وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	دورة سنة 2013 العادية

الاثنين 1 تموز 2013 عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I-(4points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points:
A (4; 2; 0), B (2; 3; 1) and C (2; 2; 2).

1) Prove that triangle ABC is right at B.
2) Show that an equation of the plane (P) determined by the three points A, B and C is $x + y + z - 6 = 0$.
3) Let (Q) be the plane passing through A and perpendicular to (AB).
   a- Determine an equation of (Q).
   b- Denote by (D) the line of intersection of (P) and (Q), show that (D) is parallel to (BC).
4) Let H(5;3;1) be a point in (Q).
   a- Show that A is the orthogonal projection of H on (P).
   b- Calculate the volume of the tetrahedron HABC.

II-(4points)

A music store sells classical and modern musical albums only.
The customers of this store are surveyed and the results are as follows:

- 20% of these customers bought each a classical album.
- Out of those who bought a classical album, 70% bought a modern album.
- 22% of the customers bought each a modern album.

A customer of the store is interviewed at random. Consider the following events:
C: «the interviewed customer bought a classical album »
M: «the interviewed customer bought a modern album ».

1) Calculate the probability $P(C \cap M)$ and verify that $P(C \cap \overline{M}) = 0.06$.
2) Prove that $P(\overline{C} \cap \overline{M}) = 0.72$.
3) Calculate the probability that the customer bought at least one album.
4) Knowing that the customer didn't buy a modern album, calculate the probability that he bought a classical album.
5) The classical album is sold for 30 000LL and the modern one is sold for 20 000LL.
   Let X be the random variable that is equal to the sum paid by a customer.
   a- Justify that the possible values of X are: 0, 20 000,30 000 and 50 000. Then, determine the probability distribution of X.
   b- During the month of June, 300 customers visited this music store. Estimate the revenue of this store during that month.

III(4 points)

In the plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A, B and C with respective affixes $z_A = i$, $z_B = 3-2i$ and $z_C = 1$.

1) Prove that the points A, B and C are collinear.
2) Consider the complex number $w = z_C - z_A$.
   Write w in exponential form and deduce that w^{20} is a real negative number.

3) Let M be a point in the plane with affix z.
   a- Give a geometric interpretation to $|z-i|$ and $|z-1|$.
   b- Suppose that $|z-i| = |z-1|$; show that the point M moves on a line to be determined.
   c- Prove that if $(z-i)\times(\bar{z}+i) = 16$, then the point M moves a circle whose center and radius to be determined.

IV-(8points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = 3 - \dfrac{4}{e^{2x}+1}$.

Let (C) be its representative curve in an orthonormal system (**unit 2 cm**).

1) Calculate $\lim\limits_{x\to-\infty} f(x)$, $\lim\limits_{x\to+\infty} f(x)$ and deduce the asymptotes to (C).

2) Prove that f is strictly increasing over $\mathbb{R}$ and set up its table of variations.

3) The curve (C) has a point of inflection W with abscissa 0. Write an equation of (T), the tangent to (C) at the point W.

4) a- Calculate the abscissa of the point of intersection of (C) with the x-axis.

   b- Draw (T) and (C).

5) a- Verify that $f(x) = -1 + \dfrac{4e^{2x}}{e^{2x}+1}$ and deduce an antiderivative F of f.
   b- Calculate, in cm^2, the area of the region bounded by the curve (C), the x- axis, the y-axis and the line with equation x = ln2.

6) The function f has over $\mathbb{R}$ an inverse function g. Denote by (G) the representative curve of g.
   a- Specify the domain of definition of g.
   b- Show that (G) has a point of inflection J whose coordinates to be determined.
   c- Draw (G) in the same system as (C).
   d- Determine g(x) in terms of x.

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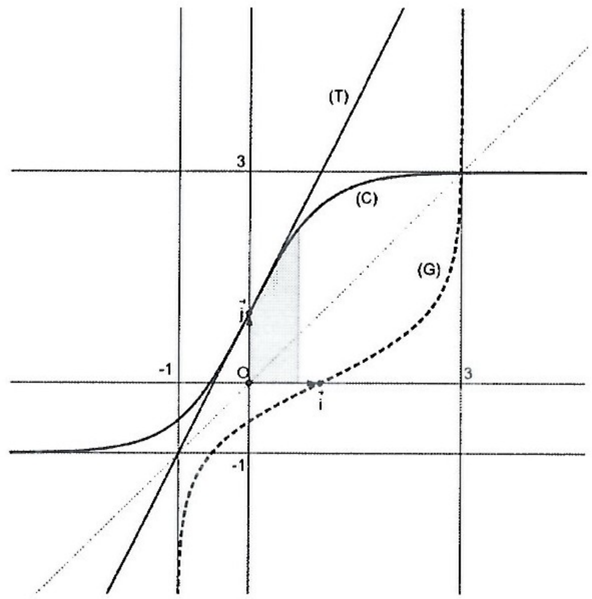
Q_1	Answers	M
1	$\overrightarrow{AB}(-2\,;1\,;1)$, $\overrightarrow{BC}(0\,;-1\,;1)$; $\overrightarrow{AB}\cdot\overrightarrow{BC} = 0$, hence triangle ABC is right at B.	0.5
2	$x_A + y_A + z_A - 6 = 0$, then A is in (P); $x_B + y_B + z_B - 6 = 0$, then B belongs to (P) and $x_C + y_C + z_C - 6 = 0$, then C is in (P) Therefore (P) : $x + y + z - 6 = 0$. **Or** $\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC}) = 0$ with M(x: y; z) any point in (P).	0.5
3. a	For any point M (x, y, z) in (Q); $\overrightarrow{AM}\,.\,\overrightarrow{AB} = 0$; (Q): $-2x + y + 2z + 6 = 0$.	0.5
3. b	A directing vector of (D) is $\vec{V} = \vec{n}_P \wedge \vec{n}_Q$, hence $\vec{V}(0; -3; 3)$, et $\overrightarrow{BC}(0; -1; 1)$ and $B \notin$ (Q), so $B \notin$ (D). Thus, (D) is parallel to (BC). **Or** : Since (BC) is perpendicular to (AB) and (AB) is perpendicular to (D) in A, (BC) and (D) being coplanar in (P) and perpendicular to the same line (AB), are parallel.	1
4. a	$A \in (P)$, $\overrightarrow{AH}(1;1;1)$ and $\vec{n}_P(1;1;1)$ hence (AH) is perpendicular to (P).	1
4. b	The volume of tetrahedron HABC is equal to $V = \frac{1}{3}$ HA × area of triangle ABC = $\frac{1}{6}\times BA\times BC\times\sqrt{3} = 1u^3$. **Or** $V = \frac{\left\lvert\overrightarrow{AH}.\left(\overrightarrow{AB}\wedge\overrightarrow{AC}\right)\right\rvert}{6} = \frac{6}{6} = 1\ u^3$.	0.5

<table>
<tr><th>Q_2</th><th>Answers</th><th>M</th></tr>
<tr><td>1</td><td>$P(C\cap M) = P(C)\times P(M/C) = 0.14.$ $P(C\cap\bar{M}) = P(C)\times P(\bar{M}/C) = 0.06.$</td><td>1</td></tr>
<tr><td>2</td><td>$P(C\cap\bar{M}) + P(\bar{C}\cap\bar{M}) = P(\bar{M}) = 1 - P(M)$ then $P(\bar{C}\cap\bar{M}) = 0.78 - 0.06 = 0.72.$</td><td>0.5</td></tr>
<tr><td>3</td><td>P (at least an album) = $1 - P(\bar{C}\cap\bar{M}) = 0.28.$</td><td>0.5</td></tr>
<tr><td>4</td><td>$P(C/\bar{M}) = \frac{P(C\cap\bar{M})}{P(\bar{M})} = \frac{0.06}{0.78} = \frac{1}{13}.$</td><td>0.5</td></tr>
<tr><td>5a</td><td>The four possible values are : 0 (the costumer did not buy anything), 20 000 (the costumer bought a modern album), 30 000 (the costumer bought a classical album), 50 000 (the costumer bought two albums).
<table>
<tr><td>x_i</td><td>0</td><td>20 000</td><td>30 000</td><td>50 000</td></tr>
<tr><td>P_i</td><td>0.72</td><td>0.08</td><td>0.06</td><td>0.14</td></tr>
</table></td><td>1</td></tr>
<tr><td>5b</td><td>$E(X) = \sum P_i X_i$ = 0 × 0. 72 + 20 000 × 0. 08 + 30 000 × 0. 06 + 50 000 × 0. 14 = 10 400 L L.
R= E(X) × 300 = 10 400 × 300 = 3 120 000 LL.</td><td>0.5</td></tr>
</table>

Q_3	Answers	M
1	$z_A - z_B = -3 + 3i$ and $z_A - z_C = -1 + i$. $z_A - z_B = 3(z_A - z_C)$ and $z_{\overrightarrow{BA}}$ $-\overrightarrow{CA}$; par suite $\overrightarrow{BA} = 3\overrightarrow{CA}$ and the three points A, B and C are collinear.	0.5
2	$w = z_{\overrightarrow{AC}} = 1 - i = \sqrt{2}\,e^{-i\frac{\pi}{4}}$, $w^{20} = \left(\sqrt{2}\right)^{20} e^{-5i\pi} = -\left(\sqrt{2}\right)^{20}$ which is real negative.	1
3. a	$\lvert z - i\rvert = \lvert z_M - z_A\rvert$ = AM ; $\lvert z - 1\rvert = \lvert z_M - z_C\rvert$ = CM.	0.5
3. b	If z_M verifies $\lvert z - i\rvert = \lvert z - 1\rvert$, so MA = MC; and the point M varies on the perpendicular bisector of segment [AC].	1
3. c	If z_M verify $(z - i)\times(\bar{z} + i) = 16 \Leftrightarrow (z_M - z_A)\times(\overline{z_M - z_A}) = 16 \Leftrightarrow \lvert z_M - z_A\rvert\times\left\lvert\overline{z_M - z_A}\right\rvert = 16$ $\Leftrightarrow \lvert z_M - z_A\rvert\times\lvert z_M - z_A\rvert = 16.$ Hence AM^2= 16; therefore the point M belongs to the circle with center A and radius 4.	1

Q_4	Answers	M
1	$\lim\limits_{x\to-\infty} f(x) = 3 - 4 = -1$ and $\lim\limits_{x\to+\infty} f(x) = 3$. Hence (C) has two asymptotes with equations y = 3 and y = – 1.	1
2	$f'(x) = \dfrac{8e^{2x}}{(e^{2x}+1)^2} > 0$; f is strictly increasing over IR. $\begin{array}{c\|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -1 & \nearrow & 3 \end{array}$	1
3	Slope of (T) = f '(0) = 2, and (T) passes through the point W (0 ; 1), then the equation of (T) is : y = 2 x + 1.	0.5
4. a	$f(x) = 0 \Leftrightarrow 3 = \dfrac{4}{e^{2x}+1} \Leftrightarrow e^{2x} = \dfrac{1}{3} \Leftrightarrow x = -\dfrac{\ln 3}{2}$.	0.5
4.b		1
5-a	$f(x) = 3 - \dfrac{4}{e^{2x}+1} = \dfrac{3e^{2x}-1}{e^{2x}+1}$ et $1 + \dfrac{4e^{2x}}{e^{2x}+1} = \dfrac{3e^{2x}-1}{e^{2x}+1}$. $F(x) = \int f(x)\,dx = \int\left(-1 + \dfrac{4e^{2x}}{e^{2x}+1}\right)dx = -x + 2\int \dfrac{2e^{2x}}{e^{2x}+1}\,dx = -x + 2\ln\left(e^{2x}+1\right) + c.$	1
5-b	A = 4A' cm^2. $A' = \int_0^{\ln 2} f(x)\,dx = \left[-x + 2\ln\left(e^{2x}+1\right)\right]_0^{\ln 2} = 2\ln 5 - 3\ln 2 = \ln\left(\dfrac{25}{8}\right)$. Thus, $A = 4\ln\left(\dfrac{25}{8}\right) cm^2$.	0.5
6. a	Dom (g) =]–1; 3[.	0.5
6. b	W (0,1) is a point of inflection of (C), then the symmetric of W with respect to the line with equation y = x is the point J (1 ; 0), which is the point of inflection of (G).	0.5
6. c	(G) is the symmetric of ©with respect to the line with equation y = x.	0.5
6. d	$y = g(x) \leftrightarrows x = f(y) \leftrightarrows \quad x = 3 - \dfrac{4}{e^{2y}+1} \quad \leftrightarrows \quad \dfrac{4}{e^{2y}+1} = 3 - x \quad \leftrightarrows \quad e^{2y} + 1 = \dfrac{4}{3-x} \quad \leftrightarrows$ $e^{2y} = \dfrac{4}{3-x} - 1 = \dfrac{1+x}{3-x}$. Thus, $2y = \ln\left(\dfrac{1+x}{3-x}\right)$; $y = g(x) = \dfrac{1}{2}\ln\left(\dfrac{1+x}{3-x}\right)$.	1