وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع: العلوم العامة

دورة العام ٢٠١٥ الاستثنائيّة
الثلاثاء ١٨ آب ٢٠١٥

عدد المسائل: ست	مسابقة في مادة الرياضيات المدة أربع ساعات	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the answer corresponding to it.

№	Questions	Answers			
		a	**b**	**c**	**d**
1	The particular solution f(x) of the differential equation y '' + 4y = 0 such that f (0) = 0 and f ' (π) = –1 is	$\frac{1}{2}\sin 2x$	$-\frac{1}{2}\sin 2x$	$-\frac{1}{2}\cos 2x$	sin 2x – cos2x
2	For all x >1, $\lim\limits_{x\to 1}\frac{\int_1^x e^{t^2}\,dt}{x-1} =$	$+\infty$	1	e	2e
3	If f is an odd function ,continuous over IR and such that $\int_{-1}^{3} f(x)\,dx = 2$, then $\int_{-3}^{-1} f(x)\,dx =$	–2	0	2	– 4
4	Let M with affix z $(z \neq 1)$ be a variable point in the complex plane referred to a direct orthonormal system. If $\frac{z+1}{z-1}$ is real, then M moves on:	the circle with center O and radius 1 except the point with affix 1	the x-axis except the point with affix 1	the line with equation y = x	the y-axis

II- (2 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the points E (– 1; 1; 0) ,

F(– 2; – 1; 0), and the line (d) with parametric equations $x = t - 1$, $y = 2t + 1$, $z = -2t$ where t is a real parameter.

Denote by (P) the plane determined by the point F and the line (d).

1) Verify that E is on (d).

2) Show that $2x - y + 3 = 0$ is an equation of (P).

3) Consider in the plane (P) the circle (C) with center F and radius FE.

a- Determine the coordinates of H, the orthogonal projection of F on (d).

b- Determine the coordinates of L, the second point of intersection of (d) and (C).

c- Write a system of parametric equations of the bisector of the angle $\widehat{EFL}$.

4) Let (Q) be the plane containing (d) and perpendicular to (P). Denote by (Δ) the perpendicular bisector of the segment [EL] in the plane (Q).

Write a system of parametric equations of (Δ).

III- (3 points)

An urn contains 5 white balls and 2 black balls. A game consists of two consecutive drawings as follows:

- A ball is selected randomly in the first drawing. If the ball selected is white, it is put back in the urn; otherwise, it is kept outside the urn.
- Two balls are selected simultaneously and randomly in the second drawing.

Consider the following events:

- W: «The ball selected in the first drawing is white»
- E: «The 2 balls selected in the second drawing are white»
- F: «The 2 balls selected in the second drawing are black»
- G: «The 2 balls selected in the second drawing are of different colors».

1) Calculate P(E/W) and $P(E/\overline{W})$. Deduce that $P(E) = \dfrac{26}{49}$.

2) Calculate P(F) and P(G).

3) Knowing that the 2 balls selected in the second drawing have the same color, calculate the probability that the ball selected in the first drawing is black.

4) In this part, we mark -3 points for each black ball selected, and +5 points for each white ball selected.

Denote by S the sum of points marked for the two balls selected in the second drawing.

Calculate the probability that S is positive.

IV- (3 points)

The plane is referred to a direct orthonormal system $(O\,;\vec{i}\,,\vec{j})$.
Denote by (C) the circle with center $I(0;\,3)$ and radius 2, and by (d) the line with equation y = – 3.
Let $L(\alpha\,;\,\beta)$ be a variable point on (C). Denote by N the orthogonal projection of L on (d) and by M the midpoint of segment [LN].

1) Write an equation of (C).
2) a- Determine the coordinates of M in terms of α and β.

b- As L moves on (C), prove that M moves on the ellipse (E) with equation $\frac{x^2}{4}+y^2=1$.

c- Draw (E).

3) Let (P) be the parabola with vertex V (0; 1) and focus $F\left(0;\frac{3}{4}\right)$.

a- Show that $y=1-x^2$ is an equation of (P).
b- Draw (P) in the same system as (E).

4) a- Calculate $\int_0^1\left(1-x^2\right)dx$.

b- Deduce the area of the region that is above the x-axis and bounded by (E) and (P).

5) Let G (– 1; 0) be a point on (P) and (Δ) the tangent at G to (P). Denote by H the point of (P) where the tangent to (P) is perpendicular to (Δ).
Prove that G, H and F are collinear.

V- (3 points)

Consider a direct equilateral triangle ODA with side equal to 1.

Let R be the rotation with center O and angle $\frac{\pi}{2}$.

Denote by B = R (A), D'= R (D). Let C be the point so that D = R(C). (C is the pre-image of D)

1) a- Make a figure.
b- Show that O is the midpoint of [CD'] and that BC= $\sqrt{3}$.

2) a- Justify that (AC) is perpendicular to (BD) and that AC = BD.
b- Show that (AD) is parallel to (BC).

3) Denote by E the point of intersection of lines (AC) and (BD). Let h be the dilation with center E that transforms A onto C.
a- Determine h (D).
b- Calculate the ratio of h.

4) Let L be the midpoint of [AD] and F = h (L).
Show that O, E, F and L are collinear.

5) Let R' be the rotation with center E and angle $-\frac{\pi}{2}$. Consider $S=h\circ R'$.

a- Determine the nature of S and so its elements.
b- Prove that S (A) = B.

VI- (7 points)

A- Let h be the function defined on IR as $h(x) = e^x - x - 1$.

Denote by (C) its representative curve in an orthonormal system.

1) a- Determine $\lim_{x\to+\infty} h(x)$.

b- Determine $\lim_{x\to-\infty} h(x)$ and show that the line (d) with equation y = – x – 1 is an asymptote to (C).

2) a- Calculate $h'(x)$ and set up the table of variations of h.

b- Draw (C) and (d).

c- Deduce that $e^x \geq x + 1$ for all x.

B- Let f be the function defined as $f(x) = \dfrac{e^x}{e^x - x}$.

Denote by (C') its representative curve in another orthonormal system.

1) Show that f is defined over IR.

2) Determine the asymptotes to (C').

3) Verify that $f'(x) = \dfrac{(1-x)e^x}{(e^x - x)^2}$ and set up the table of variations of f.

4) a- Write an equation of (T), the tangent to (C') at the point E with abscissa 0.

b- Verify that $f(x) - x - 1 = \dfrac{x(x + 1 - e^x)}{e^x - x}$.

c- Study, according to the values of x, the relative positions of (C') with respect to (T).

d- Draw (C') and (T).

C- For all natural numbers n, define the sequence (u_n) as $u_n = \int_0^n f(x)\,dx$.

1) Show that the sequence (u_n) is increasing.

2) a- For $x \geq 0$, verify that $f(x) \geq 1$.

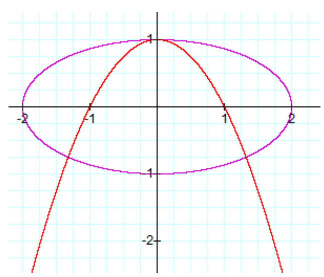
b- Is the sequence (u_n) convergent? Justify.

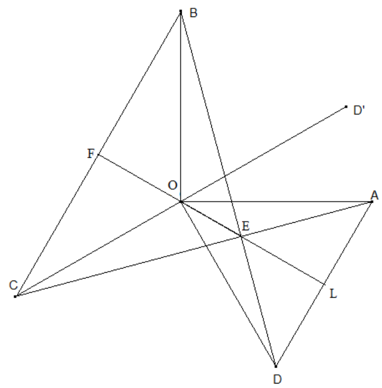
Answer Key- Math SG – Second Session - 2015

QI	Answers	N
1	y= Acos2x+Bsin2x. f (0) = 0 $\Rightarrow$ A= 0. f´(x)= –2Asin2x+2Bcos2x. f '(π)= –1 $\Rightarrow B = -\frac{1}{2}$. b)	1
2	$\lim\limits_{x\to 1}\frac{\int_1^x e^{t^2}dt}{x-1} = \frac{0}{0}$ (Indeterminate). L'H.R. $\lim\limits_{x\to 1}\frac{\int_1^x e^{t^2}dt}{x-1} = \lim\limits_{x\to 1}\frac{e^{x^2}}{1} = e$. c)	1
3	$\int_{-3}^{3} f(x)\,dx = \int_{-3}^{-1} f(x)\,dx + \int_{-1}^{3} f(x)\,dx = 0 \Rightarrow \int_{-3}^{-1} f(x)\,dx = -2.$ a)	1
4	$\frac{z+1}{z-1}$ real then $\frac{z+1}{z-1} = \frac{\bar{z}+1}{\bar{z}-1}$ which gives $z.\bar{z} - z + \bar{z} - 1 = z.\bar{z} + z - \bar{z} - 1$ and $z = \bar{z}$. Therefore, M moves on the x-axis except point with affix 1. b)	1

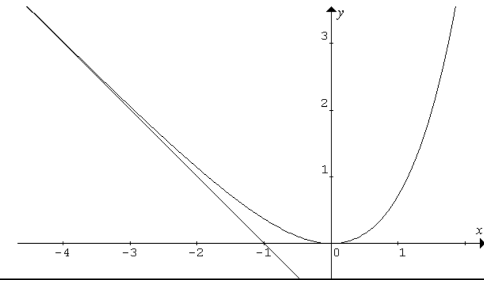
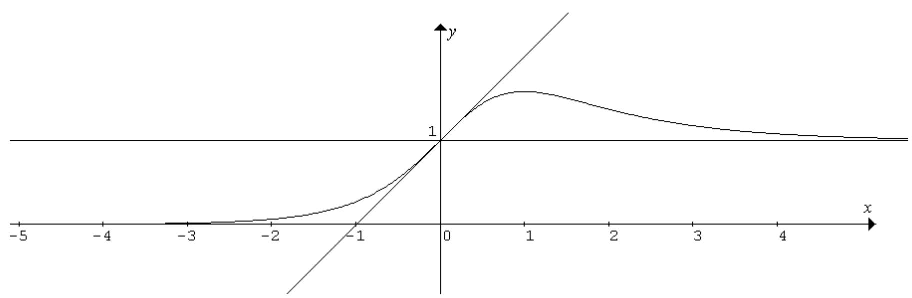
QII	Answers	N
1	For t = 0 ; E is a point on (d).	0.5
2	2(t – 1) – (2t + 1) + 3 = 0 – 4+ 1+ 3 = 0 ; $2x_F - y_F + 3 = -4+1+3 = 0$ thus $F \in (P)$ then the given equation is that of (P).	0.5
3a	$\overrightarrow{FH}(t+1; 2t+2; -2t)$; $\overrightarrow{FH}\cdot\vec{V}_{(d)} = 0$; $(t+1)+2(2t+2)+4t = 0; t = -\frac{5}{9}$ then $H\left(-\frac{14}{9}; -\frac{1}{9}; \frac{10}{9}\right)$.	1
3b	H is the midpoint of $[EL]$ thus $L\left(-\frac{19}{9}; -\frac{11}{9}; \frac{20}{9}\right)$.	0.5
3c	(FH) is the bisector of $\widehat{EFL}$. $\overrightarrow{FM} = m\overrightarrow{FH} \Leftrightarrow x = \frac{4}{9}m - 2,\ y = \frac{8}{9}m - 1,\ z = \frac{10}{9}m.$	0.5
4	A director vector of the perpendicular bisector is $\overrightarrow{n_P}(2; -1; 0)$ and the perpendicular bisector passes in point H; then system of parametric equations is: $x = 2\lambda - \frac{14}{9}; y = -\lambda - \frac{1}{9}; z = \frac{10}{9}$ where λ is a real parameter.	1

QIII	Answers	N
1	$P\left(E/W\right)=\frac{C_5^2}{C_7^2}=\frac{10}{21}$; $P\left(E/\overline{W}\right)=\frac{C_5^2}{C_6^2}=\frac{10}{15}=\frac{2}{3}$; $P(E)=P(W\cap E)+P(\overline{W}\cap E)=\frac{5}{7}\times\frac{10}{21}+\frac{2}{7}\times\frac{10}{15}=\frac{26}{49}$	2.5
2	$P(F)=P(F\cap W)+P(F\cap\overline{W})=\frac{5}{7}\times\frac{1}{21}+0=\frac{5}{147}$; $P(G)=1-\left(P(E)+P(F)\right)=\frac{64}{147}$	1.5
3	$P\left(\overline{W}/\overline{G}\right)=\frac{P\left(\overline{W}\cap\overline{G}\right)}{P\left(\overline{G}\right)}=\frac{P(\overline{W}).P\left(\overline{G}/\overline{W}\right)}{1-P(G)}=\frac{\frac{2}{7}.\frac{C_5^2}{C_6^2}}{1-\frac{64}{147}}=\frac{28}{83}$	1
4	$P(S>0)=1-P(S<0)=1-\frac{5}{147}=\frac{142}{147}$	1

QIV	Answers	N
1	$x^2+(y-3)^2=4.$	0.5
2a	$L(\alpha;\beta)$, $N(\alpha;-3)\Rightarrow M\left(\alpha;\frac{\beta-3}{2}\right).$	0.5
2b	$\frac{\alpha^2}{4}+\left(\frac{\beta-3}{2}\right)^2=\frac{\alpha^2+(\beta-3)^2}{4}=\frac{4}{4}=1$ since L is on (C). Thus M moves on (E).	1
2c		0.5
3a	$\frac{p}{2}=SF=\frac{1}{4}$ thus 2p = 1.y-axis is the focal axis, therefore $x^2=-(y-1)\Leftrightarrow y=1-x^2$. **OR**: The directrix is the line (D) with equation $y=\frac{5}{4}$. Dist(R,(D)) = RF with R(x; y)$\in$ (P) $\left\|y-\frac{5}{4}\right\|^2=x^2+\left(y-\frac{3}{4}\right)^2\Leftrightarrow y=1-x^2$.	0.5
3b	See figure in 2c.	0.5
4a	$\int_0^1\left(1-x^2\right)dx=\frac{2}{3}.$	0.5
4b	Area = $\frac{\pi ab}{2}-2\int_0^1\left(1-x^2\right)dx=\pi-\frac{4}{3}.$	1
5	y = f(x) = 1 – x^2 thus f '(– 1) = 2, $f'(x_H)=-\frac{1}{2}\Rightarrow -2x=-\frac{1}{2}\Rightarrow x=\frac{1}{4}$ thus $H\left(\frac{1}{4};\frac{15}{16}\right)$. $\overrightarrow{GH}=\frac{5}{4}\overrightarrow{GF}$ therefore G, H et F are collinear.	1

QV	Answers	N
1a		0.5
1b	$C\hat{O}D = D\hat{O}D' = 90^0$ then C, O, D´ are collinear and OC = OD = OD´, then O is the midpoint of [CD´]. OB = OC = OD´ therefore triangle CBD´ is right at B. Pythagoras gives : $CB = \sqrt{CD'^2 - BD'^2} = \sqrt{2^2 - 1} = \sqrt{3}$.	1
2a	R(C) = D et R(A) = B $\Rightarrow$ (AC) is perpendicular to (BD) and AC = BD.	0.5
2b	R(D) = D´, R(A) =B thus AD perpendicular to BD´ and (BC) is perpendicular to (BD´), then (AD) and (BC) are parallel.	1
3a	h(A) = C and (AD) are parallel to (BC), therefore h(D) = B.	0.5
3b	$\overrightarrow{BC} = K\overrightarrow{DA}$ So: $K = -\sqrt{3}$.	0.5
4	F midpoint of [BC] ; E, F and L are collinear. (OF) and (OL) are perpendicular to (BC) $\Rightarrow$ O, F and L are collinear.	1
5a	S is the similitude S $(E, \sqrt{3}, \frac{\pi}{2})$.	0.5
5b	S (A) $= hoR'(A) = h(R'(A)) = h(D) = B$ since EAD is a right isosceles triangle.	0.5

QVI	Answers		N
A	1a	$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty} x\left(\frac{e^x}{x} - 1 - \frac{1}{x}\right) = +\infty(+\infty) = +\infty$.	0.5
	1b	$\lim_{x\to-\infty} h(x) = \lim_{x\to-\infty}(e^x - x - 1) = +\infty$ and $\lim_{x\to-\infty}[h(x) - y] = \lim_{x\to-\infty} e^x = 0$, (d) is an asymptote to (C).	0.5
	2a	$h'(x) = e^x - 1$. x: - oo ... 0 ... + oo h´(x): — 0 + h(x): + oo ↘ 0 ↗ + oo	1

	2b	$\lim\limits_{x\to+\infty}\dfrac{h(x)}{x}=+\infty$ thus (C) has a vertical asymptotic direction. 	1
	2c	The minimum of h(x) is 0, so $h(x) \geq 0$, then $e^x - x - 1 \geq 0$, thus $e^x \geq x + 1$.	0.5
B	1	$h(x) \geq 0$; $e^x - x \geq 1$ so $e^x - x > 0$ thus $D_f = \mathbb{R}$	1
	2	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\dfrac{e^x}{e^x\left(1-\dfrac{x}{e^x}\right)} = 1$, $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\dfrac{e^x}{e^x - x} = \dfrac{0}{+\infty} = 0$.	1
	3	$f'(x) = \dfrac{(1-x)e^x}{\left(e^x - x\right)^2}$. x: $-\infty$ … 1 … $+\infty$ f′(x): − 0 + f(x): 0 ↘ $\dfrac{e}{e-1}$ ↗ 1	1.5
	4a	y = x + 1.	1
	4b	$f(x)-(x+1) = \dfrac{e^x}{e^x - x} - (x+1) = \dfrac{e^x - (x+1)(e^x - x)}{e^x - x} = \dfrac{x(x - e^x + 1)}{e^x - x}$	1
	4c	For $x < 0$; $f(x)-(x+1) > 0$, (C') is above (T). For $x > 0$; $f(x)-(x+1) < 0$, (C') is below (T). For x = 0 ; (C') intercept (T).	1
	4d		1
C	1	$u_{n+1} - u_n = \int_0^{n+1} f(x)dx - \int_0^{n} f(x)dx = \int_0^{n+1} f(x)dx + \int_n^{0} f(x)dx = \int_n^{n+1} f(x)dx$. knowing $f(x) > 0$ then $\int_n^{n+1} f(x)dx > 0$; So (u_n) is increasing .	1
	2a	$f(x) - 1 = \dfrac{e^x}{e^x - x} - 1 = \dfrac{e^x - e^x + x}{e^x - x} = \dfrac{x}{e^x - x} \geq 0$ thus $f(x) \geq 1$ for all $x \geq 0$.	1
	2b	$\int_0^n f(x)dx \geq \int_0^n dx$ since $n \geq 0$ then $u_n \geq n$; $\lim\limits_{n\to+\infty} u_n \geq \lim\limits_{n\to+\infty} n$; $\lim\limits_{n\to+\infty} u_n \geq +\infty$ therefore (u_n) is divergent.	1