الدورة الاستثنائية للعام 2013	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ساعتان	

This exam is formed of three exercises in three pages.
The use of non-programmable calculators is recommended.

First exercise: (6 points)

Applications of the diffraction of light

A – Measurement of the width of a slit

A laser beam of light, of wavelength in vacuum $\lambda = 632.8$ nm, falls normally on a vertical slit of width «a». The diffraction pattern is observed on a screen placed perpendicularly to the laser beam at a distance D = 1.5 m from the slit.
Let «L» be the linear width of the central fringe (Fig. 1).
The angle of diffraction θ corresponding to a dark fringe of order n is given by $\sin\theta = \frac{n\lambda}{a}$ where $n = \pm1, \pm2, \pm3\ldots$
For small angles, take $\tan\theta \approx \sin\theta \approx \theta$ in radian.

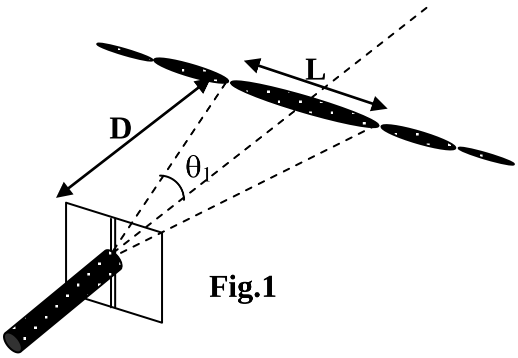


Fig.1

1) Describe the aspect of the diffraction pattern observed on the screen.
2) Write the relation among a, θ_1 and λ.
3) Establish the relation among a, λ , L and D.
4) Knowing that L = 6.3 mm, calculate the width «a» of the used slit.

B – Controlling the thickness of thin wire

A manufacturer of thin wires wishes to control the diameter of his product. He uses the same set-up mentioned in part (A) but he replaces the slit by a thin vertical wire. He observes on the screen the phenomenon of diffraction (figure 2).
For D = 2.60 m, he obtains a central fringe of constant linear width L_1 = 3.4 mm.

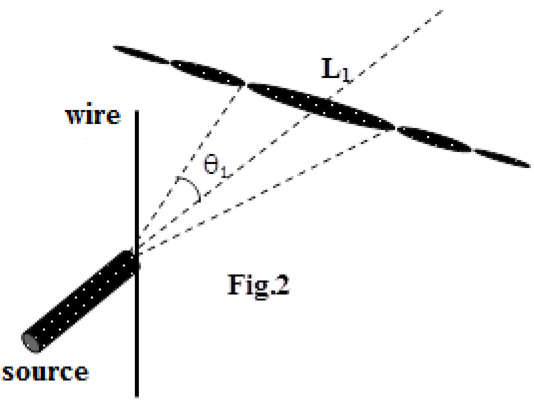


Fig.2

1) Calculate the value of the diameter «a_1» of the wire at the illuminated point.
2) The manufacturer illuminates the wire at different positions under the same precedent conditions. Specify the indicator that permits the manufacturer to check that the diameter of the wire is constant.

C – Measurement of the index of water

We place the whole set-up of part (A) in water of index of refraction n_{water} . We obtain a new diffraction pattern.
We find that for D = 1.5 m and a = 0.3 mm, the linear width of the central fringe is L_2 = 4.7 mm.
1) Calculate the wavelength λ' of the laser light in water.
2) **a)** Determine the relation among λ , λ' and n_{water} .
b) Deduce the value of n_{water} .

Second exercise: (7 points) **Mechanical oscillator**

Consider a mechanical oscillator constituted of a spring, of negligible mass, and of un-joined loops of stiffness k and a solid (S) of mass m = 0.1 kg.
The spring, placed horizontally, is fixed from one of its extremities to a fixed support and (S) is attached to the other extremity. (S) may slide without friction on a horizontal rail AB and its center of inertia G can move along a horizontal axis x'Ox. At equilibrium, G coincides with the origin O of the axis x'x (Fig. 1).

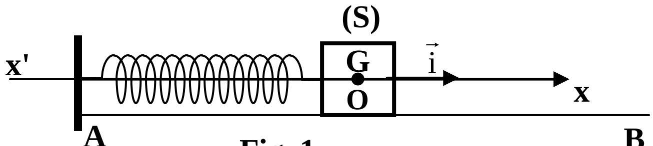


Fig. 1

The solid (S) is displaced from its equilibrium position by a distance $x_0 = \overline{OG_0}$ and we give it, at the instant $t_0 = 0$, in the positive direction an initial velocity $\vec{v}_0 = v_0\vec{i}$. Thus, (S) performs mechanical oscillations along x'Ox.

A – Theoretical study

At the instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic measure of its velocity is $v = \frac{dx}{dt}$.

Take the horizontal plane passing through G as a reference level of gravitational potential energy.

1) Write, at an instant t, the expression of the mechanical energy ME of the system (oscillator, Earth) in terms of m, x, k and v.

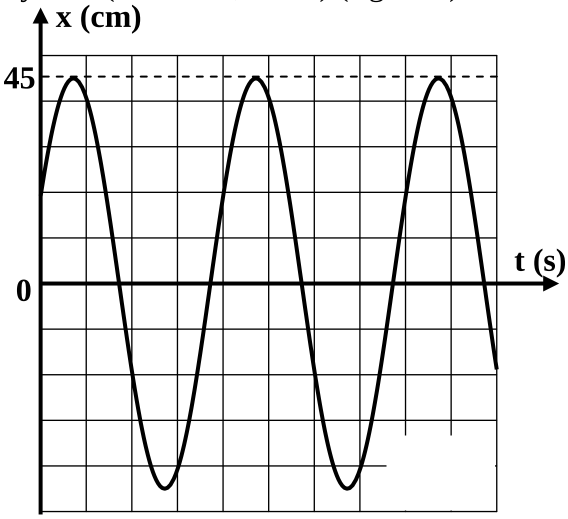
2) Establish the second order differential equation in x that describes the motion of G.

3) The solution of this differential equation has the form: $x = X_m \sin(\frac{2\pi}{T_0}t + \varphi)$, where X_m, T_0 and φ are constants. Determine the expression of the proper period T_0 in terms of m and k.

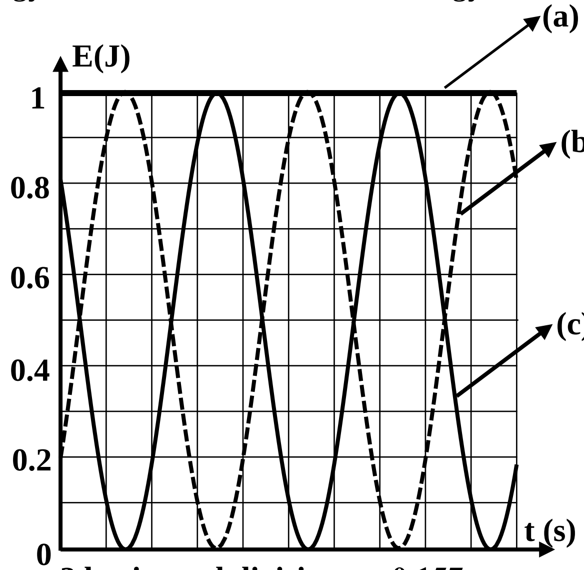
B – Graphical study of the motion

An appropriate device allows to obtain the variations with respect to time:

- of the abscissa x of G (figure 2);
- of the kinetic energy KE, of the elastic potential energy PE_e and of the mechanical energy ME of the system (oscillator, Earth) (figure 3).



1 horizontal division → 0.157 s
1 vertical division → 10 cm



2 horizontal divisions →0.157s **Fig.3**

1) Referring to figure (2), indicate the value of:
a) the initial abscissa x_0 ;
b) the amplitude X_m ;
c) the period T_0.

2) Determine the values of k and φ.

3) The curves (a), (b), and (c) of figure 3 represent the variations of the energies of the system (oscillator, Earth) as a function of time. Using this figure:
a) identify, with justification, the energy represented by each curve;
b) determine the value of the initial velocity v_0 ;

c) **i)** indicate the value of the period T of the KE and PE_e ;
ii) deduce the relation between T and T_0.

Third exercise: (7 points) Charging and discharging of a capacitor

The aim of this exercise is to determine, by two different methods, the value of the capacitance C of a capacitor. For this aim, we set-up the circuit of figure 1. This circuit is formed of an ideal generator delivering a constant voltage of value E = 10 V, a capacitor of capacitance C, two identical resistors of resistances $R_1 = R_2 = 10\ k\Omega$ and a double switch K.

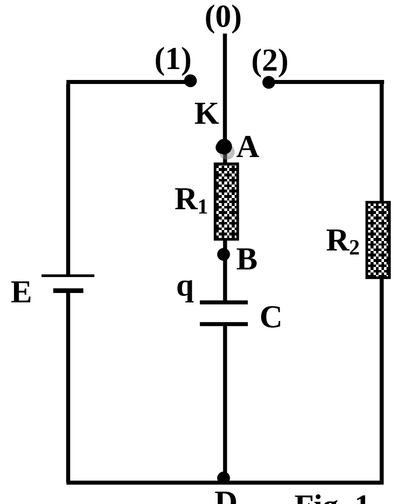


Fig. 1

A – Charging the capacitor

The switch K is in the position (0) and the capacitor is neutral. At the instant $t_0 = 0$, we turn K to position (1) and the charging of the capacitor starts.

1) Theoretical study

a) Applying the law of addition of voltages and taking the positive direction along the circuit as that of the current, show that the differential equation that describes the variation of the voltage $u_C = u_{BD}$ across the capacitor has the form: $E = R_1 C \frac{du_C}{dt} + u_C$.

b) The solution of this differential equation has the form: $u_C = A(1 - e^{\frac{-t}{\tau_1}})$ where A and τ_1 are constants. Show that A = E and $\tau_1 = R_1C$.

c) Show that at the end of charging $u_C = E$.

d) Show that the expression $u_{AB} = u_{R_1} = E\ e^{\frac{-t}{R_1C}}$.

e) Establish the expression of the natural logarithm of u_{R_1} [$\ell n\,(u_{R_1})$] as a function of time.

2) Graphical study

The variation of $\ell n\,(u_{R_1})$ as a function of time is represented by figure 2.

a) Justify that the shape of the obtained graph agrees with the expression of $\ell n(u_{R_1})$ as a function of time.

b) Deduce, using the graph, the value of the capacitance C.

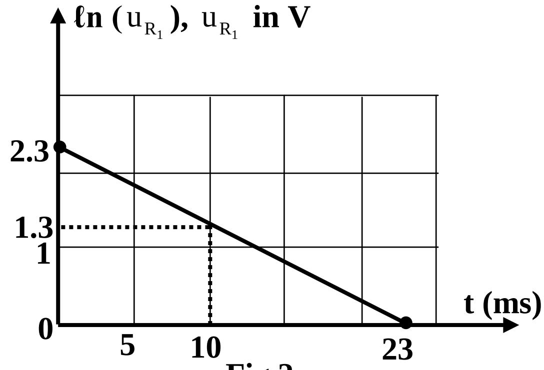


Fig.2

B – Discharging the capacitor

The capacitor being fully charged, we turn the switch K to position (2). At an instant $t_0 = 0$, taken as a new origin of time, the discharging of the capacitor starts.

1) During discharging, the current circulates from B to A in the resistor of resistance R_1. Justify.

2) Taking the positive direction along the circuit as that of the current, show that the differential equation in the voltage u_C across the capacitor has the form: $u_C + (R_1 + R_2)\,C\frac{du_C}{dt} = 0$.

3) The solution of the above differential equation has the form: $u_C = Ee^{\frac{-t}{\tau_2}}$ where τ_2 is the time constant of the circuit during discharging. Show that $\tau_2 = (R_1 + R_2)$ C.

4) The variation of the voltage u_C across the capacitor and the tangent to the curve $u_C = f(t)$ at the instant $t_0 = 0$, are represented in figure 3. Deduce, from this figure, the value of the capacitance C.

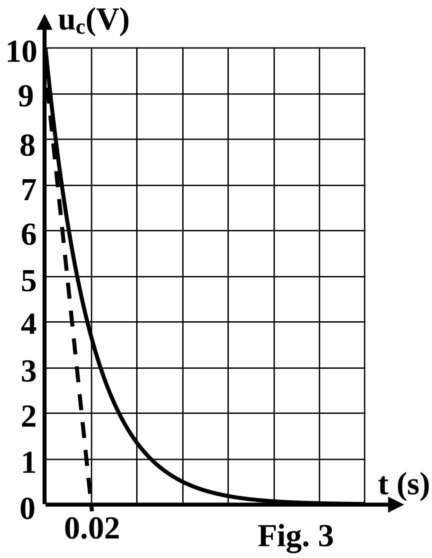


Fig. 3

First exercise (6 points)

Part of the Q	Answer	Mark
A-1	We observe: - alternating bright and dark fringes; - the width of the central fringe is double that of any other bright fringe; - The direction of the pattern of fringes is perpendicular to that of the slit.	¾
A-2	$\sin\theta_1 = \frac{\lambda}{a} \approx \theta_1$	¼
A-3	We have $\tan\theta_1 = \frac{L}{2D}$, for small θ_1, $\tan\theta_1 \approx \theta_1 = \frac{L}{2D}$. and $\sin\theta_1 \approx \theta_1$ so $\frac{\lambda}{a} = \frac{L}{2D}$.	**1**
A-4	$a = \frac{2D\lambda}{L} = \frac{2\times1.5\times632.8\times10^{-9}}{6.3\times10^{-3}} = 3\times10^{-4}$ m = 0.3 mm.	¾
B-1	The diameter of the wire $= \frac{2\times2.6\times632.8\times10^{-9}}{3.4\times10^{-3}}$ $= 0.967\times10^{-3}$ m = 0.967 mm	¾
B-2	The linear width of the central fringe. Because if L = constant ⇒ a = constant	½
C.1	Apply the same ralation we obtain : $\frac{\lambda'}{a} = \frac{L_2}{2D}$ $\Rightarrow \lambda' = \frac{aL_2}{2D} = \frac{0.3\times10^{-3}\times4.7\times10^{-3}}{2\times1.5} = 470\times10^{-9}$ m	¾
C-2-a	$\lambda' = \frac{V}{\nu}$ and $\lambda = \frac{C}{\nu} \Rightarrow \frac{\lambda'}{\lambda} = \frac{V}{C} = \frac{1}{n_{water}} \Rightarrow \lambda' = \frac{\lambda}{n_{water}}$	¾
C-2-b	$n_{water} = \frac{\lambda}{\lambda'} = \frac{623.8}{470} = 1.346$	½

Second exercise (7 points)

Part of the Q	Answer	Mark
A-1	ME = ½ kx^2 + ½ mv^2	½
A-2	No friction then $\frac{dME}{dt} = 0 = kxx' + mvv' \Rightarrow x'' + \frac{k}{m}x = 0$.	¾
A-3	$x = X_m\sin(\frac{2\pi}{T_0}t+\varphi) \Rightarrow v = \frac{2\pi}{T_0}X_m\cos(\frac{2\pi}{T_0}t+\varphi)$ $\Rightarrow x'' = -(\frac{2\pi}{T_0})^2X_m\sin(\frac{2\pi}{T_0}t+\varphi)$ $\Rightarrow -(\frac{2\pi}{T_0})^2X_m\sin(\frac{2\pi}{T_0}t+\varphi) + \frac{k}{m}X_m\sin(\frac{2\pi}{T_0}t+\varphi) = 0$ $\Rightarrow T_0 = 2\pi\sqrt{\frac{m}{k}}$.	1
B-1-a	x_0 = 20 cm	¼
B-1-b	X_m = 45 cm	¼
B-1-c	$T_0 = 4\times0.157 = 0.628$s	½
B-2	$T_0 = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \frac{4\pi^2m}{T_0^2}$ $\Rightarrow$ k = 10 N/m. For t_0=0, $x = x_0 = X_m\sin\varphi \Rightarrow \sin\varphi = \frac{x_0}{X_m} = \frac{20}{45} = 0.44 \Rightarrow \varphi = 0.46$ rad or $\varphi = \pi - 0.46$ rad ; but $v_0 = X_m\,\omega\cos\varphi > 0$ according to figure 2 $\Rightarrow \cos\varphi > 0$ and $\varphi = 0.46$ rad.	1 ½
B-3-a	The curve (a) represents ME bacause ME= cte. PE_{e_0} = ½k$(x_0)^2$= ½(10)×$(0.2)^2$ = 0.2 J $\Rightarrow$ (b) represents PE_e. The curve (c) represents KE	1
B-3-b	KE_0 = ½ m$(v_0)^2$ = 0.8 J $\Rightarrow v_0$ = 4 m/s.	½
B-3-c-i	T = 2×0.157 = 0.314 s	¼
B-3-c-ii	T_0 = 0.628 s $\Rightarrow T = \frac{T_0}{2}$	½

Third exercise (7 points)

Part of the Q	Answer	Mark
A-1-a	$u_{AD} = u_{AB} + u_{BD} \Rightarrow E = R_1.i + u_C$ with $i = C\frac{du_C}{dt}$ we obtain : $E = R_1 C\frac{du_C}{dt} + u_C$	¾
A-1-b	$\frac{du_C}{dt} = \frac{A}{\tau_1}e^{-\frac{t}{\tau_1}}$, Substitute in the differenial equation, we obtain $E = R_1 C\frac{A}{\tau_1}e^{-\frac{t}{\tau_1}} + A(1-e^{-\frac{t}{\tau_1}}) \Rightarrow E = Ae^{\frac{-t}{R_1C}}(\frac{R_1C}{\tau_1} - 1) + A$ Then A = E and $\tau_1 = R_1.C$	**1**
A-1-c	At the end of the charge , $t \to \infty \Rightarrow e^{-\frac{t}{\tau_1}} \to 0 \Rightarrow u_C = A = E$. or : At the end of thecharge $i = 0 \Rightarrow u_{R1} = 0 \Rightarrow uc = E$	½
A-1-d	$u_{R_1} = R_1 i = R_1 C\frac{du_C}{dt} = R_1 C\frac{E}{\tau_1}e^{-\frac{t}{\tau_1}} = Ee^{\frac{-t}{R_1C}}$ or : $u_G = u_{R_1} + u_C \Rightarrow E = u_{R1} + E - Ee^{\frac{-t}{R1C}} \Rightarrow u_{R_1} = Ee^{\frac{-t}{R_1C}}$	½
A-1-e	$u_{R1} = Ee^{\frac{-t}{R_1C}} \Rightarrow \ln u_{R_1} = \ln E - t/\tau_1$	¼
A-2-a	$\ln(u_R) = \ln E - \frac{t}{R_1C}$ decreasing linearly with time, or it has the form of $\ln(u_R) = at + b$ with $a < 0$.	½
A-2-b	The slope of this straight line is $-\frac{1}{R_1C} = \frac{2.3-1.3}{0-0.01} = -100\ s^{-1} \Rightarrow$ $\frac{1}{R_1C} = 100\ s^{-1}$ and $C = \frac{1}{10^6}F = 1\ \mu F$.	**1**
B-1	Because the armature B of the capacitor is charged positively.	¼
B-2	$u_C = u_{R_1} + u_{R_2} = (R_1+R_2)\,i$ with $i = -C\frac{du_C}{dt}$, we obtain : $u_C + (R_1+R_2)C\frac{du_C}{dt} = 0$.	¾
B-3	replace $u_C = E\,e^{-\frac{t}{\tau_2}}$ in the differential equation, we obtain: $E\,e^{-\frac{t}{\tau_2}} + (R_1+R_2)C(-\frac{E}{\tau_2}e^{-\frac{t}{\tau_2}}) = 0 \Rightarrow \tau_2 = (R_1+R_2)C$	½
B-4	The slope of the tangent to the curve $u_C = f(t)$ at the instant $t_0 = 0$ meets the time axis at a point of abscissa $\tau_2 = 0.02s$ $\tau_2 = (R_1+R_2)C \Rightarrow \frac{0.02}{20000} \Rightarrow C = 10^{-6}F$.	