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دائرة الامتحانات

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ملاحظة:- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the point A (6 ; 3 ; 2), the plane (P) with equation $x-y+2z-7=0$ and the line (d) with parametric equations:

$$\begin{cases} x = t \\ y = t-3 \\ z = -1 \end{cases}$$ where t is a real parameter.

1) Show that A is a point in (P) and that (d) is parallel to (P).

2) a- Verify that the point C (1 ; –2 ; –1) is on (d).

b- Determine a system of parametric equations of line (L) passing through C and perpendicular to (P).

c- Show that the point E $(3;-4;3)$ is the symmetric of C with respect to (P).

d- Deduce a system of parametric equations of line (Δ) symmetric of line (AC) with respect to (P).

II- (4 points)

An urn contains seven balls: four red balls and three green balls.

A player selects randomly and simultaneously three balls from this urn.

1) a- Calculate the probability that the player selects exactly two red balls.

b- Show that the probability that the player selects at least two red balls is equal to $\frac{22}{35}$.

2) After selecting three balls, the player scores:

- 9 points if he gets three red balls;
- 6 points if he gets exactly two red balls;
- 4 points if he gets exactly one red ball;
- zero if he gets three green balls.

Denote by X be the random variable that is equal to the score of the player.

a- Determine the probability distribution of X.

b- Knowing that the player scored more than 2 points, calculate the probability that his score is multiple of 3.

III- (4 points)

The complex plane is referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$.

A- Consider the points A and B with respective affixes $z_A = 2+2i$ and $z_B = \left(1+\sqrt{3}\right)\left(-1+i\right)$.

1) Determine the exponential form of the complex number $\frac{z_B}{z_A}$.

2) Prove that the triangle OAB is right at O.

B- To every point M with affix z ($z \neq 0$), associate the point M′ with affix z′ such that $z' = 1+i-\frac{2}{z}$.

Let z = x + iy with x and y are two real numbers.

1) Express, in terms of x and y, the real part and the imaginary part of the complex number z'.

2) Prove that if the real part of z' is zero, then M moves on a circle whose center and radius are to be determined.

IV- (8 points)

A- Consider the function g defined over]0; +∞ [as $g(x) = x^2 - 2\ln x$.

1) Determine $\lim_{x\to 0} g(x)$ and $\lim_{x\to+\infty} g(x)$.

2) Set up the table of variations of g and deduce that $g(x) > 0$.

B- Let f be the function defined over]0;+∞ [as $f(x) = \frac{x}{2} + \frac{1+\ln x}{x}$ and let (C) be its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Determine $\lim_{x\to 0} f(x)$ and deduce an asymptote to (C).

2) a- Determine $\lim_{x\to+\infty} f(x)$ and show that the line (Δ) with equation $y = \frac{x}{2}$ is an asymptote to (C).

b- Study, according to the values of x, the relative positions of (C) and (Δ).

3) Show that $f'(x) = \frac{g(x)}{2x^2}$ and set up the table of variations of f.

4) Calculate the coordinates of the point B on (C) where the tangent (T) is parallel to (Δ).

5) Show that the equation f(x) = 0 has a unique solution α, then verify that $0.34 < \alpha < 0.35$.

6) Plot (Δ), (T) and (C).

7) Let h be the function defined over $]0;+\infty[$ as $h(x) = \frac{1+\ln x}{x}$.

a- Find an antiderivative H of h.

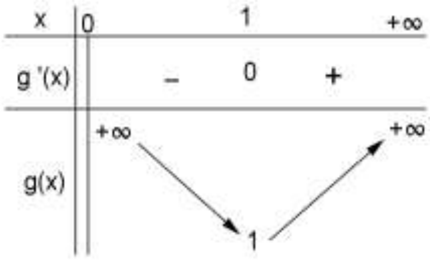
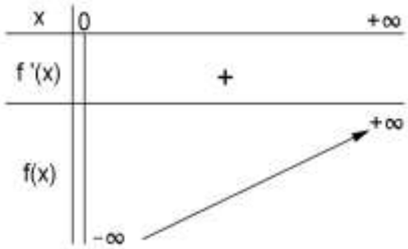
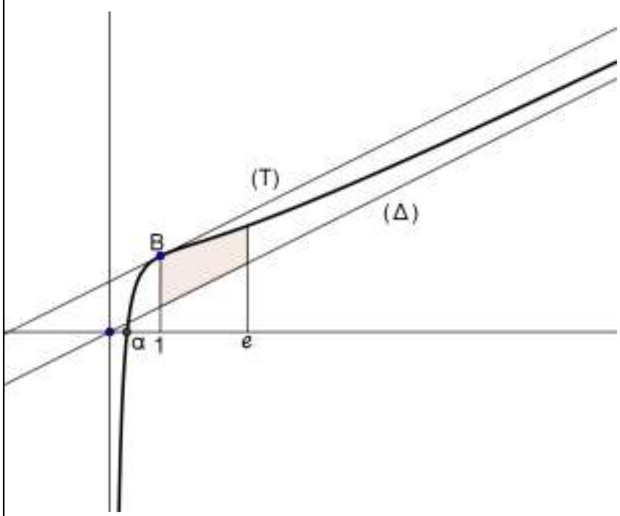
b- Deduce the measure of the area of the region bounded by (C), (Δ) and the lines with equations x = 1 and x = e.

Bareme LS En Session 2- 2013

Q1	Answers	M
1	$x_A - y_A + 2z_A - 7 = 0$ then $A \in (P)$. and t –t+3 –2 –7= $-6 \neq 0$. Hence (d) is parallel to (P).	1
2a	For: $x = x_C = 1$, t = 1 , $y = y_C = -2$, and $z = z_C = -1$; hence C $\in (d)$.	0.5
2b	$\overrightarrow{v_L}$ is parallel to $\overrightarrow{n_P}(1;-1;2)$ and (L) passes through C, hence a system of parametric equations of (L) is : x = m +1, y = – m –2, z = 2m –1 where m is a real parameter.	0.5
2c	(L) intersects (P) at point I (m +1, – m –2, 2m –1), and I $\in (P)$, hence m = 1and $I(2;-3;1)$. Moreover, I is the midpoint of [EC], hence : $x_E = 2x_I - x_C = 3$, $y_E = 2y_I - y_C = -4$ and $z_E = 2z_I - z_C = 3$. **OR :** $\overrightarrow{CE}(2;-2;4)$, $\overrightarrow{CE} = 2\overrightarrow{n_P}$, (CE) is perpendicular to(P) and the point $I(2;-3;1)$ midpoint of [CE], is in (P), hence C and E are symmetric with respect to (P).	1.5
2d	The line (Δ) passes through A and E, hence $\overrightarrow{AM} = k\overrightarrow{AE}$, x = –3k + 6, y = –7k +3, z = k + 2.	0.5

Q2	Answers	M
1a	P{2R,1G}= $\frac{C_4^2 \times C_3^1}{C_7^3} = \frac{18}{35}$.	1
1b	P{2R,1G}+P{3R}= $\frac{18}{35} + \frac{C_4^3}{C_7^3} = \frac{22}{35}$.	1
2a	P(X= 9) = P(3R) = $\frac{C_4^3}{C_7^3} = \frac{4}{35}$. P(X= 6) = P{2R,1G)}= $\frac{18}{35}$. P(X = 4) = P{1R,2G} = $\frac{C_4^1 \times C_3^2}{C_7^3} = \frac{12}{35}$. P(X= 0) = P(3G) = $\frac{C_3^3}{C_7^3} = \frac{1}{35}$.	1
2b	P(Score multiple of 3/ Score > 2) = $\frac{22}{35} \div \frac{34}{35} = \frac{11}{17}$.	1

Q3	Answers	M
A1	$\frac{z_B}{z_A} = \frac{(1+\sqrt{3})(-1+i)}{2(1+i)} = \frac{1+\sqrt{3}}{2}i = \frac{1+\sqrt{3}}{2}e^{i\frac{\pi}{2}}$.	1
A2	$\overrightarrow{OA} \cdot \overrightarrow{OB} = -2(1+\sqrt{3}) + 2(1+\sqrt{3}) = 0$. **OR** : $\frac{z_B}{z_A}$ is pure imaginary hence $(\overrightarrow{OA}, \overrightarrow{OB}) = \arg\left(\frac{1+\sqrt{3}}{2}i\right) = \frac{\pi}{2}$ then $(OB) \perp (OA)$.	0.5
B1	$z' = 1 + i - \frac{2}{x+iy} = 1 + i - \frac{2x-2iy}{x^2+y^2}$. $\text{Re}(z') = 1 - \frac{2x}{x^2+y^2}$, $\text{Im}(z') = 1 + \frac{2y}{x^2+y^2}$.	1
B2	$\text{Re}(z') = 1 - \frac{2x}{x^2+y^2} = 0 \Rightarrow x^2 + y^2 - 2x = 0 \Leftrightarrow (x-1)^2 + y^2 = 0$ hence M moves on the circle with center (1 ; 0) and radius 1 deprived from O.	1.5

Q_4	Answers	M
A.1	$\lim\limits_{x\to 0} g(x) = +\infty$ $\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty} x\left(x - 2\frac{\ln x}{x}\right) = +\infty$. (since $\lim\limits_{x\to+\infty} \frac{\ln x}{x} = 0$)	0.5
A.2	$g'(x) = 2x - \frac{2}{x} = \frac{2(x+1)(x-1)}{x}$. g(x) has a minimum equal to 1, hence $g(x) > 0$ for $x > 0$.  	1
B.1	$\lim\limits_{x\to 0} f(x) = -\infty$ then the line with equation $x = 0$ is an asymptote to (C).	0.5
B.2.a	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(\frac{x}{2} + \frac{\ln x}{x} + \frac{1}{x}\right) = +\infty$ and $\lim\limits_{x\to+\infty}\left[f(x) - \frac{x}{2}\right] = \lim\limits_{x\to+\infty}\left(\frac{\ln x}{x} + \frac{1}{x}\right) = 0$ .Hence the line (Δ) with equation $y = \frac{x}{2}$ is an asymptote to (C).	0.5
B.2.b	$f(x) - \frac{x}{2} = \frac{1}{x}(\ln x + 1) = 0$ for $x = \frac{1}{e}$ then $y = \frac{1}{2e} \Rightarrow A\left(\frac{1}{e}; \frac{1}{2e}\right)$ is the point of intersection of (Δ) and (C). For $x > \frac{1}{e}$, (C) is above (Δ) and for $0 < x < \frac{1}{e}$ (C) is below (Δ).	1
B.3	$f'(x) = \frac{1}{2} + \frac{\left(\frac{1}{x}\right)(x) - \ln x - 1}{x^2} = \frac{1}{2} - \frac{\ln x}{x^2} = \frac{g(x)}{2x^2}$.  	1
B.4	$f'(x_B) = \frac{1}{2} \Rightarrow \frac{x^2 - 2\ln x}{2x^2} = \frac{1}{2} \Rightarrow x = 1$ hence $B\left(1; \frac{3}{2}\right)$.	0.5
B.5	f is continuous and strictly increasing from $-\infty$ to $+\infty$ hence the equation $f(x) = 0$ has a unique solution α .Moreover $f(0.34) = -0.061 < 0$ and $f(0.35) = 0.032 > 0$ hence $0.34 < \alpha < 0.35$.	1
B.6	 	1
B7a	$H(x) = \int h(x)\,dx = \frac{(\ln x)^2}{2} + \ln x + c$ $= \frac{(1+\ln x)^2}{2} + k$.	0.5
B7b	$A = \int_1^e \left(f(x) - \frac{x}{2}\right)dx$ $= \int_1^e \frac{1+\ln x}{x}dx$ $= H(e) - H(1) = \frac{3}{2}\ u^2$.	0.5