عدد المسائل أربع	مسابقة في الرياضيات	الإسم:
	المدة ساعتان	الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو تخزين المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

The following table shows, the medical care expenses, in millions LL, of a big industrial company between the years 2000 and 2007.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Rank of the year x_i	0	1	2	3	4	5	6	7
Expenses y_i (in millions LL)	115.1	115.2	121.7	129.5	137.9	144.9	156.5	163.8

1) a - Write an equation of the regression line ($D_{y/x}$) of y in terms of x.
   b- Assume that the growth in the medical care expenses follows the same pattern for the following years; estimate the expenses in 2015.
2) The company wishes that, starting from the year 2008, the medical care expenses grow only by 2% per year and continue to increase in the same manner from one year to another. This growth is modeled by a sequence (u_n) where u_n denotes the medical care expenses, in millions LL, for the year (2007 + n).Thus $u_0 = 163.8$.
   a-Verify that $u_1 = 167.076$ and calculate u_2.
   b-Justify that (u_n) is a geometric sequence and specify its common ratio.
   c-Does the company save money in 2015 by following the new model? Justify.

II- (4 points)

The 60 students of a language class have the choice to learn Arabic, English or French. (Each student chooses only one language).

- 25% of the students learn Arabic out of whom 6 are girls;
- 30% of the students learn English out of whom10 are boys;
- The class contains a total of 25 girls.

1) Copy and complete the following table :

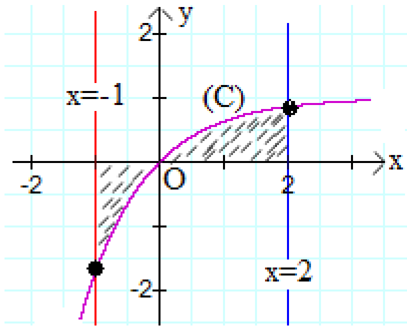
	Learn Arabic	Learn English	Learn French	Total
Girls			11	
Boys				
Total				60

2) A student is randomly chosen from this class. Consider the following events :
   - E: « the chosen student learns English» ;
   - B : « the chosen student is a boy ».

   a - Calculate the probabilities P(E) , P(B) and $P(E\cap B)$.
   b - Show that $P(E\cup B) = \dfrac{43}{60}$ and calculate $P(\overline{E}\cap\overline{B})$.
3) In what follows, a group of two students is chosen randomly from this class.
   a- Calculate the probability of the event S « The two chosen students learn the same language ».
   b- The two chosen students are boys. Calculate the probability that they learn French.

III-(4 points)

Answer by true or false to each of the following statements.
Justify the answer.



1) If the curve (C) to the right represents the function f defined on IR as $f(x) = 1 - e^{-x}$, then the area of the region bounded by (C), the x-axis and the lines with equations x = –1 and x = 2, is equal to $\left(e + \frac{1}{e^2} + 1\right)$ square units.

2) The set of solutions of the inequality $\ln(2x-1) - \ln(10-4x) < 0$ is $\left]-\infty; \frac{11}{6}\right[$.

3) Rami deposited a capital of 20 000 000 LL in a bank at 8% annual interest compounded annually. In the same time, Sami deposited a capital of 22 000 000 LL in another bank at 7% annual interest compounded annually. The future value in Rami's account will be, for the first time, greater than that of Sami after 11 years.

IV- (8 points)

A- Consider the function g defined on I = $[0\,;+\infty[$ as $g(x) = 1 + \frac{x-3}{8}e^x$.

1) Show that $g'(x) = \frac{1}{8}(x-2)e^x$.
2) Set up the table of variations of the function g and verify that $g(x) > 0$ for all x in I.

B- Let f be the function defined on $[0\,;+\infty[$ as $f(x) = x + 1 + \frac{x-4}{8}e^x$.

Denote by (C) the representative curve of f in an orthonormal system .

1) Calculate f(0) and f(4) and determine $\lim\limits_{x\to+\infty} f(x)$.
2) Verify that $f'(x) = g(x)$ and set up the table of variations of f.
3) Write an equation of the tangent (T) to (C) at the point with abscissa 3.
4) Plot (T) and (C).
5) The line (d) with equation $y = \frac{9}{8}x$ intersects (C) in two points with respective abscissas α and β ($\alpha < \beta$). Verify that $0.7 < \alpha < 0.9$.

C-

In what follows, take $\alpha = 0.813$ and $\beta = 3.919$.

A company produces objects. The total cost of production, in millions LL, is given by :

$f(x) = x + 1 + \frac{x-4}{8}e^x$ where x is the number, in thousands, of objects produced, with $0 \le x \le 4$.

1) Calculate the fixed cost.
2) Calculate the marginal cost of producing 2000 objects . Give an economical interpretation to the result.
3) Each produced object is sold for 1125 LL and suppose that the production is sold entirely.

   a- Show that the function of the revenue R is given by $R(x) = \frac{9}{8}x$.

   b- For what level of production will the company realize profit? Justify.

ES SESSION 2- MATH 2013

Q_1	Answers	M
1.a	The equation of line $(D_{y/x})$: $y = 7.445x + 109.516$.	1
1.b	An estimation of the expenses is : $y = 7.445\times 15 + 109.516 = 221\,191\,000$ LL.	1
2.a	$u_1 = u_0\,(1+0.02) = 167.076$. $u_2 = u_1\,(1+0.02) = 170.417$.	2
2.b	$u_n = u_{n-1}\,(1+0.02) = 1.02\,u_{n-1}$. Hence (u_n) is a geometric sequence with common ratio $r = 1.02$.	1.5
2.c	2015 = 2007+8, then n = 8. $u_8 = u_0.r^8 = 163.8\,(1.02)^8 = 191.917\,806$ that is 191 91780LL. The company saves 29 274 000 LL.	1.5

<table>
<tr><th>Q_2</th><th>Answers</th><th>M</th></tr>
<tr><td>1</td><td>
<table>
<tr><th></th><th>Study Arabic</th><th>Study English</th><th>Study French</th><th>Total</th></tr>
<tr><td>Girls</td><td>6</td><td>8</td><td>11</td><td>25</td></tr>
<tr><td>Boys</td><td>9</td><td>10</td><td>16</td><td>35</td></tr>
<tr><td>Total</td><td>15</td><td>18</td><td>27</td><td>60</td></tr>
</table>
</td><td>1</td></tr>
<tr><td>2.a</td><td>$P(E) = \frac{18}{60} = \frac{3}{10}$. $P(B) = \frac{35}{60} = \frac{7}{12}$. $P(E\cap B) = \frac{10}{60} = \frac{1}{6}$.</td><td>1.5</td></tr>
<tr><td>2.b</td><td>$P(E\cup B) = P(E) + P(B) - P(E\cap B) = \frac{43}{60}$. $P(\bar{E}\cap\bar{B}) = \frac{6+11}{60} = \frac{17}{60}$. OR:
$\bar{E}\cap\bar{B} = \overline{E\cup B}$ $P(\bar{E}\cap\bar{B}) = 1 - \frac{43}{60} = \frac{17}{60}$.</td><td>1</td></tr>
<tr><td>3.a</td><td>$P(S) = \frac{C_{15}^2}{C_{60}^2} + \frac{C_{18}^2}{C_{60}^2} + \frac{C_{27}^2}{C_{60}^2} = \frac{203}{590}$.</td><td>1.5</td></tr>
<tr><td>3.b</td><td>$P(FF/BB) = \frac{C_{16}^2}{C_{35}^2} = \frac{24}{119}$.</td><td>2</td></tr>
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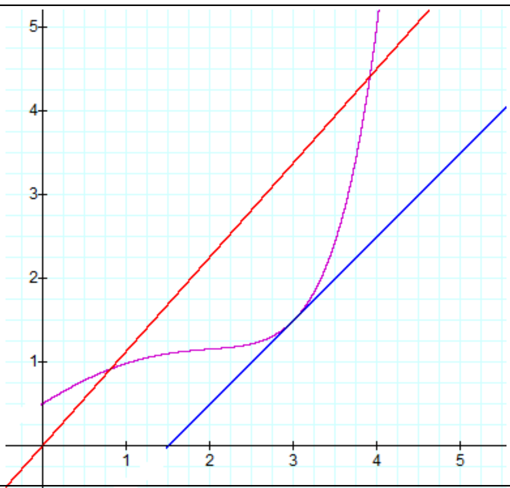
Q_3	Answers	M
1	$A = -\int_{-1}^{0}\left(1-e^{-x}\right)dx + \int_{0}^{2}\left(1-e^{-x}\right)dx = -\left[x+e^{-x}\right]_{-1}^{0} + \left[x+e^{-x}\right]_{0}^{2} = \left(e + \frac{1}{e^2} - 1\right)$ square units. **F**	2.5
2	Domain of definition : $2x-1 > 0$ and $10 - 4x > 0$, hence $\frac{1}{2} < x < \frac{5}{2}$. $\ln(2x-1) < \ln(10-4x)$; $2x-1 < 10-4x$; $x < \frac{11}{6}$. therefore $\frac{1}{2} < x < \frac{11}{6}$. **F**	2
4	$C_n = C_0\,(1+i)^n \Rightarrow 20\,000\,000(1+0.08)^n > 22\,000\,000(1+0.07)^n \Rightarrow$ $\left(\frac{1.08}{1.07}\right)^n > 1.1$, hence $n\ln\left(\frac{1.08}{1.07}\right) > \ln 1.1 \Rightarrow n > 10.245 \Rightarrow n = 11$. **T**	2.5

Q4	Answers	M
A1	$g'(x)=\frac{1}{8}\left[e^x+(x-3)e^x\right]=\frac{x-2}{8}e^x.$	1
A2	$\begin{array}{cccccc} x & 0 & & 2 & & +\infty \\ \hline g'(x) & & - & 0 & + & \\ \hline g(x) & \frac{5}{8} & \searrow & 1-\frac{e^2}{8}\approx 0.076367 & \nearrow & +\infty \end{array}$ The minimum of g(x) is positive hence g(x) > 0 for all $x\in[0,+\infty[$.	2
B1	f(0) = $\frac{1}{2}$, f(4) = 5 . $\lim\limits_{x\to+\infty} f(x)=+\infty.$	1
B2	$f'(x)=1+\frac{e^x+(x-4)e^x}{8}=1+\frac{(x-3)e^x}{8}=g(x).$ $\begin{array}{ccccc} x & 0 & & & +\infty \\ \hline f'(x) & & & + & \\ \hline f(x) & \frac{1}{2} & & \nearrow & +\infty \end{array}$	1.5
B3	y = f '(3)(x – 3) +f(3) = x + 1 – $\frac{e^3}{8}$.	1
B4		2
B5	f(0.7) – y(0.7) = 0.86 – 0.78 > 0; f(0.9) – y(0.9) = 0.94 – 1.01<0 hence 0.7 < α < 0 .9.	1
C1	C (0) = 1– 0.5 = 0.5. Therefore the fixed costs are 0.5 × 1 000 000 = 500 000 LL.	1
C2	Let $M_C(x)$ the marginal cost. $M_C(x)$ =f '(x) = g(x). $M_C(2)$= 0.076367. The minimum cost of production of a supplementary unit (1000 objects) is equal to 76367 LL and it occurs after the production of 2000 objects.	1.5
C3a	$R(x)=\frac{1125}{1000000}.x.1000=1.125x=\frac{9}{8}x.$	1
C3b	For $x\in]0.813\ ;\ 3.919[$ the line of the revenue function is above the curve (C) of the function of the cost, hence for a production between 814 and 3918 objects the company realizes profit.	1