

الاسم:	مسابقة في الرياضيات	عدد المسائل ثلاث
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ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو تخزين المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (5 points)

A cosmetic company is launching two new products on the market: a perfume and a lotion for women.

The price of a bottle of perfume and a bottle of lotion is 100 000LL.

The company sells the product in boxes containing each a bottle of perfume and two bottles of lotion and offers 10% discount on the price of the perfume and 15% discount on that of the lotion.

The price of a box, after the discount, is equal to 122 000 LL.

- 1) a- Calculate the initial price of a bottle of perfume and that of a bottle of lotion.
b- Calculate the price of each of the two bottles after the discount.
- 2) The company offers a supplementary discount of 5% on the price of the perfume and 8% on that of the lotion to the customers who buy a large quantity of boxes. Calculate the price of 50 boxes after the two discounts.

II- (5 points)

In a laboratory, a cage contains 20 guinea pigs distributed as shown in the following table:

Sex \ Color	Color	
	White	Black
Male	7	6
Female	5	2

A-

An employee catches, randomly, one guinea pig from this cage.

- 1) Calculate the probability that the caught guinea pig is a black male.
- 2) The caught guinea pig is a male, what is the probability that it is black?

B-

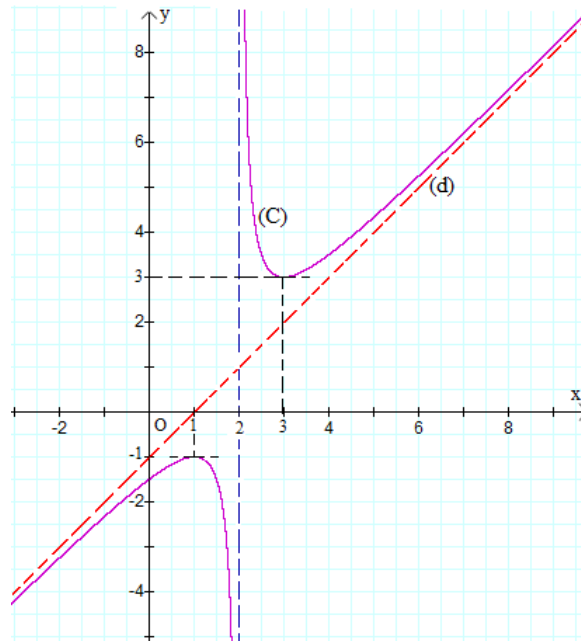
In this part, the employee catches randomly two guinea pigs from this cage successively and without replacement.

- 1) Show that the probability that the two caught guinea pigs are males is equal to $\frac{39}{95}$.
- 2) What is the probability that the two caught guinea pigs are two males of different colors?

III- (10 Points)

Consider the function f defined over $]-\infty; 2[\cup]2; +\infty[$ as: $f(x) = ax + b + \frac{1}{x-2}$,

and let (C) be its representative curve in an orthonormal system.



A- Use the graph to answer the following questions.

1) Determine :

a- $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$.

b- $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow +\infty} f(x)$.

2) Compare $f'(0)$ and $f'(1)$.

3) Solve the inequality $f(x) > 3$.

4) Give the number of solutions of the equation $f(x) = -3$.

5) Determine an equation of the oblique asymptote (d), then deduce a and b.

6) Set up the table of variations of the function f .

B- Let $f(x) = x - 1 + \frac{1}{x-2}$.

1) Calculate $f'(x)$.

2) Write an equation of the tangent to (C) at the point with abscissa $x = 0$.

LH SESSION 2-MATH 2013

Q ₁	Answers	G
1a	Let x be the price of the perfume and y that of the body lotion. $\begin{cases} x + y = 100000 \\ 0.9x + 2 \times 0.85y = 122000 \end{cases}$ $x = 60\,000$ and $y = 40\,000$. The price of the perfume is 60 000LL and that of the lotion is 40 000LL.	2.5
1b	The price of the perfume becomes $0.9 \times 60000 = 54000$ LL and the price of a lotion bottle is $40000 \times 0.85 = 34000$ LL.	1
2	The price of the perfume after the two discounts $54000 \times 0.95 = 51\,300$ LL. The price of the lotion after the two discounts is $34000 \times 0.92 = 31\,280$ LL. The price of 50 boxes after the two discounts is $50(51\,300 + 2 \times 31\,280) = 5\,693\,000$ LL.	1.5

Q ₂	Answers	G
A1	$P(\text{BM}) = \frac{6}{20} = \frac{3}{10}$.	1
A2	$P(\text{B/M}) = \frac{6}{13}$.	1
B1	$P(\text{MM}) = \frac{13}{20} \times \frac{12}{19} = \frac{39}{95}$.	1
B2	$P(\text{Mw, Mb}) + P(\text{Mb, Mw}) = 2 \times \frac{6}{20} \times \frac{7}{19} = \frac{21}{95}$.	2

Q ₃	Answers	G																						
A1a	$\lim_{\substack{x \rightarrow 2 \\ x < 2}} f(x) = -\infty$; $\lim_{\substack{x \rightarrow 2 \\ x > 2}} f(x) = +\infty$.	1																						
A1b	$\lim_{x \rightarrow -\infty} f(x) = -\infty$; $\lim_{x \rightarrow +\infty} f(x) = +\infty$.	1																						
A2	$f'(1) = 0$ and $f'(0) > 0$ then $f'(0) > f'(1)$.	1																						
A3	$f(x) > 3$ for $x > 2$ and $x \neq 3$.	1																						
A4	$f(x) = -3$ has two solutions since the line with equation $y = -3$ intersects (C) in two points.	1																						
A5	The equation of (d) is $y = mx + n$ and (d) passes through the points (1 ; 0) and (0 ; -1), therefore $n = -1$ and $m = 1$; $y = x - 1$; but the equation of the oblique asymptote is $y = ax + b$ hence $a = 1$ and $b = -1$.	1.5																						
A6	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>$-\infty$</td> <td>1</td> <td>2</td> <td>3</td> <td>$+\infty$</td> </tr> <tr> <td>$f'(x)$</td> <td></td> <td>+</td> <td>0</td> <td>-</td> <td>-</td> <td>0</td> <td>+</td> </tr> <tr> <td>$f(x)$</td> <td></td> <td></td> <td>$-\infty$</td> <td></td> <td>$+\infty$</td> <td></td> <td>$+\infty$</td> </tr> </table>	x	$-\infty$	1	2	3	$+\infty$	$f'(x)$		+	0	-	-	0	+	$f(x)$			$-\infty$		$+\infty$		$+\infty$	1.5
x	$-\infty$	1	2	3	$+\infty$																			
$f'(x)$		+	0	-	-	0	+																	
$f(x)$			$-\infty$		$+\infty$		$+\infty$																	
B1	$f'(x) = 1 - \frac{1}{(x-2)^2}$.	1																						
B2	$y = f'(0)x + f(0) = \frac{3}{4}x - \frac{3}{2}$	1																						