وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة المتوسطة

دورة العام ٢٠١٦ العادية
الثلاثاء ٣١ أيار ٢٠١٦

عدد المسائل: خمسة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I - (2 points)

Consider the three numbers A, B and C:

$$A=\frac{9}{2}-\frac{9}{2}\times\frac{1}{3} \quad ; \quad B=\frac{10^{14}\times 2^{10}}{5\times 4\times 10^{12}\times 2^{9}} \quad ; \quad C=\left(2+\sqrt{5}\right)^2+\left(1-2\sqrt{5}\right)^2 .$$

1) By writing all the steps of calculation, show that A, B and C are natural numbers.

2) Verify that $A\times B=C$.

II – (4 points)

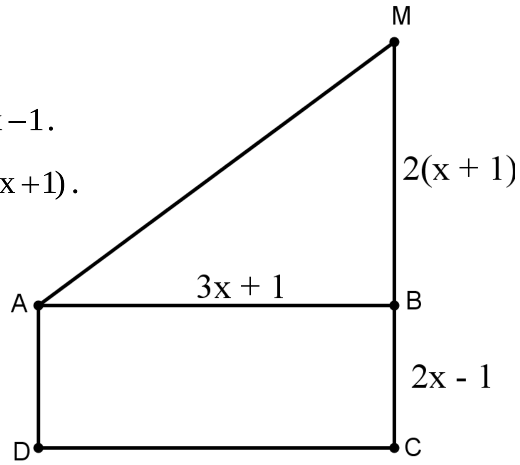
Given the expression: $E(x)=(3x+1)(2x-1)-(3x+1)(x+1)$.

1) Show that $E(x)=(3x+1)(x-2)$.

2) Solve the equation E(x) = 0.

3) In the adjacent figure:

- x *is a length expressed in cm such that x* > 1.
- ABCD is a rectangle such that $AB=3x+1$ and $BC=2x-1$.
- ABM is a triangle, right angled at B, such that $MB=2(x+1)$.



Denote by S the area of ABCD and S' that of ABM.

a. Express S and S' in terms of x.

b. Verify that S – S′ = E(x).

c. Calculate x so that S = S′.

III – (3 points)

1) Solve the following system: $\begin{cases} 6x+4y=20\,000 \\ 2x+8y=15\,000 \end{cases}$

2) A bookshop offers 40% discount on the price of a copybook and 60% discount on that of a pencil.

The sum of the original prices of 2 copybooks and 8 pencils is 15 000 L.L.

The sum of prices, after discount, of one copybook and one pencil is 2 000 L.L.

a. Prove that the previous information can be modeled by the above system.

b. Find the price of a copybook and that of a pencil after the discount.

IV - (6 points)

In an orthonormal system of axes (x'Ox, y'Oy), consider the points A(–2; 0) and B(1; 3).

Let (d) be the line with equation $y = -x + 4$.

1) **a.** Plot the points A and B.

b. Verify, by calculation, that point B is on the line (d), then draw (d).

c. Determine the equation of line (AB) and verify that (AB) is perpendicular to (d).

d. The line (d) intersects x'Ox at E and y'Oy at F. Calculate the coordinates of points E and F.

2) Let (C) be the circle circumscribed about the triangle ABF.

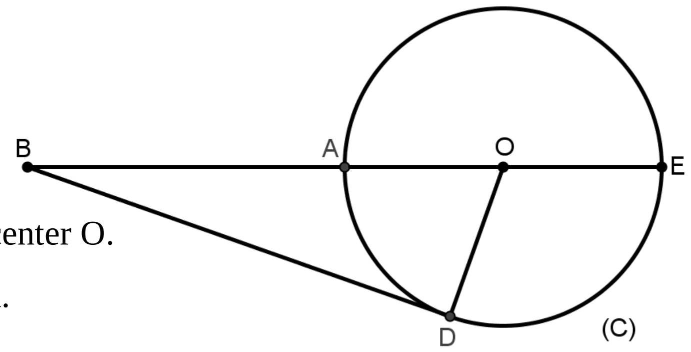
a. Determine the coordinates of point I, the center of (C). Calculate the radius of (C).

b. Verify that O is a point on the circle (C).

3) **a.** Calculate AB.

b. Calculate, rounded to the nearest degree, the measure of $\widehat{BAF}$.

V - (5 points)



In the adjacent figure:

- AE = 4 cm.
- (C) is the circle with diameter [AE] and center O.
- B is the symmetric of E with respect to A.
- (BD) is tangent to (C) at D.

1) Copy the figure.

2) Calculate BD.

3) The parallel through point A to (OD) intersects the line (BD) at M and (ED) at L.

a. Show that D is the midpoint of [EL].

b. Deduce that M is the centroid of triangle EBL.

4) **a.** Prove that the two triangles BDE and BAD are similar.

b. Calculate $\frac{DE}{DA}$.

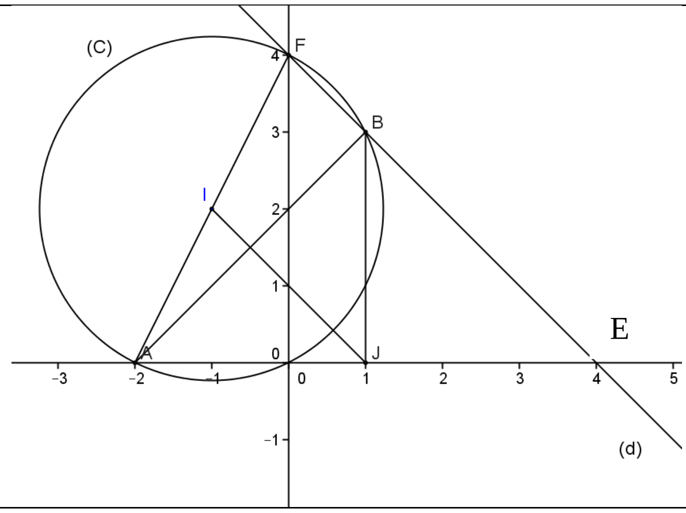
5) Let F be the translate of A by the translation with vector $\overrightarrow{ED}$.

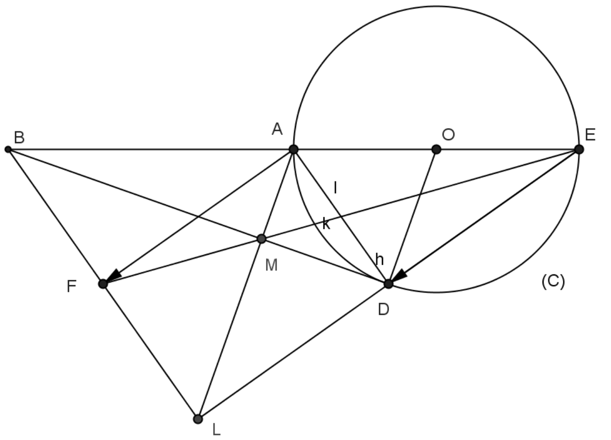
a. Prove that ADLF is a rectangle.

b. Prove that F is the midpoint of [BL].

c. Deduce that the points E, M and F are collinear.

دورة عام ٢٠١٦ العادية اللغة الانكليزية	الشهادة المتوسطة مشروع معيار التصحيح	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات

	Question I	
	Answers	**note**
1	$A = \frac{9}{2} - \frac{9}{2} \times \frac{1}{3} = \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3,$ ¼ + ¼ $B = \frac{10^{14} \times 2^{10}}{5 \times 4 \times 10^{12} \times 2^{9}} = \frac{10^2 \times 2}{20} = \frac{200}{20} = 10$ ¼ + ¼ $C = \left(2+\sqrt{5}\right)^2 + \left(1-2\sqrt{5}\right)^2 = 4 + 4\sqrt{5} + 5 + 1 - 4\sqrt{5} + 20 = 30$ ¼ + ¼ +¼	**1¾**
2	$A \times B = 3 \times 10 = 30$ $c = 30$ Then : $A \times B = C$	¼
	Question II	
1	E(x) = (3x +1)(2x−1 – x – 1) = (3x+1)(x – 2). ½ + ¼	¾
2	(3x+1)(x – 2) = 0 Then x = $\frac{-1}{3}$ or x = 2 ¼ +¼	½
3.a	S= (3x +1)(2x−1) S'= (3x +1)(x+1) ½ + ¾	**1¼**
3.b	S-S'= (3x +1)(2x−1) - (3x +1)(x+1) = E(x)	½
3.c	S=S' then S-S' = 0 ; E(x) = 0 x=-1/3 (rejected) x = 2 (accepted) ¼ + ¼ +¼ + ¼	**1**
	Question III	
1	x= 25000 ; y = 1250 ¾ + ¼	1
2.a	1st equation: 2x + 8y = 15000 ¼ 2nd equation: (1-0.4)x + (1-0.6)y = 2000 then 6x + 4y = 20000 ¾	1
2.b	The price of a copybook = 2500 ×(0.6) = 1500 L.L ½ The price of a pencil = 1250× (0.4) = 500 L.L ½	1
	Question IV	
1.a		½
1.b	$x_B = 1$ $-x_B + 4 = 3 = y_B$ ¼ + ½ (For drawing the line)	¾
1.c	Equation of (AB) : y = x + 2 ½ + ½ slope (AB) = 1 and slope (d) = -1 then slope(AB) × slope(d) = -1 ¼ So (AB) is perpendicular to (d)	**1¼**
1.d	E(4;0) and F(0 ; 4) ¼ + ¼	½

2.a	$\widehat{ABF} = 90°$ (ABF is inscribed in a semicircle of diameter [AF]) ¼ I midpoint of [AF] then I(-1; 2) ½ R = $\frac{AF}{2} = \sqrt{5}$ or AI = IB = IF = R = $\sqrt{5}$ ½	**1¼**
2.b	OI = $\sqrt{5}$ or $\widehat{AOF} = 90°$ then O is a point of the circle.	½
3.a	AB = $\sqrt{18} = 3\sqrt{2}$	½
3.b	$\cos\widehat{BAF} = \frac{AB}{AF} = \frac{3\sqrt{2}}{2\sqrt{5}}$ ¼ Then $\widehat{BAF} = cos^{-1}\left(\frac{3\sqrt{2}}{2\sqrt{5}}\right)$=18,43°$\approx 18°$ ¼ + ¼	¾
	Question V	
1		½
2	The triangle ABD is right at D Then by Pythagoras theorem $BD^2 = OB^2 - OD^2$ BD = $\sqrt{32}$ = $4\sqrt{2}$	½
3.a	In the triangle ALE : (AL) // (OD) and O midpoint of [AE] Then by the converse of the midpoint theorem, D midpoint of [EL].	½
3.b	In the triangle BEL : [LA] and [BD] are two medians intersect at M, then M centroid.	½
4.a	The 2 triangles BDE and BAD are similar since : $\hat{B}$ is a common angle ½ $\widehat{ADB} = \widehat{AED} = \frac{\widehat{AD}}{2}$ ½	1
4.b	The ratio of similarity : $\frac{BE}{BD} = \frac{DE}{DA} = \frac{BD}{BA}$ ¼ $\frac{DE}{DA} = \frac{BD}{BA} = \frac{4\sqrt{2}}{4} = \sqrt{2}$ ¼	½
5.a	$\overrightarrow{AF} = \overrightarrow{ED} = \overrightarrow{DL}$ then AFLD is a parallelogram ¼ $\widehat{ADL} = 90°$ then AFLD is a rectangle ¼	½
5.b	AD = FL (Opposite sides in a rectangle) AD = $\frac{BL}{2}$ (Midpoint theorem) ¼ Then BL = 2 FL , B, F and L are collinear ((AD) // (BL) and (AD) // (FL)) ¼ Then F midpoint of [BL]	½
5.c	[EF] 3rd median in the triangle EBL then E, M and F are collinear.	½