وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	الشهادة الثانوية العامة فرع: الاجتماع والاقتصاد	دورة العام ٢٠١٥ الاستثنائيّة الخميس ٢٠ آب ٢٠١٥
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الاجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (4 points)

A company produces and sells a certain product.
The table below shows the demand y in hundreds of units, in terms of the unit price x in thousands LL.

Unit price x_i in thousands LL	8	9	10	12	14
Quantity demanded y_i in 100 of units	12	10	6	5	4

Assume that the preceding pattern remains valid as the price increases.

1) a- Calculate the means $\bar{x}$ and $\bar{y}$.

b- An equation of the regression line $\left(D_{y/x}\right)$ is: $y = -1.3017x + b$.

Deduce from the preceding question that b = 21.198.

2) Determine the coefficient of correlation r and give an interpretation to the value thus obtained.

3) a- Express, in terms of the unit price x, the elasticity of the demand.

b- For an increase of 1% on the unit price x_0, the demand will decrease by 4%.

Calculate x_0.

c- Estimate, in LL, the revenue for a unit price of 12 500 LL.

II- (4 points)

A pastry produces and sells chocolate bars.
In order to promote the sale of these bars, the manager of this pastry decides to put gift coupons in 50% of the bars produced. Among the bars containing gift coupons(winning bars), 60% contain one gift coupon while the others contain 2 gift coupons.

1) A client buys a chocolate bar. Consider the following events:

- G: «The client buys a winning bar»
- U: «The client finds one gift coupon only»
- D: «The client finds two gift coupons».

a- Show that the probability that the client finds one gift coupon only is 0.3.

b- Let X be the random variable equal to the number of gift coupons obtained by the client.
Determine the probability distribution of X.

2) In this part, suppose that the pastry has produced 200 chocolate bars.
A client buys randomly and simultaneously two chocolate bars.

a- Calculate the probability that the client does not find any gift coupon.

b- Calculate the probability that the client finds at least one gift coupon.

c- Calculate the probability that the client finds two gift coupons.

III- (4 points)

Wassim wants to buy a car for 30 000 000 LL. He paid 5 000 000 LL down payment and decides to borrow the rest as a loan from a bank "A". This loan should be paid back in equal monthly payments for 3 years at an annual interest rate of 7% compounded monthly.

A-

1) a- Determine the value of each monthly payment.
   b- Calculate the amount of interest to be paid by Wassim over 3 years.

2) At the same time, Wassim deposits in another bank "B" a capital of 25 000 000 LL over a period of 3 years at an annual interest rate of 6% compounded quarterly.
   Calculate the interest earned by Wassim in bank "B" over 3 years.

3) Did Wassim make a good decision when he chose to borrow the loan to buy the car? Justify.

B-

Suppose that the lifetime of this car is 10 years and its salvage value is 5 000 000 LL.

1) Calculate the constant annual depreciation of this car.
2) What would be the price of this car in 5 years?

IV- (8 points)

Consider the function f defined over IR as $f(x) = 3-(x+1)e^{-x+1}$ and denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

A-

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-1)$.
   b- Determine $\lim\limits_{x\to+\infty} f(x)$. Deduce an asymptote (d) to (C).
   c- Study, according to the values of x, the relative positions of (C) and (d).

2) a- Show that $f'(x) = xe^{-x+1}$ and set up the table of variations of f.
   b- Draw (d) and (C).

3) a- Show that: $\int (x+1)e^{-x+1}dx = (-x-2)e^{-x+1} + k$ where k is a real constant.
   b- Deduce the area of the region bounded by (C), (d) and the two lines with equations x = 0 and x = 3.

B-

A factory produces watches. The total cost of production, in millions LL, is modeled as $C_T(x) = 6-(x+2)e^{-x+1}$, where x is expressed in hundreds of watches ($0 \le x \le 4$).

1) Calculate the total cost of producing 300 watches.

2) 75% of the watches are sold for 40 000 LL each watch , and the remaining are donated.

   a- Prove that the profit function is expressed as $P(x) = 3x-6+(x+2)e^{-x+1}$.
   b- Show that P'(x) = f(x) and set up the table of variations of the function P over [0; 4].
   c- Calculate P(1) and give an interpretation to the obtained result.
   d- What is the minimum number of watches that should be produced by this factory in order to realize profit?

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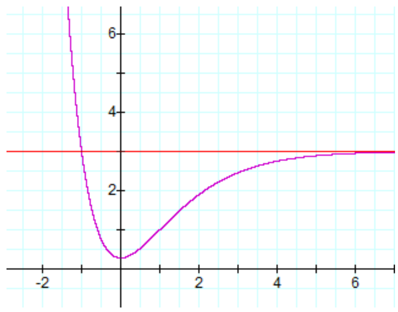
QI	Answers	M
1a	$\overline{x}=10.6$, $\overline{y}=7.4$	1
1b	$b=\overline{y}-a\overline{x}$ thus b = 7.4 + 1.3017 × 10.6 = 21.198	1
2	r = – 0.912. There is a strong negative correlation (-1 < r < -0.86)	1.5
3a	$E(x)=-x\dfrac{d'(x)}{d(x)}=\dfrac{1.3017x}{-1.3017x+21.198}$.	1
3b	$E\left(x_0\right) = 4 \Leftrightarrow \dfrac{1.3017x_0}{-1.3017x_0+21.198}=4 \Leftrightarrow x_0=13.026.$	1.5
3c	$R(x)=xd(x)=x(-1.3017x+21.198)$, for x =12.5. $R(12.5)=12.5\times d(12.5)=12.5(-1.3017(12.5)+21.198)=61.584375$ The revenue will be 61.58437 × 1000 × 100 = 6 158 437.5 LL.	1

QII	Answers	M
1a	P (U ∩ G) = P (U/G) × P(G) = 0.6 × 0.5 = 0.3.	1
1b	X(Ω)= {0 ;1 ; 2}. P(X= 0) = P ($\overline{G}$) = 0. 5. P(X=1) =P (U∩G) = 0. 3. P(X=2) = P(D∩G) = 0.5×0.4 = 0.2. **OR** P(X=2) = 1- [P(X= 0) + P(X=1)] = 0.2	1.5
2a	$P(0\,\text{gift coupons})=\dfrac{C_{100}^2}{C_{200}^2}=\dfrac{4950}{19900}=\dfrac{99}{398}=0.248$	1.5
2b	P (at least 1 coupon) =1– P (0 coupon) = $\dfrac{299}{398}=0.751$.	1.5
2c	$P\left(2\text{coupons}\right) = \dfrac{C_{40}^1\times C_{100}^1}{C_{200}^2}+\dfrac{C_{60}^2}{C_{200}^2}=\dfrac{577}{1990}=0.2899$	1.5

QIII	Answers	M
A1a	$[C(1+i)^n]\times i=R[\left(1+i\right)^n-1] \Rightarrow R=\dfrac{[C(1+i)^n]\times i}{[\left(1+i\right)^n-1]}$ with $i=\dfrac{7}{100}\times\dfrac{1}{12}$ and $n=t\times k=36$ ⇒R = 771 927 LL OR: $C_n=R\times\dfrac{[1-(1+i)^{-n}]}{i}$ thus ⇒R = 771, 927 LL	1.5
A1b	Interest = 2 789 387 – 25 000 000 = 2 789 387LL.	1
A2a	Future value : $F=25000000\left(1+\dfrac{0.06}{4}\right)^{12}=2\ 989\ 0454$ LL. I = 29 890 454 – 25 000 000 = 4 890 454LL	1.5
A2b	Yes he did the right decision because the interest earned in bank B is greater than the interest that he has to pay to bank A.	1
B1	Annual depreciation = $\dfrac{30000000-5000000}{10}=2\ 500\ 000.$	1
B2	In five years, the price of the car becomes: 30 000 000 – 2 500 000 ×5 =17 500 000 LL.	1

<table>
<tr><th>QIV</th><th>Answers</th><th>M</th></tr>
<tr><td>A1a</td><td>$\lim\limits_{x\to-\infty} f(x) = 3 + (\infty)e^{1+\infty} = +\infty.\quad f(-1) = 3.$</td><td>1</td></tr>
<tr><td>A1b</td><td>$f(x) = 3 - \dfrac{x+1}{e^{x-1}}$; $\lim\limits_{x\to+\infty} \dfrac{x+1}{e^{x-1}} = \lim\limits_{x\to+\infty} \dfrac{1}{e^{x-1}} = \dfrac{1}{+\infty} = 0$;
thus $\lim\limits_{x\to+\infty} f(x) = 3$, then the line (d) with equation y=3 is an asymptote to (C).</td><td>1</td></tr>
<tr><td>A1c</td><td>Let $h(x) = f(x) - 3 = -(x+1)e^{1-x}$; Sign of $h(x)$ = sign of $-(x+1)$ since $e^{1-x} > 0$
if $x < -1; -(x+1) > 0;$ so $h(x) > 0$ (C) is above (d)
if $x > -1; -(x+1) < 0;$ so $h(x) < 0$ (C) is above (d)
if $x = -1; -(x+1) = 0;$ so (d) intersects (C) at point $(-1;3)$</td><td>1</td></tr>
<tr><td>B2a</td><td>$f'(x) = 0 - [e^{1-x} + (x+1)e^{1-x}] = xe^{1-x}.\qquad f(0) = 3 - 2.718 = 0.281$
<table>
<tr><td>x</td><td>$-\infty$ 0 $+\infty$</td></tr>
<tr><td>f '(x)</td><td>–   0   +</td></tr>
<tr><td>f(x)</td><td>$+\infty$ ↘ $3 - e$ ↗ 3</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>B2b</td><td>
</td><td>1.5</td></tr>
<tr><td>A3.a</td><td>$\left((-x-2)e^{-x+1} + k\right)' = -e^{-x+1} + (-1)e^{-x+1}(-x-2) = (x+1)e^{-x+1}$</td><td>1</td></tr>
<tr><td>A3.b</td><td>Area = $\int_0^3 (3 - f(x))dx = \int_0^3 (x+1)e^{1-x}dx = \left[-(x+2)e^{1-x}\right]_0^3 = \left(-5e^{-2} + 2e\right)u^2$.</td><td>1</td></tr>
<tr><td>B1</td><td>300 watches correspond to x= 3. $C_T(3) = 6 - 5e^{-2}$ = f (3)= 5.323 therefore 5 323 000 LL.</td><td>1</td></tr>
<tr><td>B2a</td><td>$1000000\,R(x) = \dfrac{75}{100} \times 40000 \times (100 \times x)$. Thus $R(x) = 3x$
Therefore: $P(x) = R(x) - C_T(x) = 3x - 6 + (x+2)e^{1-x}$</td><td>1.5</td></tr>
<tr><td>B2b</td><td>$P'(x) = 3 + e^{1-x} - (x+2)e^{1-x} = 3 - (x+1)e^{1-x} = f(x)$;
$P'(x) > 0$ since (C) is above x-axis.
p(0) = – 0.563 and p(4) = 6.29
<table>
<tr><td>x</td><td>0   4</td></tr>
<tr><td>P'(x)</td><td>+</td></tr>
<tr><td>P(x)</td><td>–0.563 ↗ 6.29</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>B2c</td><td>P(1) = –3+3 = 0 and P is continuous and strictly increasing over[0; 4] . Hence, for selling of 100 watches, the company breaks-even.</td><td>1</td></tr>
<tr><td>B2d</td><td>The company must sell a minimum of 101 watches in order to achieve profit.</td><td>1</td></tr>
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