ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (2 points)

The following 3 questions are independent of each other :
(All steps of calculation must be shown in each exercise)

1) Given : $A = \dfrac{5\sqrt{18} - 2\sqrt{98}}{2\sqrt{3} \times \sqrt{24} - 4\sqrt{2}}$.

    a. Write A as a fraction in its simplest form.

    b. Write A in scientific notation.

2) α is an acute angle. Prove that : $(1 - \sin^2 \alpha) \cdot \tan^2 \alpha = \sin^2 \alpha$.

3) Determine the real number x so that the following table represents a proportion:

$3 + \sqrt{5}$	x
$\dfrac{7}{5}$	$3 - \sqrt{5}$

II- (3 points)

40 students were surveyed about the number of books they read last month.
The following table represents the results of the survey:

Number of read books	0	2	4	6	7	Total
Frequency	2	7	10	9	12	40
Relative frequency in%			25			100
Central angle of a circle graph			90°			360°

1) Determine the mean of this series.
2) Copy the table above, then complete it.
3) What is the number of students who have read **at least** 6 books?

III- (3 points)

Given that $A(x) = (2x - 3)^2 - (x - 6)^2$ and $B(x) = 2(x - 3)^2 + 9 - x^2$.

1) a. Expand and reduce A(x).

    b. Calculate $A(1 + \sqrt{2})$ .Write the answer in the form $a + b\sqrt{2}$ where a and b are two integers .

2) Factorize A(x).

3) Verify that $B(x) = (x - 3)(x - 9)$.

4) Let $F(x) = \dfrac{A(x)}{B(x)}$

    a. For what values of x, is F(x) defined?

    b. Simplify F(x),then solve the equation $F(x) = -1$.

IV- (2 points)

1) Solve the following system, showing the calculation details :
$$\begin{cases} x - y = -6 \\ 8x - 5y = 0 \end{cases}$$

2) In the opposite figure, the unit of length is the centimeter:
   - The points O, K and P are collinear
   - The points O, B and A are collinear
   - (KB) and (PA) are parallel
   - OK = x, OP = y, KB=5, KP =6 and PA=8.
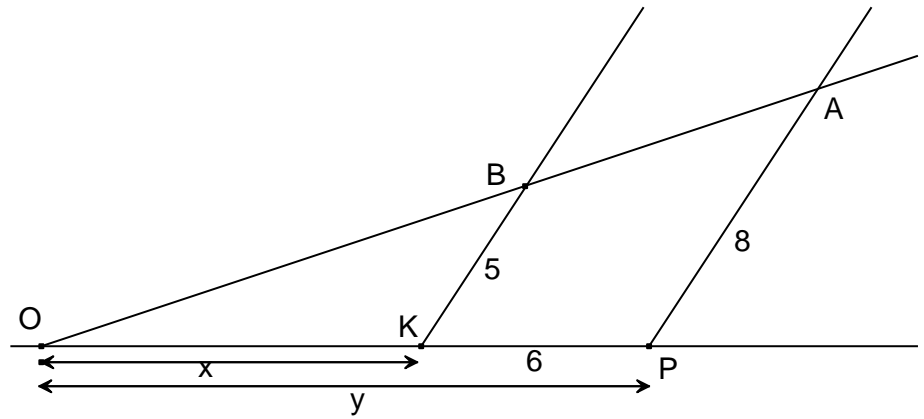
Calculate the length OP.

V-(5.5 points)

In an orthonormal system of axes x'Ox; y'Oy , where the unit of length is the centimeter, consider the line (d) with equation $y = -x - 4$ and the points $A(-1;-3)$, $B(-7;3)$ and C (3;1).

1) a. Verify that A and B are two points on (d).
   b. Plot the points A, B and C . Draw (d).

2) a. Calculate BC.

   b. Knowing that $AB = 6\sqrt{2}$ and $AC = 4\sqrt{2}$, prove that ABC is a right triangle.

3) Let J be the center of the circle circumscribed about the triangle ABC. Calculate the coordinates of J.

4) Prove that the line (d') with equation y = x + 4 is the perpendicular bisector of [AB].

5) a. Calculate the coordinates of the vector $\overrightarrow{BC}$.

   b. Let D be the translate of A under the translation with vector $\overrightarrow{BC}$. Calculate the coordinates of D.

6) Let A' be the symmetric of A with respect to J.
   a. Prove that ABA'C is a rectangle.
   b. Prove that C is the midpoint of [DA'].

VI- (4.5 points)

In the opposite figure :
   - A,E and B are three collinear points
   - AE = 8 and EB = 4
   - (C_1) is the circle with diameter [EB] and center J
   - (C_2) is the circle with diameter [EA] and center I
   - M is a variable point on (C_1)
   - The line (ME) intersects (C_2) at another point N.
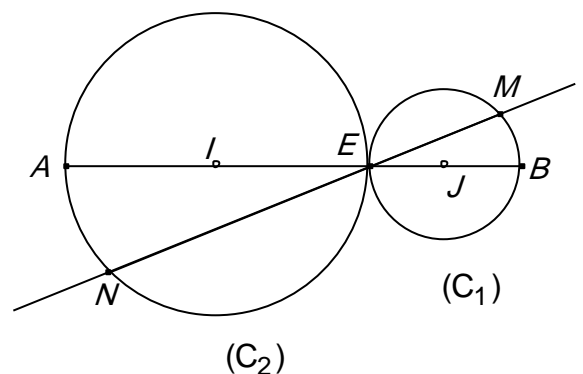
1) Copy this figure .
2) Show that the lines (MB) and (NA) are parallel.
3) Prove that the triangles ANE and BME are similar. Determine the ratio of similarity.
4) Let P be the point defined as $\overrightarrow{MP} = \overrightarrow{ME} + \overrightarrow{MB}$.
   a. Prove that the quadrilateral EPBM is a rectangle.
   b. Deduce that P is a point on (C_1).
5) Denote by K the intersection point of (ME) and (IP) .
   Prove that [MK] is a median in the triangle IMP.
6) The lines (BP) and (AN) intersect at L.
   As M moves on (C_1) , prove that L moves on a circle with diameter to be determined.

Questions		Answers Keys	Grades
I	**1.a**	$A = \dfrac{5\sqrt{18} - 2\sqrt{98}}{2\sqrt{3} \times \sqrt{24} - 4\sqrt{2}} = \dfrac{15\sqrt{2} - 14\sqrt{2}}{12\sqrt{2} - 4\sqrt{2}} = \dfrac{1}{8}$	0.75
	1.b	$A = 0.125 = 1.25 \times 10^{-1}$	0.25
	2	$\tan^2 \alpha = \dfrac{\sin^2 \alpha}{\cos^2 \alpha}, \left(1 - \sin^2 \alpha\right) = \cos^2 \alpha.$ Hence $\tan^2\alpha\,(1-\sin^2\alpha) = \sin^2\alpha.$	0.5
	3	$x = \dfrac{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}{\dfrac{7}{5}} = \dfrac{20}{7}$	0.5
II	**1**	$\overline{X} = \dfrac{0 \times 2 + 2 \times 7 + 4 \times 10 + 6 \times 9 + 7 \times 12}{40} = 4.8$	0.75
	2	<table><tr><td>Number of read books</td><td>0</td><td>2</td><td>4</td><td>6</td><td>7</td><td>Total</td></tr><tr><td>Frequency</td><td>2</td><td>7</td><td>10</td><td>9</td><td>12</td><td>40</td></tr><tr><td>% Relative frequency</td><td>5</td><td>17.5</td><td>25</td><td>22.5</td><td>30</td><td>100</td></tr><tr><td>Central angle</td><td>18°</td><td>63°</td><td>90°</td><td>81°</td><td>108°</td><td>360°</td></tr></table>	2
	3	The number of students is 21	0.25
III	**1.a**	$3x^2 - 27$	0.5
	1.b	$6\sqrt{2} - 18$	0.5
	2	$(2x-3-x+6)(2x-3+x-6) = 3(x-3)(x+3)$	0.5
	3	$B(x) = 2(x-3)^2 - (x^2 - 9) = (x-3)[2(x-3) - (x+3)] = (x-3)(x-9)$	0.5
	4.a	F(x) is defined for $x \neq 3$ and $x \neq 9$	0.25
	4.b	$F(x) = \dfrac{3(x+3)}{x-9}$; $\dfrac{3(x+3)}{x-9} = -1$; then $4x = 0$; $x = 0$	0.75
IV	**1**	$\begin{cases} x - y = -6 \\ 8x - 5y = 0 \end{cases}$, ..., x=10, y=16	1
	2	Refer to Thales: $\dfrac{x}{y} = \dfrac{5}{8}$ and PK=6 :y-x=6 So,we get the given system.OP = y =6 .or……	1
V	**1.a**	The coordinates of A and B verify the equation of (d).	0.5

	1.b	Figure A, B, C and (d)	0.75
	2.a	$BC = \sqrt{104} = 2\sqrt{26}$	0.5
	2.b	$BC^2 = AB^2 + AC^2$; ABC is right.	0.5
	3	J midpoint of [BC] $\quad x_J = \dfrac{-7+3}{2} = -2,\ y_J = \dfrac{3+1}{2} = 2\ \ ; J(-2;2).$	0.5
	4	The perpendicular bisector of [AB] is perpendicular to [AB] at its midpoint (-4 ; 0) .Its slope =1 and its equation is y = x + 4.	0.75
	5.a	$\overrightarrow{BC}\,(10;-2)$	0.5
	5.b	$\overrightarrow{AD} = \overrightarrow{BC},\ D(9;-5)$	0.5
	6.a	ABA'C parallelogram + right angle: rectangle.	0.5
	6.b	A',, C and D are collinear . CA'=CD or……	0.5
VI	**1**		0.25
	2	(MB) and (NA) are perpendicular to (MN).	0.5
	3	$\widehat{BME} = \widehat{ENA} = 90^0\ \widehat{AEN} = \widehat{MEB}$, $\dfrac{EN}{EM} = \dfrac{EB}{EA} = \dfrac{1}{2}$	1
	4.a	EPBM is a parallelogram+ right angle: rectangle..	0.5
	4.b	BPE is right, [EB] diameter then P is a point on (C_1)	0.5
	5	In the triangle IMP, [MK] is a median	0.75
	6	(BP) is perpendicular to (MB) then perpendicular to (AN) so $\widehat{ALB} = 90^o$ and L moves on the circle with diameter [AB].	1