وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة المتوسطة	دورة العام 2015 الاستثنائية الثلاثاء 11 آب 2015
عدد المسائل: خمسة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I- (2 points)

Given the number a = $\dfrac{7+\sqrt{125}+\sqrt{20}}{14}$.

1) Write a in the form $x+y\sqrt{5}$ where x and y are two rational numbers.

2) Compare $a+1$ and a^2.

3) Verify that $a^3=2a+1$.

II - (4 points)

1) a. Verify that $x^2+4x+3=(x+2)^2-1$.

b. Factorize x^2+4x+3 .

2) Given an isosceles triangle ABC with vertex A so that its area is equal to x^2+4x+3 and BC = 2x+2 $(x>0)$. Let [AH] be an altitude in this triangle.

a. Show that $AH=x+3$.

b. Calculate AB^2 in terms of x.

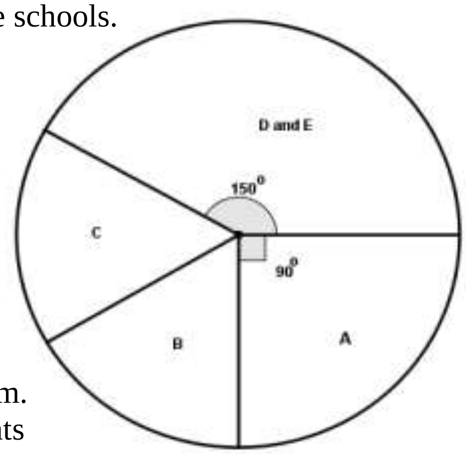
3) a. Find x such that the area of ABC is equal to 8. [Use **1)a.**]

b. For x = 1, calculate $\sin \hat{A}BC$ and deduce, rounded to the nearest degree, the measure of angle $\hat{A}BC$.

III - (4 points)

The Brevet students of five schools A, B, C, D and E sit for the official exam.
The adjacent circle graph represents the distribution of students in these schools.

- The total number of students is 240
- The angle that represents the students of D and E together is 150°
- The angle that represents the students of A is 90°
- The number of students of B is equal to that of C.



1) Verify that the number of students of A is 60.
2) Calculate the number of students of B and that of C.
3) Show that the number of students of D and E together is 100.
4) 20% of the students of A and 15% of the students of B failed, calculate the total number of students of A and B who passed the exam.
5) Three times the number of students of D minus the number of students of E is equal to 180.

a. Write a system of two equations with two variables to represent the number of students of D and E.

b. Solve the system and verify that the number of students of D is 70.

IV- (5points)

In an orthonormal system of axes x'Ox,y'Oy, consider the points A(0 ; 2) and B(- 4 ; 0) .

1) Plot the points A and B.

2) Show that $y = \frac{1}{2}x + 2$ is an equation of the line (AB).

3) Let [OH] be an altitude in triangle OAB.

a. Find an equation of line (OH).

b. Verify that the coordinates of H are $\left(-\frac{4}{5};\frac{8}{5}\right)$.

4) The parallel through B to y'Oy intersects (OH) at E.

a. Calculate the coordinates of point E.

b. Calculate OE and HE.

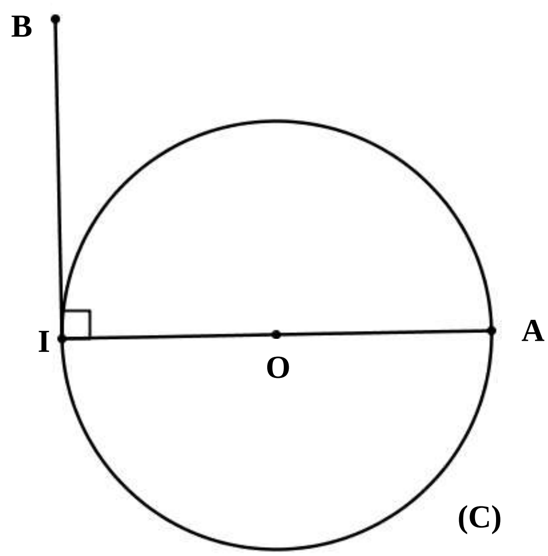
5) Let (C) be the circle circumscribed about triangle OBE and (d) the tangent at O to (C).
The two lines (d) and (EA) intersect at F.

Prove that $\frac{EA}{EF} = \frac{4}{5}$.

V- (5 points)

In the adjacent figure:

- (C) is a circle with center O and diameter [IA] so that IA = 8
- B is a point on the tangent at I to (C) so that IB = 6.



1) Copy the figure that will be completed later.

2) Let (C') be the circle with diameter [IB].The two circles (C) and (C') intersect at I and another point E.

a. Prove that A, E and B are collinear.

b. Calculate AB.

3) a. Write in two different triangles the ratios equal to $\cos \widehat{IBA}$.

b. Show that BE = 3.6

c. Deduce the length AE, then calculate IE.

4) The tangent at B to (C') intersects (IE) at F.

a. Show that the two triangles EBF and EIB are similar.

b. Deduce the value of $EI \times EF$.

5) Let L be the translate of B under the translation with vector $\overrightarrow{IA}$.
Prove that the four points A, E, F and L are on the same circle with diameter to be determined.

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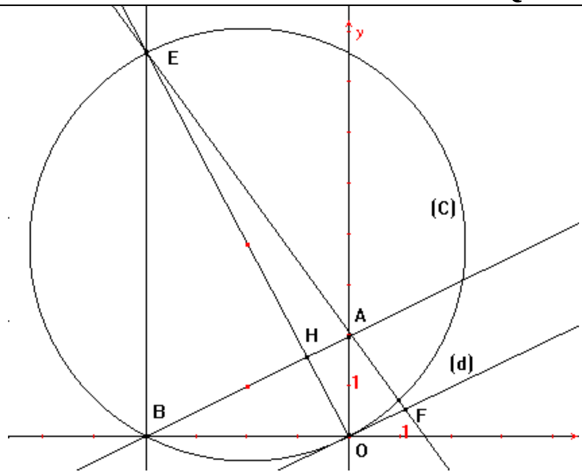
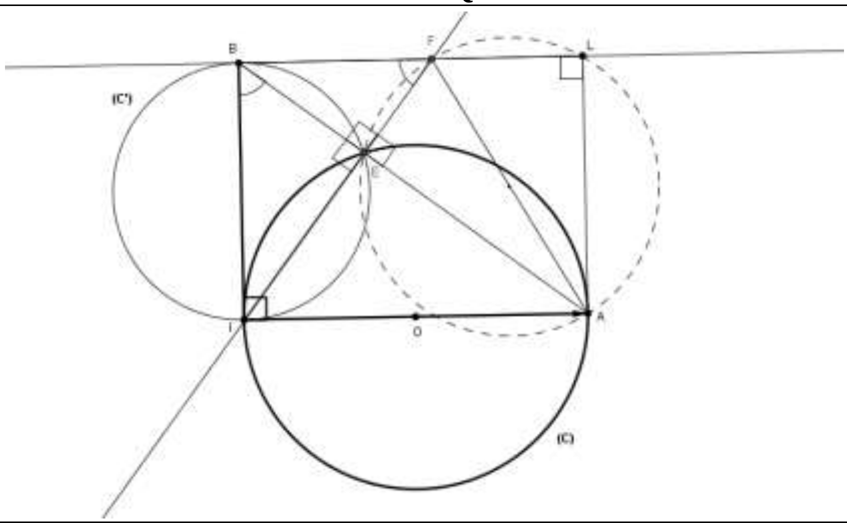
	Question I	
	Answers	Grade
1	$a=\frac{7+7\sqrt{5}}{14}=\frac{1+\sqrt{5}}{2}=\frac{1}{2}+\frac{1}{2}\sqrt{5}$ **(0.25) + (0.25)**	0.5
2	$a+1=\frac{3+\sqrt{5}}{2}$; $a^2=\frac{3+\sqrt{5}}{2}$ so a+ 1 = a^2 **(0.25) + (0.5) + (0.25)**	1
3	$a^3=\frac{8+4\sqrt{5}}{4}=2+\sqrt{5}$; 2a+1 = $2+\sqrt{5}$ Or $a^3=a^2$. a = (a+1)a = a^2 + a = a+1 +a= 2a +1**(0.25) + (0.25)**	0.5
	Question II	
1.a	$(x+2)^2-1=x^2+4x+4-1=x^2+4x+3$	0.5
1.b	$x^2+4x+3=(x+1)(x+3)$	0.5
2.a	Area of ABC $=\frac{BC\times AH}{2}$; $x^2+4x+3=\frac{2(x+1)\times AH}{2}=,(x+1)(x+3)=\frac{2(x+1)\times AH}{2}$ so AH = x+3**(0.25) + (0.25) + (0.25)**	0.75
2.b	$AB^2=(x+3)^2+(x+1)^2=2x^2+8x+10$	0.5
3.a	$(x+2)^2-1=8;(x+2)^2=9, x+2=3$ ou $x+2=-3$ So x = 1 since x= -5 not accepted**(0.25) + (0.5) + (0.25)**	1
3.b	$\sin\hat{B}=\frac{AH}{AB}=\frac{2}{\sqrt{5}}=0.89$, so $\hat{B}\approx 63^\circ$, **(0.5) + (0.25)**	0.75
	Question III	
1	Number of students of $A=240\times\frac{90}{360}=60$	0.5
2	Number of students of $B=240\times\frac{60}{360}=40$, Number of students of $C=40$ **(0.25) + (0.25)**	0.5
3	Number of students of C and E $=240\times\frac{150}{360}=100$ or another method…	0.25
4	Number of students who failed in A and B = $60\times\frac{20}{100}+15\times\frac{40}{100}=18$ **(0.25) + (0.25)** Number of students who passed in A and B =100 – 18 = 82 students. **(0.5)**	1
5.a	x + y = 100**(0.25)** 3x – y = 180.**(0.75)**	1
5.b	4x = 280, x = 70 and y = 30**(0.5) + (0.25)**	0.75

Question IV		
1	 Fig.	0.5
2	y= $\frac{1}{2}x+2$ is the equation of (AB)**slope (0.5) + b(0.25) or(verificationof a point (0.25)**	0.75
3.a	y = - 2x is the equation of (OH) **slope (0.5) + equation (0.25)**	0.75
3.b	$\frac{1}{2}x$ + 2 = -2 x so x = $\frac{-4}{5}$ et y = $\frac{8}{5}$**(0.5) + (0.25)**	0.75
4.a	$x_E = x_B = -4\ et\ y_E = -2x_E = -2(-4) = 8$ therefore $E(-4\,;8)$**(0.25) + (0.25)**	0.5
4.b	$OE = \sqrt{16+64} = 4\sqrt{5}\,;HE = \sqrt{\frac{256}{5}} = \frac{16\sqrt{5}}{5}$**(0.25) + (0.5)**	0.75
5	(d) // (AB) so : $\frac{EA}{EF} = \frac{EH}{EO}$(Thales') so : $\frac{EA}{EF} = \frac{\frac{16\sqrt{5}}{5}}{4\sqrt{5}} = \frac{4}{5}$//**(0.25) + ratio(0.5) + (0.25)**	1
Question V		
1		0.25
2.a	IÊB = IÊA = 90° so IÊB+IÊA = 180° therefore the 3 points are collinear.	0.5
2.b	By applying Pythagorean AB^2=100 then AB=10.	0.5
3.a	cos$\widehat{IBA}$ =$\frac{IB}{AB}$in triangle IBA, andcos$\widehat{IBA}$=$\frac{BE}{BI}$in triangle IBE.**(0.25) + (0.25)**	0.5
3.b	$\frac{IB}{AB}$= $\frac{BE}{BI}$so$\frac{6}{10}$ =$\frac{BE}{6}$thenBE=3.6**(0.25) + (0.25)**	0.5
3.c	AE = AB – BE =6.4**(0.25)** By applying Pythagorean in triangle IAE we get IE^2= 23.04 therefore IE= 4.8**(0.25)**	0.5
4.a	$\widehat{BEF}$ = $\widehat{BEI}$= 90°**(0.25)+(0.5)** $\widehat{BFE}$ =$\widehat{IBE}$ having same complement $\widehat{EBF}$so the two triangles are similar.	0.75
4.b		0.5
5	Locating point L, AIBLis a rectangle,A, E, L and F are on the same circle of diameter [AF]**(0.25) + (0.25) + (0.25) + (0.25)**	1