وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: العلوم العامة	دورة العام ٢٠١٦ العاديّة الخميس ٩ حزيران ٢٠١٦

عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: أربع ساعات	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2.5 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of each question and give, with justification, the answer corresponding to it.

N°	Questions	Answers			
		a	b	c	d
1	The equation: $\arccos(3x-1)+\arccos x=\frac{\pi}{2}$ is satisfied for x =	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{5}$
2	z is a complex number. If $z=-\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}}$; then $z^2=$	$4e^{i\frac{\pi}{4}}$	$8e^{i\frac{\pi}{4}}$	$(2+\sqrt{2})e^{-i\frac{\pi}{4}}$	$4e^{-i\frac{\pi}{4}}$
3	$\int_{-a}^{a}\frac{x^2}{x^2+1}dx=$	$2\arctan(a)$	$2[a-\arctan(a)]$	0	$a-\arctan(a)$
4	If F is a function defined as : $F(x)=\int_0^x\sqrt{1+t^2}dt$; then $F'(1)=$	$\sqrt{2}$	2	1	$\frac{1}{2}$
5	A sequence (U_n) is defined as $U_0=5$ and $U_{n+1}=\sqrt{2+U_n}$. If (U_n) is convergent, then its limit is:	0	2	-1	$\sqrt{2}$

II- (2.5 points)

In the space referred to a direct orthonormal system $\left(O\,;\,\vec{i}\,,\vec{j}\,,\vec{k}\right)$, consider the plane (P) with equation: $x+y-z+1=0$, the point $A\,(1\,;\,0\,;-1)$ and the line (d) defined as : $x=t-1\,;\,y=t\,;\,z=-t+3$ (t is a real parameter).

1) a- Show that the line (d) is perpendicular to plane (P).

b- Determine the coordinates of H, the point of intersection of (d) and (P).

2) Verify that the point $K(0\,;-1\,;\,0)$ is the orthogonal projection of A on (P).

3) Denote by (Δ) the line passing through H, contained in the plane (P) and perpendicular to the line (KH).

a- Verify that $\vec{V}(-2\,;1\,;-1)$ is a direction vector of the line (Δ).

b- Write a system of parametric equations of line (Δ).

4) Consider in the plane (P) the circle (C) with center H and radius $\sqrt{6}$. This circle intersects the line (Δ) in two points T and S. Determine the coordinates of T and S.

III- (3 points)

Given:

- A bag B_1 containing **one** 20 000 LL bill and **three** 50 000 LL bills.
- A bag B_2 containing **two** 20 000 LL bills and **two** 100 000 LL bills.
- A six-sided fair die (numbered 1 through 6).

1) The die is rolled once.
If this die shows 5 or 6, one ball is randomly selected from bag B_1, otherwise one ball is randomly selected from bag B_2 .

Consider the following events:
A : «obtain a 20 000 LL bill»
B : «obtain a 50 000 LL bill»
C : «obtain a 100 000 LL bill»
E : «the die shows the number 5 or 6».

a- Verify that the probability of the event A is $P(A)=\dfrac{5}{12}$.

b- Which bill is the most probably to be selected? Justify.

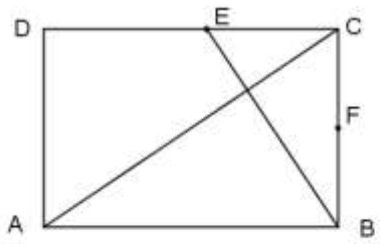
2) All bills from B_1 and B_2 are now placed in the same bag B and the same die is rolled.
If the die shows 5 or 6, then two bills are randomly and simultaneously selected from bag B, otherwise three bills are randomly and simultaneously selected from B.

a-Verify that the probability to have a total sum less than 80 000 LL is $\dfrac{13}{84}$.

b- The total sum obtained is less than 80 000 LL. What is the probability that the die shows the face numbered 3?

IV- (3 points)

ABCD is a direct rectangle such that AB = 3, AD = 2 and $\left(\overrightarrow{AB}, \overrightarrow{AD}\right) = \frac{\pi}{2}[2\pi]$.



F is the midpoint of segment [BC].

The perpendicular through B to the line (AC) intersects (DC) at E.

Let S be the similitude that maps A onto B and B onto F.

1) Determine the ratio (scale factor) k and an angle α of S.
2) Justify that the image of line (AC) under S is (BE).
3) Determine the image of (BC) under S. Deduce the point H image of C under S.
4) Determine the image of the rectangle ABCD under S.
5) The plane is referred to a direct orthonormal system $(A;\vec{u},\vec{v})$ with $\vec{u} = \frac{1}{3}\overrightarrow{AB}$ and $\vec{v} = \frac{1}{2}\overrightarrow{AD}$.
   Write the complex form of S, then deduce the affix of its center W.

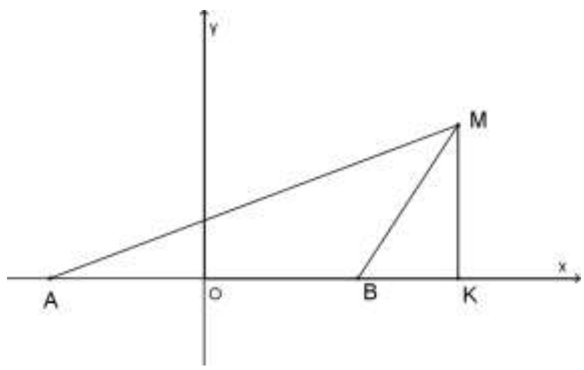
6) Let M be a point in the plane with affix $z = 3\cos\theta + 2i\sin\theta$ $\left(\text{with } 0 < \theta < \frac{\pi}{2}\right)$.

   a- Prove that M moves on an ellipse (Γ) with center A, having B and D as two of its vertices.

   b- Write an equation of (Γ') the image of the ellipse (Γ) under S.

V- (2 points)

In the plane referred to an orthonormal system $(O;\vec{i},\vec{j})$, consider the points A(–1;0) and B(1;0). Let M(x ; y) be any point in the plane such that $|x| \geq 1$. Denote by K the orthogonal projection of M on the x-axis.
Suppose that: $MK^2 = AK \times BK$.



1) Prove that M moves on the hyperbola (H) with equation $x^2 - y^2 = 1$.

2) a- Find the coordinates of the vertices and foci of (H).

   b- Write the equations of the asymptotes to (H).

   c- Draw (H)

3) Consider the point G(0;–1) and the parabola (P) with equation $y = \frac{1}{2}x^2$.

   Let L be the common point to (P) and (H) so that $x_L > 0$.

   Prove that (GL) is a common tangent to (P) and (H).

VI- (7 points)

Part A

Consider the differential equation $(E): y' + y = 1 + x + e^{-x}$.

1) Verify that $u = x + xe^{-x}$ is a particular solution of (E).
2) Let $y = z + u$ where z is a function of x.
   a- Form the differential equation (E') satisfied by z.
   b- Solve (E') and deduce the general solution of (E).
   c- Find the particular solution of (E) verifying $y(0) = 1$.

Part B

Let h be the function defined over $\mathbb{R}$ as $h(x) = 1 - xe^{-x}$.

1) Calculate $h'(x)$ and set up the table of variations of the function h.
2) For all real numbers x, verify that $h(x) > 0$.

Part C

Let f be the function defined over $\mathbb{R}$ as $f(x) = x + (x+1)e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$; graphical unit is **2 cm** .

1) Calculate $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to -\infty} \frac{f(x)}{x}$. Give a graphical interpretation.
2) Let (d) be the line with equation $y = x$.
   a- Discuss, according to the values of x, the relative positions of (C) and (d).
   b- Show that (d) is an asymptote to (C) at $+\infty$.

3) a- Verify that $f'(x) = h(x)$ and set up the table of variations of the function f.
   b- Prove that the curve (C) has an inflection point W whose coordinates are to be determined.
   c- Let E be the point on (C) where the tangent (D) to (C) is parallel to the line (d).
   Determine the coordinates of E.

4) Prove that the equation f(x) = 0 has a unique root α, then verify that $-0.7 < \alpha < -0.6$.

5) Draw (d), (D) and (C).

6) Let g be the inverse function of f, and denote by (G) the representative curve of g in the system $(O; \vec{i}, \vec{j})$.
   a- Draw (G).
   b- Solve the inequality $\ln(-g(x)) > 0$.

7) Calculate, in cm^2, the area of the region bounded by the curve (G), the line (d) and the x-axis.

Marking Scheme- Math GS – First Session - 2016

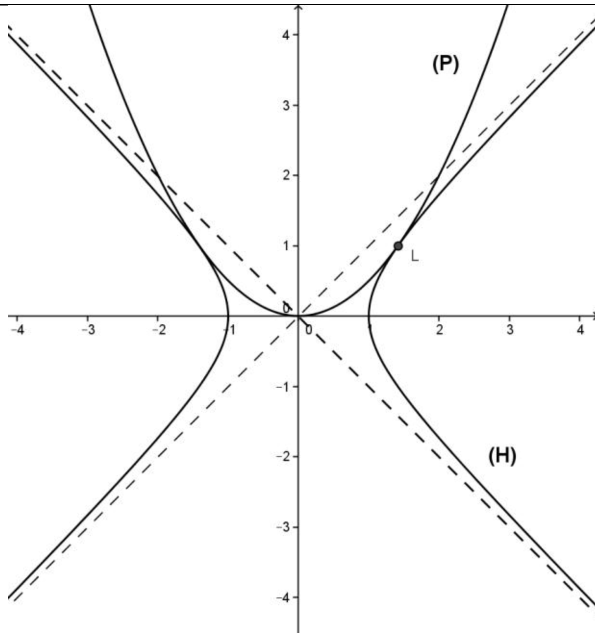
QI	Solution	G
1	$\arccos(3x-1)=\frac{\pi}{2}-\arccos x$; $\frac{1}{3}\le x\le\frac{2}{3}$; $\cos(\arccos(3x-1))=\sin(\arccos x)$ $3x-1=\sqrt{1-x^2}$; thus $x=0$ or $x=\frac{3}{5}$ ($x=0$ rejected) **Or** by calculator. (c)	1
2	$z=-\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}}$; $z^2=2\sqrt{2}-i\sqrt{2}=4e^{-i\frac{\pi}{4}}$ OR let $\theta=\arg(z)$ then $\arg(z)=\frac{7\pi}{8}$ and $\lvert z\rvert=2$ hence $z^2=4e^{-i\frac{\pi}{4}}$ (d)	1
3	$\int_{-a}^{a}\frac{x^2}{x^2+1}dx=2\int_{0}^{a}\frac{x^2}{x^2+1}dx=2\int_{0}^{a}\left(1-\frac{1}{x^2+1}\right)dx=2(a-\arctan a).$ (b)	1
4	$F(x)=\int_{0}^{x}\sqrt{1+t^2}dt,\ F'(x)=\sqrt{1+x^2}$; $F'(1)=\sqrt{2}$ (a)	1
5	$\lim\limits_{n\to\infty}u_n=L,\ L=\sqrt{2+L}$, $L^2-L-2=0$ $(L\ge 0)$, L=2 ou L=-1(rej) (b)	1

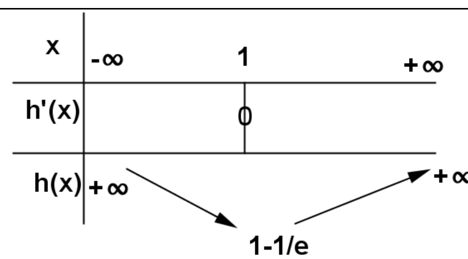
QII	Solution	G
1a	$\overrightarrow{v_d}(1,1,-1)=\overrightarrow{n_P}$ then (d) is perpendicular to plane (P).	0.5
1b	t-1+t+t-3+1=0 thus t=1 . H(0,1,2)	1
2	$\overrightarrow{AK}(-1,-1,1)=-\vec{n}_{(P)}$ and $K\in(P)$	1
3a	$\overrightarrow{V_{(\Delta)}}=\overrightarrow{AH}\wedge\overrightarrow{n_{(P)}}$ **OR** $\vec{V}_{(\Delta)}.\overrightarrow{AH}=0$ et $\vec{V}_{(\Delta)}.\vec{V}_{(d)}=0$	1
3b	$x=-2k,\ y=k+1,\ z=-k+2.$	0.5
4	$R=\sqrt{6}$; $M\in(\Delta)$ $HM^2=R^2$ then $6k^2=6$, k= ±1 $T(-2,2,1),S(2,0,3)$	1

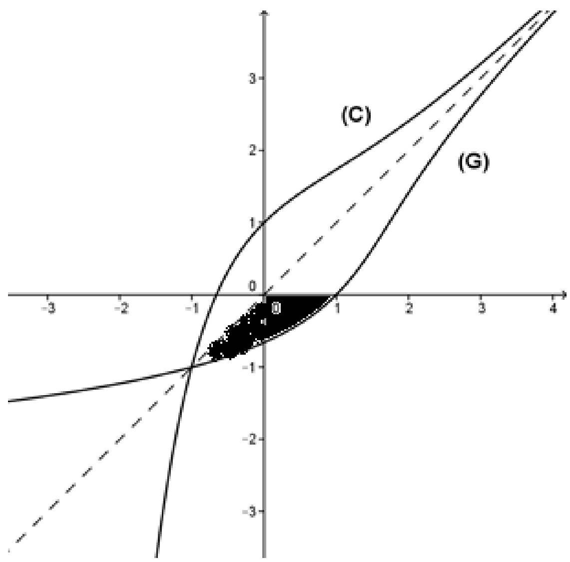
QIII	Solution	G
1a	$P(A)=P(E\cap A)+P(\overline{E}\cap A)=P(E)\times P(A/E)+P(\overline{E})\times P(A/\overline{E})=\frac{2}{6}\times\frac{1}{4}+\frac{4}{6}\times\frac{2}{4}=\frac{5}{12}$	1.5
1b	$P(B)=\frac{1}{4}$ and P(C)=1- [P(B)+P(C)]= $\frac{1}{3}$ thus the bill 20 000 is the most probable to be obtained.	1.5
2a	$P(S<80\ 000)=\frac{2}{3}\left(\frac{C_3^2}{C_8^2}\right)+\frac{2}{6}\left(\frac{C_3^1\times C_3^1}{C_8^2}\right)+\frac{4}{8}\left(\frac{C_3^3}{C_8^3}\right)=\frac{13}{84}$	1.5
2b	$P(\text{face}3/S<80000)=\dfrac{\frac{1}{6}\times\frac{C_3^3}{C_8^3}}{\frac{13}{84}}=\frac{1}{52}$	1.5

QIV	Solution	G
1	$\left.\begin{matrix} A \to B \\ B \to F \end{matrix}\right\}$ BF = KAB, $K=\frac{1}{3}, \alpha = \left(\overrightarrow{AB}, \overrightarrow{BF}\right) = \frac{\pi}{2}$	0.5
2	S(AC) is a line passing through B and perpendicular to (AC) so it is (BE).	0.5
3	S(BC) is the line (Δ) passing through F and perpendicular to (BC) $\left.\begin{matrix} (AC) \to (BE) \\ (BC) \to (\Delta) \end{matrix}\right\}$ thus S(C) =H=$(\Delta) \cap (BE)$	1.5
4	S(ABCD)=BFHP with P being the fourth vertex of the rectangle BFHP	0.5
5	$z' = \frac{1}{3}iz + 3,\ z_W = \frac{27}{10} + \frac{9}{10}i$	1
6a	$z = 3\cos\theta + 2i\sin\theta = x + iy$ then $\cos\theta = \frac{x}{3}$ and $\sin\theta = \frac{y}{2}$, thus$(\Gamma): \frac{x^2}{9} + \frac{y^2}{4} = 1$ Center A(0,0). Vertices M(3,0)=B and N(0,2)=D.	1
6b	$z' = \frac{1}{3}i(x+iy) + 3$, $x = 3y'; y = 9 - 3x'$, thus $(\Gamma'): \frac{(x-3)^2}{4/9} + y^2 = 1$ **OR** : The center of (Γ') is B(3,0), the two vertices are F and H such that BF=1 and FH=2/3. Equation : $\frac{(x-3)^2}{4/9} + y^2 = 1$	1

QV	Solution	G
1	$MK^2 = AK \times BK$, $y^2 = \lvert x-1 \rvert \times \lvert x+1 \rvert = x^2 - 1$, $x^2 - y^2 = 1$	1
2a	Rectangular hyperbola. Vertices : A(-1,0) and B(1,0) . Foci: F($\sqrt{2}$,0) and F'(-$\sqrt{2}$,0).	1
2b	Asymptotes : $y = x$, $y = -x$.	0.5

2c		0.5
3	$y^2 - 2y + 1 = 0$ $y = 1$ thus $x = \pm\sqrt{2}$. $L(\sqrt{2},1)$ is a common point to (P) and (H). $G(0,-1)$; $L(\sqrt{2},1)$. $(GL): y = \sqrt{2}x - 1$	1

QVI		Solution	G
A	1	$u(x) = x + xe^{-x}$, $u'(x) = 1 + e^{-x} - xe^{-x}$ $.1 + e^{-x} - xe^{-x} + x + xe^{-x} = 1 + x + e^{-x}$ thus, u(x) is a solution of the differential equation.	0.5
	2a	$z' + z = 0$	0.5
	2b	$z = Ce^{-x}$ thus $y = Ce^{-x} + x + xe^{-x}$	0.5
	2c	y(0)=C=1thus $y = x + (x+1)e^{-x}$	0.5
B	1	$h(x) = 1 - xe^{-x}$, $h'(x) = -e^{-x} + xe^{-x} =$ e^{-x} . $h'(x) > 0$ si x>0 $\lim\limits_{x\to-\infty} (x+1)e^{-x} = -\infty$ 	1
	2	The minimum of h is positive thus h(x)>0 for all real numbers x.	0.5
C	1	$\lim\limits_{x\to-\infty} f(x) = -\infty$ since $\lim\limits_{x\to-\infty} (x+1)e^{-x} = -\infty$. $\lim\limits_{x\to-\infty} \frac{f(x)}{x} = +\infty$ thus the curve (C) has an asymptotic direction parallel to y'Oy.	1

	2a	$f(x)-y=(1+x)e^{-x}$. If x=-1, (C) intersects (d) in A(-1,-1), if x>-1 (C) is above (d). If x<-1 (C) is below (d).	1
	2b	$\lim_{x\to+\infty} f(x)=+\infty$ since $\lim_{x\to+\infty}\frac{1+x}{e^x}=0$ and since $\lim_{x\to+\infty}[f(x)-x]=\lim_{x\to+\infty}\frac{1+x}{e^x}=0$ then y=x is an asymptote to (C)	0.5
	3a	$f'(x)=h(x)$ thus $f'(x)>0$ for all x	0.5
	3b	$f''(x)=h'(x)$ but $h'(x)=0$ for x=1 thus W(1,1+2e^{-1}) is a point of inflection..	1
	3c	$f'(x)=1$ thus $-xe^{-x}=0$ so $x=0$ and E(0,1)	1
	4	f is defined, continuous and strictly increasing from $-\infty$ to $+\infty$ thus the equation f(x) =0 has a unique root α . $f(-0.7)\times f(-0.6)=-0.0958\times 0.1288=-0.01<0$ hence $-0.7<\alpha<-0.6$	1
	5		1
	6a	(G) is the symmetric of (C) with respect to the line with equation y=x	1
	6b	$\ln(-g(x))>0; -g(x)>0$ and $-g(x)>1$ thus $g(x)<-1$ therefore $x\in\left]-\infty,-1\right[$	1
	7	A= $\int_{-1}^{0}[f(x)-x]=\int_{-1}^{0}(e^{-x}+xe^{-x})dx=4(e-2)\,cm^2$	1.5