote to (C) .	0.5
	b	$\lim\limits_{x\to+\infty} f(x) = (1)(+\infty) = +\infty$; f(2.5) ≈ 5.22.	1
2		$f'(x) = e^x - \left(\dfrac{2e^x(x+1) - 2e^x}{(x+1)^2}\right)$ $= \left(\dfrac{(x+1)^2 - (x+1) + 2}{(x+1)^2}\right)e^x$ $= \left(\dfrac{x^2+1}{(x+1)^2}\right)e^x > 0$ for all x 	1
3		Let $\phi(x) = f(x) - x$. $\phi(1.8) \approx -0.07 < 0$ and $\phi(1.9) \approx 0.17 > 0$	1
4	a	If x =0, then f(0) = -1 ,so (C) cuts the y-axis in (0,-1) If f(x) = 0, then x = 1, so (C) cuts the x-axis in (1; 0).	0.5
	b		1
5	a	On $]-1;+\infty[$, f is continuous and strictly increasing, it has an inverse function f^{-1}.	0.5
	b	(C') and (C) are symmetrical with respect to y = x. See graph.	1
6		Due to symmetry w.r.t. y = x The area of the region bounded by (C'), the line (d), the y-axis and the Line with equation $x = -1$ is = 0.53 + area of right isosceles triangle with side 1 = 1.03 units of area	1.5