وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحاناتللشهادة الثانوية العامة فرع العلوم العامة	دورة العام 2015 العادية الأربعاء 10 حزيران 2015
عدد المسائل : ست	مسابقة في مادة: الرياضيات المدة: أربع ساعات	الاسم: الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات أو رسم البيانات.

- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الإلتزام بترتيب المسائل الوارد في المسابقة)

I- (2 points)

In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points I (2, 1,2) and F(1, 1,1) . Let (d) be the line defined by: $x = 2m$; $y = m - 1$; $z = m + 2$, where m is a real parameter. (P) is the plane determined by the line (d) and the point I .

1) Verify that $x - y - z + 1 = 0$ is an equation of (P).

2) E is the orthogonal projection of I on (d). Find the coordinates of E.

3) Consider, in the plane (P), the circle (C) with center I, and tangent to (d).
   a- Prove that F is on (C).
   b- Write a system of parametric equations for the line (Δ) , the tangent at F to (C).

II-(3 points)

Consider two urns U and V such that:

- Urn U contains four white balls and two red balls.
- Urn V contains two white balls and three red balls.

A- A player selects, randomly, one ball from the urn U and one ball from the urn V.
He scores +3 points for a red ball selected from urn U and +1 point for a red ball selected from urn V; he scores -1 point for a white ball selected from urn U and -2 points for a white ball selected from urn V.
Let X be the random variable that is equal to the algebraic sum of points scored by the player.

1) Find the four possible values of X and prove that the probability $P(X = 0) = \frac{2}{5}$.
2) Determine the probability distribution of X.

B- In this part, the player selects at random, one ball from the urn U and puts it in the urn V. Then, he selects two balls simultaneously and randomly from the urn V.
Consider the following events:
W: «the ball selected from the urn U is white»,
D: «the two balls selected from the urn V have different colors»,

1) Verify that $P(D/W) = \frac{3}{5}$, then calculate $P(D \cap W)$.
2) CalculateP(D).
3) Knowing that the two balls selected from urn V have the same color, what is the probability that the ball selected from the urn U is white?

III- (2 points)

Consider the sequence (u_n) defined by $u_1 = \frac{1}{2}$ and for all natural numbers $n \geq 1$: $u_{n+1} = \frac{n+1}{2n} u_n$.

1) a- Use mathematical induction to prove that $u_n > 0$ for all $n \geq 1$,

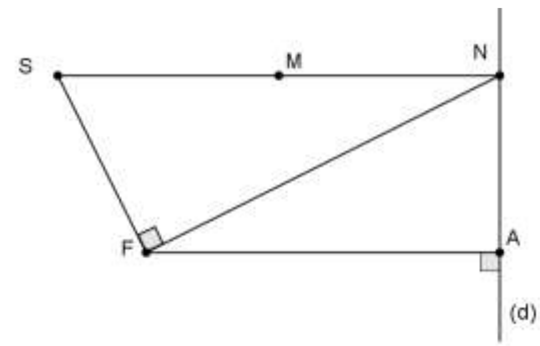
b- Prove that the sequence (u_n) is decreasing. Deduce that (u_n) is convergent.

2) Let (v_n) be the sequence defined, for all $n \geq 1$, by $v_n = \ln\left(\frac{u_n}{n}\right)$.

a- Prove that (v_n) is an arithmetic sequence whose common difference $d = -\ln 2$ and determine its first term.

b- Express v_n in terms of n, then verify that $u_n = \frac{n}{2^n}$.

IV- (3 points)



In the given figure:

- A and F are two fixed points with AF= 4.
- (d) is the line perpendicular to (AF) at A,
- N is a variable point on (d),
- (NS) is the line parallel to (AF),
- NFS is right triangle at F,
- M is the midpoint of [NS].

A-

1) a- As N varies on (d) , prove that M moves on a parabola (P) with focus F and directrix (d).

b- Determine the vertex V of (P).

2) (Δ) is the parallel through F to (d) . E is a point on (Δ) so that FE = 4.

a- Show that E is on (P).

b- Prove that (EA) is tangent to (P).

B-

The plane is referred to the direct orthonormal system $(V;\vec{i},\vec{j})$ with $\vec{i} = \frac{1}{2}\overrightarrow{VA}$.

1)a-Verify that $y^2 = -8x$ is an equation of (P) .

b- Draw (P).

2) T is a point with affix z and L is a pointwith affix z' such that : $z' = 3z - \bar{z}$.

Let $z = x + iy$ and $z' = x' + iy'$. (x , y , x' and y' are real numbers)

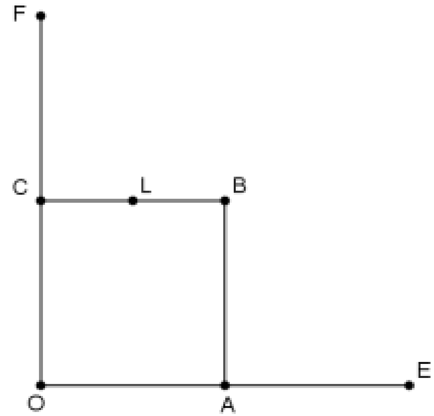
a- Express x' and y' in terms of x and y.

b- As T varies on the circle with center O and radius 1, prove that L moves on an ellipse (E) having A and F as two of its vertices.

V- (3 points)

In the figure below, OABC is a direct square so that:

OA = 2, and $\left(\overrightarrow{OA};\overrightarrow{OC}\right)=\frac{\pi}{2}\ [2\pi]$.



Let E be the symmetric of O with respect to A , F the symmetric of O

with respect to Cand L the midpoint of segment [BC].

S is the similitude that maps O onto E and C onto O.

1) Calculate the ratio k and an angle α of S.

2) a- Determine the image of line (BC) under S .

b- Prove that the image of the line (OB) under S is the line (EF).

c- Determine S(B), then S(L).

3) The complex plane is referred to a direct orthonormal system $\left(O;\frac{1}{2}\overrightarrow{OA},\frac{1}{2}\overrightarrow{OC}\right)$.

a- Write the complex form of S.

b- Deduce the affix of the center I of S.

c- Prove that I is the intersection point of (OL) and (EC).

VI- (7 points)

A-

Consider the differential equation (E): $y'' - 4y' + 4y = 4x - 4$.

Let $y = z + x$.

1) Write the differential equation (E') satisfied by z and solve (E').

2) Determine the particular solution of (E) whose representative curve has at the pointwith abscissa 0 a tangent with equation $y = x - 1$.

B-

Consider the function f defined on $\mathbb{R}$ by $f(x) = (2x-1)e^{2x} + x$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

(d) is the line with equation $y = x$.

1) a) Determine $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty} \frac{f(x)}{x}$.

   b) Study, according to the values of x, the relative positions of (C) and (d).

   c) Show that (d) is an asymptote to (C) as x tends to $-\infty$.

2) a) Show that (C) has an inflection point whose coordinates are to be determined.

   b) Set up the table of variations of f' and deduce that f is strictly increasing on $\mathbb{R}$.

3) (C) intersectsthe x-axis at a point with abscissa k.

   a) Verify that $0.4 < k < 0.5$.

   b) Plot (C) identifying its point of intersection with the y-axis.

4) Find, over $\mathbb{R}$, an antiderivative F of f.

5) f has an inverse function g on $\mathbb{R}$. Denote by (G) the representative curve of g.

   a) Write an equation of (D), the tangentto (G) at the point with abscissa -1.

   b) Prove that (G) has an inflection point S whose coordinates are to be determined.

   c) Draw (G) in the same system as (C).

   d) Let $E = \int_{-1}^{0} g(x)dx$. Express E in terms of k.

دورة العام 2015 العادية	امتحاناتللشهادة الثانوية العامة	وزارة التربية والتعليم العالي
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		دائرة الامتحانات

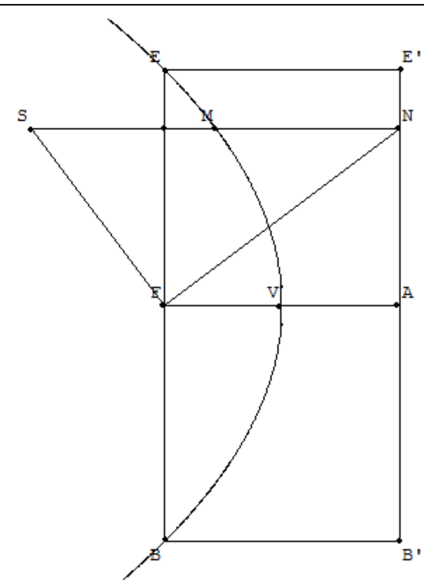
أسس التصحيح

مسابقة في مادة:الرياضيات
المدة:أربع ساعات

Q1	Answers	M
1	• Let A(0; -1; 2) be a point on (d) and let M(x; y; z) be a variable point in (P), then: $\overrightarrow{AM}.(\overrightarrow{IA}\times\overrightarrow{V_d})=0$ gives x-y-z+1=0. OR (d) lies in (P) since 2m-(m-1)-(m+2)+1=0 and I(2,1,2) is in (P) since 2-1-2+1=0	1
2	E(2m,m-1,m+2) $\in$ (d) $\overrightarrow{IE}(2m-2, m-2, m)$ and $\overrightarrow{V}_{(d)}(2,1,1)$ $\overrightarrow{IE}.\overrightarrow{V}_{(d)}=4m-4+m-2+m=0$ $m=1$ So, $E(2,0,3)$	1
3a	Radius of the circle is IE = $\sqrt{2}$, IF= $\sqrt{1+0+1}=\sqrt{2}$	1
3b	$\overrightarrow{V}_\Delta=\overrightarrow{IF}\wedge\overrightarrow{N_p}\ -\vec{i}-2\vec{j}+\vec{k}$ therefore $(\Delta):\begin{cases}x=-t+1\\y=-2t+1\\z=t+1\end{cases}$	1

Q2		Answers	M
A	1	The values of X are: 3+1= 4, 3–2=1, –1+1= 0 and –1–2 = –3. $P(X=0)=P(W_U;R_V)=\frac{4}{6}\times\frac{3}{5}=\frac{2}{5}.$	0.5
	2	P(X= –3) = P(2W) = $\frac{4}{6}\times\frac{2}{5}=\frac{4}{15}$; $P(X=1)=P(R_U;W_V)=\frac{2}{6}\times\frac{2}{5}=\frac{2}{15}$; $P(X=4)=P(R;R)=\frac{2}{6}\times\frac{3}{5}=\frac{1}{5}.$	1.5
B	1	P(D/W) = $\frac{C_3^1.C_3^1}{C_6^2}=\frac{3}{5}$ and $P(D\cap W)=$ P(D/W) .P(W)= $\frac{3}{5}.\frac{4}{6}=\frac{2}{5}$	2
	2	P(D) = $\frac{4}{6}.P(D/W)+\frac{2}{6}.P(D/R)=\frac{4}{6}.\frac{3}{5}+\frac{2}{6}.\frac{C_2^1.C_4^1}{C_6^2}=\frac{26}{45}$	1
	3	P(W/$\overline{D}$)= $=\frac{P(W\cap\overline{D})}{P(\overline{D})}=\frac{P(W)-P(w\cap D)}{1-P(D)}=\frac{12}{19}$	1

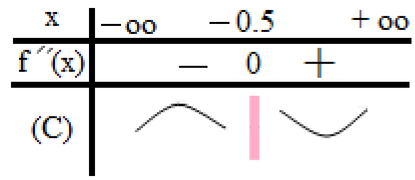
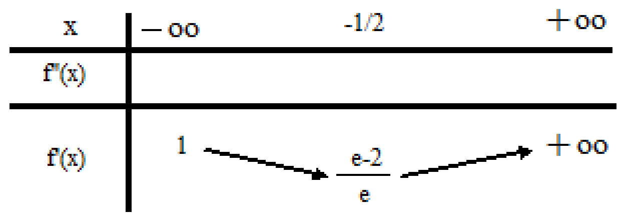
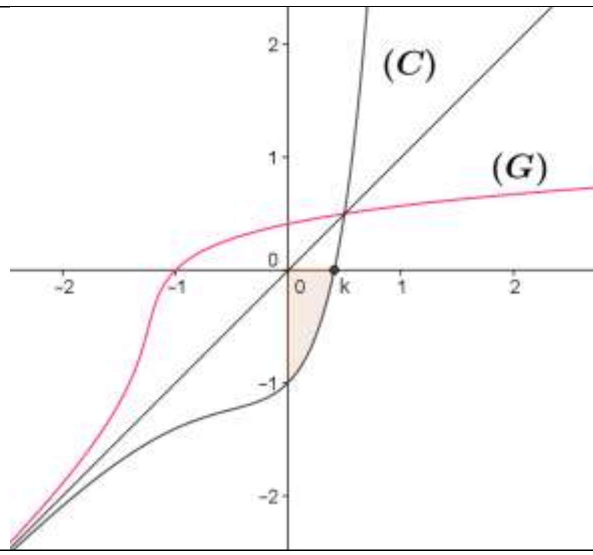
Q3	Answers	M
1a	$u_1=\frac{1}{2}>0$; Suppose $u_n>0$, then $u_{n+1}=\frac{n+1}{2n}u_n>0$.	1
1b	$u_{n+1}-u_n=\frac{1-n}{2n}u_n\le 0$; hence (u_n) is decreasing. (u_n) is decreasing and has a lower bound 0 then it is convergent.	1
2a	$v_{n+1}=\ln\left(\frac{u_{n+1}}{n+1}\right)=\ln\left(\frac{1}{n+1}\times\frac{n+1}{2n}u_n\right)=\ln\left(\frac{1}{2}\times\frac{u_n}{n}\right)=v_n-\ln 2.$ (v_n) is an arithmetic sequence with first term $v_1=-\ln 2$ and $d=-\ln 2$.	1
2b	$v_n=v_1+(n-1)d=-n\ln 2.$ $\ln\left(\frac{u_n}{n}\right)=-n\ln 2\Leftrightarrow\frac{u_n}{n}=e^{-n\ln 2}\Leftrightarrow u_n=\frac{n}{e^{n\ln 2}}=\frac{n}{(e\ln 2)^n}=\frac{n}{2^n}.$	1

Q4	Answers	M
A1a	MF = ½ SN = MN = d ($M\to(d)$) hence M moves on (P).	1
A1b	V is the midpoint of [FA].	0.5
A2a	EF=4 equals distance from E to (d). Thus E is on (P).	0.5
A2b	E' is the orthogonal p[rojection of E on (d). (EE') perpendicular to (d). (EA) is the bisector of $\widehat{FEE'}$. Therefore (EA) is tangent to (P).	1
B1a	P = 4 = FA. Therefore, $y^2=-8x$ (P)	0.5
B1b		0.5

B2a	$x' + iy' = 3x + 3iy - x + iy$ $\begin{cases} x' = 2x \\ y' = 4y \end{cases}$	1
B2b	$x^2 + y^2 = 1$ then $\frac{x'^2}{4} + \frac{y'^2}{16} = 1$ is the equation of (E) with $a = 4$ and $b = 2$. Therefore $A(2,0)$ and $F(-2,0)$ are 2 vertices of (E).	1.5

Q5	**Answers**	**M**
1	$k = \frac{OE}{CO} = \frac{4}{2} = 2$, $\alpha = \left(\overrightarrow{OC}; \overrightarrow{EO}\right) = \frac{\pi}{2} (\text{mod } 2\pi)$.	1
2a	S(C) = O then S(BC)) is the line passing in O and perpendicular to (BC) thusS((BC))= OC.	0.5
2b	S((OB)) is the line passing in E and perpendicular to (OB) thusitis (EB). But E, B and F are collinearsoitis (EF).	1
2c	$S(B) = S((OB) \cap (BC)) = (EF) \cap (OC) = F$ L is the midpoint of $[BC]$ so S(L)=C since C is the midpoint of $[OF]$	1
3a	$z' = 2iz + b$ but S(C) is O thus b=4. Therefore $z' = 2iz + 4$.	1
3b	$z_I = \frac{4}{1-2i} = \frac{4}{5} + \frac{8}{5}i$	0.5
3c	$z_{\overline{IE}} = -\frac{4}{5} z_{\overline{EC}}$ and $z_{\overline{OI}} = \frac{4}{5} z_{\overline{OL}}$, so I is the intersection of (EC) and (OL).	1

Q6		Answers	M
A	1	$z'' - 4z' + 4z = 0$; $z = (ax + b)e^{2x}$	1
	2	$y(0) = -1$ and $y'(0) = 1$; $a = -2$ et $b = 1$; $f(x) = x + (2x - 1)e^{2x}$	1.5
B	1a	$\lim_{x\to+\infty} f(x) = +\infty$ and $\lim_{x\to+\infty} \frac{f(x)}{x} = +\infty$	1
	1b	$f(x) - x = (2x - 1)e^{2x}$; if $x < \frac{1}{2}$ then (C) is below (d); if $x > \frac{1}{2}$ then (C) is above (d); if $x = \frac{1}{2}$ (d) intersects (C).	1
	1c	$\lim_{x\to-\infty} f(x) - x = \lim_{x\to-\infty} (2x - 1)e^{2x} = 0$. So : $y = x$ is an oblique asymptote at $-\infty$	0.5

<table>
<tr><td></td><td>2a</td><td>$f''(x) = 4(2x+1)e^{2x}$

Thus $I\left(-\frac{1}{2};-\frac{e+4}{2e}\right)$ is a point of inflection.</td><td>1</td></tr>
<tr><td></td><td>2b</td><td>
$f'(x) \geq \frac{e-2}{2} > 0$ so f is strictly increasing over $\mathbb{R}$.</td><td>1</td></tr>
<tr><td></td><td>3a</td><td>Comme $f(0,4).f(0,5) < 0$ donc $0,4 < x < 0,5$.</td><td>0.5</td></tr>
<tr><td></td><td>3b</td><td></td><td>1.5</td></tr>
<tr><td></td><td>4</td><td>$F(x) = \frac{x^2}{2} + e^{2x}(x-1) + c$</td><td>1</td></tr>
<tr><td></td><td>5a</td><td>$(D): y = x+1$</td><td>1</td></tr>
<tr><td></td><td>5b</td><td>By symmetrywith respect to y=x, $H\left(-\frac{e+4}{2e};-\frac{1}{2}\right)$ point of inflection of (G)</td><td>0.5</td></tr>
<tr><td></td><td>5c</td><td>(G0 issymmetric of (C) with respect to y=x.See figure</td><td>1</td></tr>
<tr><td></td><td>5d</td><td>$E = \int_{-1}^{0} g(x)dx = \int_{0}^{k} -f(x)dx = -F(k) + F(0) = -\frac{k^2}{2} - (k-1)e^{2k} + 1$.</td><td>1.5</td></tr>
</table>