ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المُرشّح الإجابة بالترتيب الذي يناسبه (دون الإلتزام بترتيب المسائل الوارد في المسابقة).

I - (4 points)

A company posts an advertisement for a new product on an internet site for 15 days.

The following table gives the number of persons, in tens, who saw the advertisement after its post date.

Rank of the day $\mathbf{x_i}$	**1**	**2**	**3**	**4**	**5**
Number of persons $\mathbf{y_i}$ (in tens)	**8**	**12**	**25**	**34**	**56**

1) Calculate the linear correlation coefficient and give an interpretation to the value thus found.

2) Write an equation of the regression line $D_{y/x}$ of y in terms of x.

3) Assume that the previous model remains valid. Estimate the number of persons who saw the advertisement on the 10^{th} day after its post date.

4) In fact 1250 persons saw the advertisement on the 10^{th} day. Calculate the percentage error in the previous estimation.

5) Another model of adjustment for this data is given by $y = 1.5x^2 - 2x + 7$.
On the 10^{th} day, which one among the two previous adjustments gives a better estimation? Justify.

II - (4 points)

Consider two identical boxes E and F.

Box E contains 9 red balls and 3 green balls . Box F contains 3 red balls and 5 green balls.

A game consists of selecting at random one of the two boxes, then drawing a ball from the selected box.

A- Consider the following events:

E : « the player selects box E »,

F : « the player selects box F »,

R : « the player draws a red ball ».

1) Calculate the probabilities P(R/E) and $P(E \cap R)$.

2) Prove that $P(R) = \dfrac{9}{16}$.

3) Knowing that the player has drawn a red ball, what is the probability that the ball is drawn from box E?

B- The player replaces the ball drawn in the selected box and plays the same game all over again.

Denote by X the random variable equal to the number of red balls drawn after the two rounds of the game.

1) Determine the three possible values of X.

2) Show that $P(X= 2) = \dfrac{81}{256}$.

3) Determine the probability distribution of X.

III - (4 points)

A sports club had 400 subscribers in 2007. It is noticed that from one year to another the club loses 40% of its subscribers and receives 100 new subscribers.

Denote by a_n the number of subscribers in this club in the year (2007 + n) for all natural numbers n.

That is $a_o = 400$.

1) Show that, for all natural numbers n, $a_{n+1} = 0.6\,a_n + 100$.

2) Consider the sequence (u_n) defined, for all natural numbers n, by $u_n = a_n - 250$.

a- Show that (u_n) is a geometric sequence whose first term and common ratio are to be determined.

b- Find u_n and then a_n in terms of n.

c- Prove that the sequence (a_n) is decreasing.

3) Assume that this model remains valid.

a- In which year will the number of subscribers become less than 260 for the first time?

b-Will the number of subscribers become less than 250? Justify.

IV - (8 points)

Consider the function f defined over $[0; +\infty[$ by $f(x) = (2x+6)e^{-x}$ and denote by (C) its representative curve in an orthonormal system.

A-1) Calculate $\lim\limits_{x\to+\infty} f(x)$ and deduce an asymptote to (C).

2) Verify that $f'(x) = -2(x+2)e^{-x}$ and set up the table of variations of the function f.

3) Draw (C).

4) a- Show that the function F defined over $[0 ; +\infty[$ by $F(x) = (-2x-8)e^{-x}$ is an anti-derivative of f.

b- Deduce the area of the region bounded by (C), the x-axis and the lines with equations $x = 0$ and $x = 1$.

B- A company produces batteries. The demand is given by f(x) expressed in thousands of batteries with x

being the unit price expressed in thousands of LL. $(0.1 \le x \le 4)$

1) What is the demand corresponding to a price of 1 000 LL?

2) a- Calculate the elasticity E(x) of the demand.

b- Calculate the unit price for which the elasticity is equal to 1.

3) Justify that the revenue is given by $R(x) = \left(2x^2 + 6x\right)e^{-x}$ and that it is expressed in millions LL.

4) The following table represents the variations of R.

x	**0.1**		**1.303**		**4**
R(x)		↗		↘	

a- Complete the table and calculate the maximum revenue.

b- How many demanded batteries correspond to this maximum revenue?

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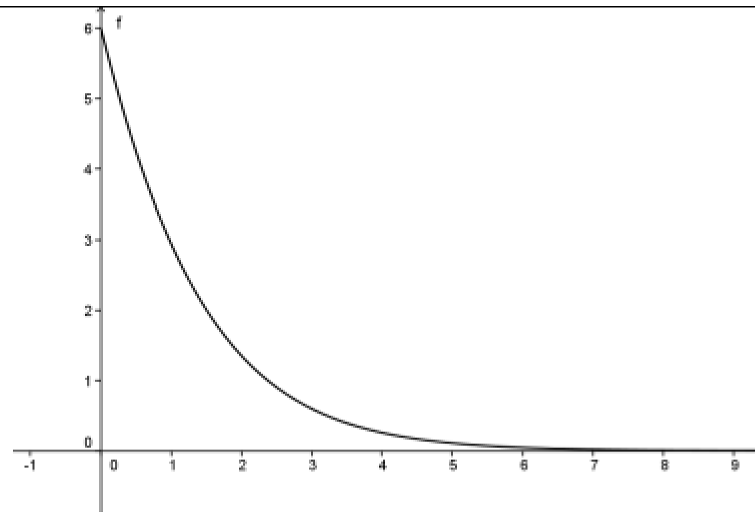
Q_1	Answers	Mark
1	r = 0.969 . There is a strong positive correlation between the two variables.	1
2	y = 11.8x –8.4.	1
3	For x=10, y = 109.6 . So 1096 persons.	1
4	$\frac{1250-1096}{1250}\times 100 = 12.32\%$.	2
5	The second model gives y = 137 . So 1 370 persons. 1 250 – 1 096 > 1 370 – 1250, The second model gives a better estimation.	2
Q_2	Answers	Mark
A.1	$P(R/E)=\frac{9}{12}=\frac{3}{4}$. $P(R\cap E)=\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$.	1.5
A.2	$P(R)=P(R\cap E)+P(R\cap F)=\frac{3}{8}+\frac{1}{2}\times\frac{3}{8}=\frac{9}{16}$.	1
A.3	$P(E/R)=\frac{P(E\cap R)}{P(R)}=\frac{3}{8}\times\frac{16}{9}=\frac{2}{3}$.	1
B.1	The possible values of X are 0, 1 and 2.	0.5
B.2	P(X=2) = P(RR)= $\left(\frac{9}{16}\right)^2=\frac{81}{256}$.	1
B.3	P(X= 0) = P(GG), P(X=1) = P(GR) + P(RG), P(X=1) = P(GR) + P(RG) $=2\times\frac{9}{16}\times\frac{7}{16}=\frac{63}{128}$. x_i: 0 ; 1 ; 2 p_i: $\left(\frac{7}{16}\right)^2=\frac{49}{256}$; $2\times\frac{9}{16}\times\frac{7}{16}=\frac{63}{128}$; $\left(\frac{9}{16}\right)^2=\frac{81}{256}$	2
Q_3	Answers	Mark
1	$a_{n+1}=(1-0.4)\times a_n+100=0.6a_n+100.$	1
2a	$u_{n+1}=a_{n+1}-250=0.6a_n+100-250=0.6(a_n-250)=0.6a_n$. ($u_n$) is a geometric sequence with common ratio 0.6 and first term u_0 = 150.	1.5
2b	$u_n=u_0\times r^n=150(0.6)^n$ and $a_n=150(0.6)^n+250.$	1
2c	(u_n) is a geometric sequence with common ratio r with $0<r<1$ and first term positive hence (u_n) is decreasing. $a_{n+1}-a_n=u_{n+1}-u_n<0$ since (u_n) is decreasing. Therefore (a_n) is decreasing.	1
3a	$150(0.6)^n+250<260 \Leftrightarrow 150(0.6)^n<10 \Leftrightarrow (0.6)^n<\frac{1}{15}$, so $n>\frac{\ln(1/15)}{\ln 0.6} \Leftrightarrow n>5.3.$ Then in 2013, the number of subscribers of the club will become less than 250 for the first time.	1.5
3b	The number of subscribers decreases but cannot be less than 250 since $a_n-250=150(0.6)^n>0.$ The answer is **NO.**	1

<table>
<tr><th>Q4</th><th>Answers</th><th>Mark</th></tr>
<tr><td>A1</td><td>$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(2xe^{-x}+6e^{-x}\right)=0$, the line with equation y = 0 is an asymptote to (C).</td><td>1</td></tr>
<tr><td>A2</td><td>$f'(x)=\left[2e^{-x}-(2x+6)e^{-x}\right]=-2(x+2)e^{-x}$.
<table><tr><td>x</td><td>0 $+\infty$</td></tr><tr><td>f '(x)</td><td>–</td></tr><tr><td>f(x)</td><td>6 ↘ 0</td></tr></table></td><td>2</td></tr>
<tr><td>A3</td><td></td><td>1.5</td></tr>
<tr><td>A.4a</td><td>$F'(x)=\left[-2e^{-x}-(-2x-8)e^{-x}\right]=(2x+6)e^{-x}=f(x)$.</td><td>1</td></tr>
<tr><td>A.4b</td><td>$A=\left[-2(x+4)e^{-x}\right]_0^1=-10e^{-1}+8=4.321$.</td><td>1.5</td></tr>
<tr><td>B.1</td><td>For a price of 1000LL, x = 1, f(1) = 2.943 . Then the demand is 2934 batteries.</td><td>1</td></tr>
<tr><td>B.2a</td><td>$E(x)=\frac{-xf'(x)}{f(x)}=\frac{2x(x+2)e^{-x}}{2(x+3)e^{-x}}=\frac{x(x+2)}{x+3}$.</td><td>1</td></tr>
<tr><td>B.2b</td><td>x(x + 2) = x + 3 ; $x^2+x-3=0$; x = $\frac{-1+\sqrt{13}}{2}$ =1.303 or x = $\frac{-1-\sqrt{13}}{2}$ (rejected)
The unit price is 1303 LL.</td><td>1</td></tr>
<tr><td>B.3</td><td>The revenue R(x) = xf(x)= $x(2x+6)e^{-x}=\left(2x^2+6x\right)e^{-x}$. Since the demand is in thousands of batteries and the price in thousands LL , the revenue is in thousands of batteries × thousands LL, thus in millions LL.</td><td>1.5</td></tr>
<tr><td>B.4a</td><td><table><tr><td>x</td><td>0.1   1.303   4</td></tr><tr><td>R(x)</td><td>0.56 ↗ 3.047 ↘ 1.026</td></tr></table>The maximum revenue is 3 047 000LL</td><td>1.5</td></tr>
<tr><td>B.4b</td><td>For x = 1.303. f(1.303) = 2.338 . Therefore 2338 demanded batteries.</td><td>1</td></tr>
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