

الاسم:
الرقم:

مسابقة في مادة الرياضيات
المدة: ساعتان

عدد المسائل: ستة

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (2.5points)

All steps of calculation must be shown in each exercise.

1) Given $A = 6 \times 10^2 + 10^2 + 4 \times 10^{-2} + 10 - 6$.

- Write A in the form of a decimal number.
- Write A in scientific notation .
- Write A as the sum of an integer and a fraction less than 1, in its simplest form.

2) Show that the number $D = \frac{4}{2 + \sqrt{3}} \div \frac{2 - \sqrt{3}}{2}$ is a natural number.

3) Given the two numbers $B = \frac{5}{8} + \frac{3}{8} \times \frac{4}{6} - \left(\frac{3}{2} - 1\right)^2$ and $C = 2\sqrt{75} - 4\sqrt{27} + 4\sqrt{12}$.

- Write B as a fraction in its simplest form.
- Write C in the form $a\sqrt{3}$ where a is an integer.

II- (1.5points)

1) Solve the following system $\begin{cases} 2a - b = 4 \\ a + b = 5 \end{cases}$

2) Given the two polynomials $P(x) = (2a - b)x^2 + 5x - \frac{2}{3}$ and $Q(x) = 4x^2 + (a + b)x + c$.

Calculate a, b and c so that P(x) and Q(x) are identical.

III- (3.5points)

Part A

Given $E(x) = (3x - 1)^2 - (3x - 1)(x + 2)$.

- Expand and reduce E(x).
- Calculate x such that E(x) = 3.
- Factorize E(x).

Part B

In the next figure:

- x represents a length in cm so that $x > 0.5$
- ABCD is a square with side $3x - 1$.
- DEFG is a rectangle such that:

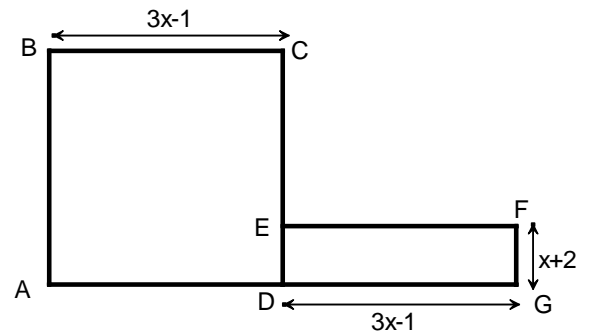
$FG = x + 2$ and $EF = 3x - 1$.

1) Calculate, in terms of x, the area S of ABCD and the area S' of DEFG.

2) Solve the equation $S - S' = 0$.

3) Determine all the integers x so that:

$S - S' > 6x^2 - 5x - 12$.



IV- (5.5points)

In an orthonormal system of axes $x'Ox; y'Oy$, consider the points $A(-2 ; -2)$, $E(2 ; 6)$ and $B(6 ; -2)$.

- 1) a-Plot the points A,E and B .
b-Verify that the equation of (AE) is $y= 2x+2$.
- 2) Determine the equation of (AB).
- 3)Verify that $AE =BE$.
- 4) Let K be the midpoint of the segment [AE].
a-Calculate the coordinates of K.
b-Let (d) be the perpendicular to (AE) at K. Determine the equation of (d).
- 5) The line (d) intersects the perpendicular bisector of [AB] at I.
a- show that I is the center of circle circumscribed about the triangle ABE.
b- Calculate the coordinates of I.
- 6) Let J be the symmetric of E with respect to I.Show that (AJ) is parallel to (d).

V- (2points)

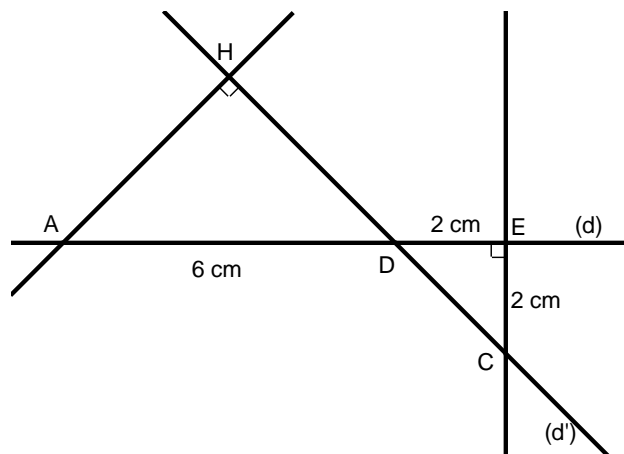
Given a parallelogram ABCD with center O. The points E and F are such that $\overline{AE} = \overline{DA}$ and $\overline{CF} = \overline{OC}$.

- 1) Draw a figure.
- 2) Prove that $\overline{EB} = \overline{AC}$ and $\overline{OF} = \overline{AC}$.
- 3) The lines (EF) and (OB) intersect at K. Prove that K is the midpoint of the segment [EF].

VI- (5points)

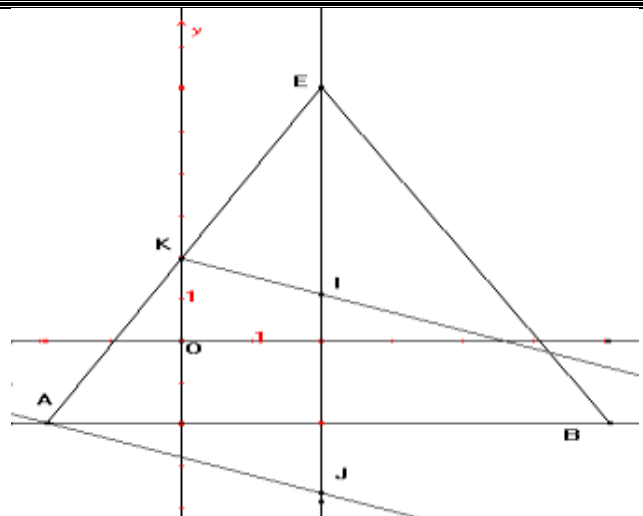
In the next figure:

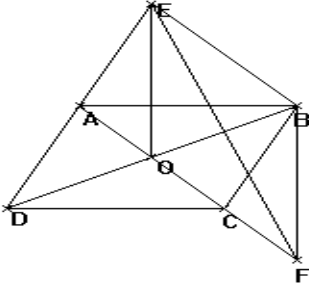
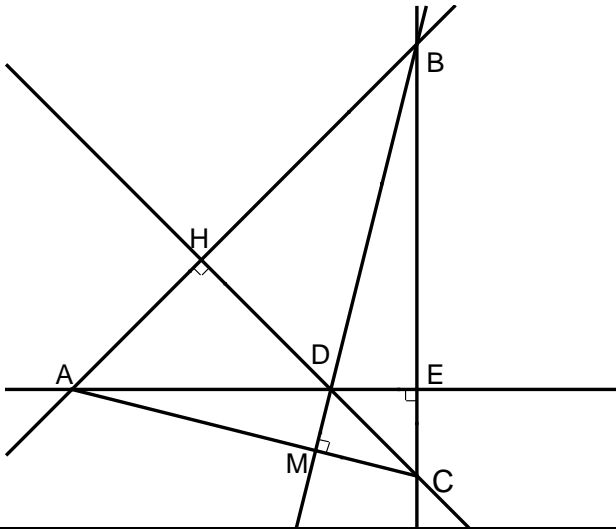
- (d) and (d') are two lines intersecting at D
- EDC is a right triangle at E
- $AD = 6\text{ cm}$ and $DE = EC = 2\text{ cm}$
- (AH) is perpendicular to (d').



- 1) Copy the figure.
- 2) What is the nature of the triangle ADH ?
Calculate the exact lengths of [DH], [DC] and [AC].
- 3) Prove that the points A, H, E and C are on the same circle whose diameter to be determined.
- 4) Let M be the orthogonal projection of D on (AC).
Prove that the two triangles AHC and DMC are similar. Calculate the product $CM \times CA$.
- 5) Calculate $\tan \widehat{ACH}$. Deduce the value of the angle \widehat{ACH} rounded to the nearest degree.
- 6) The lines (AH) and (CE) intersect at B, prove that (MD) passes through B.

I	1.a	704,04	0.25	
	1.b	$7,0404 \times 10^2$	0.25	
	1.c	$704 + \frac{1}{25}$	0.25	
	2	$\frac{4}{2+\sqrt{3}} \times \frac{2}{2-\sqrt{3}} = 8$	0.5	
	3.a	$\frac{5}{8} + \frac{1}{4} - \frac{1}{4} = \frac{5}{8}$	0.5	
	3.b	$10\sqrt{3} - 12\sqrt{3} + 8\sqrt{3} = 6\sqrt{3}$	0.75	
II	1	$\begin{cases} 2a - b = 4 \\ a + b = 5 \end{cases} \quad a=3, \quad b=2$	0,75	
	2	$a=3, \quad b=2 \quad \text{et} \quad c = -\frac{2}{3}$	0,75	
III	A	1	$E(x) = 9x^2 - 6x + 1 - (3x^2 + 5x - 2) = 6x^2 - 11x + 3$	0.5
		2	$x(6x - 11) = 0$, alors $x=0$ ou $x = \frac{11}{6}$.	0.5
		3	$E(x) = (3x-1)(3x-1-x-2) = (3x-1)(2x-3)$	0.5
	B	1	$S = (3x-1)^2 \quad \text{et} \quad S' = (3x-1)(x+2)$	0.5
		2	$(3x-1)(2x-3) = 0$ alors $x = \frac{1}{3}$, $x = \frac{3}{2}$, $x = \frac{1}{3}$ inacceptable donc $x = \frac{3}{2}$.	0.75
		3	$6x^2 - 11x + 3 > 6x^2 - 5x - 12$. $-6x > -15$ donc $x < \frac{15}{6}$ alors $x=1$ ou $x=2$.	0.75
IV	1.a	Points A,B et E.	0,5	
	1.b	Les coordonnées de A et E vérifient l'équation.	0,5	
	2	$y_A = y_B = -2$, Eq. de (AB) : $y = -2$.	0,5	



IV	3	$AE = 4\sqrt{5}$ et $BE = 4\sqrt{5}$	0,75
	4a	$K(0;2)$	0,5
	4b	Equation de (d) : $y = -\frac{1}{2}x + 2$	0,75
	5.a	(d) est la médiatrice de [AE] donc I est le point d'intersection des médiatrices des côtés du triangle AEB alors c'est le centre du cercle circonscrit à ce triangle.	0,5
	5.b	$I(2;1)$	0,75
	6	Dans le triangle EAJ ,K et I sont les milieux de [AE] et [JE] donc (AJ) parallèle à (d).Ou....	0,75
V	1		0,5
	2	$\overline{AE} = \overline{DA} = \overline{CB}$ alors AEBC est un parallélogramme. $\overline{EB} = \overline{AC}$; $\overline{AO} = \overline{OC} = \overline{CF}$ donc $\overline{OF} = \overline{AC}$	1
	3	$\overline{EB} = \overline{AC}$ et $\overline{OF} = \overline{AC}$ donc $\overline{EB} = \overline{OF}$ alors OBEF est un parallélogramme alors K milieu de [EF].	0,5
VI	1		0,25
	2	ADH est rectangle isocèle (angle de 45^0), $DH = 3\sqrt{2}$; $DC = 2\sqrt{2}$ $AC = 2\sqrt{17}$	1,75
	3	AHC et AEC sont deux triangles rectangles .A,H,E et C sont sur un même cercle de diamètre [AC]	0,5
	4	$\square AHC = \square DMC = 90^\circ$ et $\square HCA$ angle commun. $CM \times CA = CD \times CH = 2\sqrt{2} \times 5\sqrt{2} = 20$.	1
	5	$\tan \square ACH = \frac{AH}{HC} = \frac{3\sqrt{2}}{5\sqrt{2}} = \frac{3}{5}$ $\square ACH = \tan^{-1} \frac{3}{5} \square 30,9 \square 31^\circ$	0,75
	6	Dans le triangle ABC, D est l'orthocentre donc (MD) est la troisième hauteur	0,75