المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة: الرياضيات الشهادة: الثانوية العامة الفرع: علوم الحياة نموذج رقم -١- المدّة : ساعتان

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

In the space referred to an orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the points E(2 ; 2 ; 0) and F(0 ;0 ;-2), the plane (P) with equation x+y+z – 1=0 and the line (d) with parametric equations

$$\begin{cases} x = -t - 1 \\ y = t + 5 \ (t \in \mathbb{R}). \\ z = 3t + 9 \end{cases}$$

Denote by H the orthogonal projection of E on (P)

1)
   a- Verify that E is a point on (d).
   b- Determine the coordinates of A ,the intersection point of (d) and (P).

1)
   a-Verify that F is the symmetric of E with respect to (P).
   b-Write a system of parametric equations of the line (Δ) bisector of the angle EAF .

2) Let (Q) be the plane containing F and parallel to (P) and K the intersection point of (d) and the plane (Q).
   a) Write an equation of the plane (Q).
   b) Verify that A is the midpoint of [EK].

II- (4points)

U_1 and U_2 are two boxes so that :

U_1 contains 10 balls : 6 red and 4 black .
U_2 contains 10 balls: 5 red and 5 black .

A die numbered 1 through 6 is rolled .

. If this die shows 1 or 2 , then two balls are randomly selected at a time from the box U_1 .

.Otherwise , two balls are randomly selected one after another with replacement from the box U_2

Consider the following events :

U_1 :''The selected box is U_1 .''
U_2 :''The selected box is U_2 .''
R :''The selected balls are red ''.

1) calculate $P(R / U_1), P(R \cap U_1)$

2) verify that P(R)= $\frac{5}{18}$.

3) The two balls selected are red , calculate the probability that they come from U_1 .

4) Let X be the random variable that is equal to the number of the red balls selected .

a) verify that $P(X=1)=\frac{23}{45}$.

b) Determine the probability distribution of X

III- (4points)

The complex plane is referred to an orthonormal system $(O;\vec{u},\vec{v})$.

Denote by A, Band C the points with respective affixes $z_A=2-3i$, $z_B=i$ et $z_C=6-i$.

1) Calculate $\frac{z_B-z_A}{z_C-z_A}$. Deduce the nature of the triangle ABC.

For each point M with affix distinct from i , we associate the point M' with affix :

$z'=\frac{i(z-2+3i)}{z-i}$.

2) If z= 1– i , determine the exponential form of z'.

3)a- If $z'=2i$, find the algebraic form of z .(Denote by E the image point of z obtained).

b- Verify that E is a point on the line (AB).

4)Prove that if M moves on the perpendicular bissetor of [AB] then M' moves on a circle with center O and a radius to be detremined .

IV- (8points)

Consider the function defined over ℝ by : $f(x)=\ln(e^{2x}-e^{x}+1)-1$. (C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1. Determine the limit of f at $-\infty$ and deduce an asymptote to (C).
2. a.Show that the line (D) with equation y=2x – 1 is an asymptote to (C).
   b.Discuss according to x , the relative position of (C) and (D).
3. Calculate $f'(x)$ and set up the table of variations of f .
4. Determine the coordinates of A ,where the tangent to (C) is parallel to (D).
5. Draw (D) and (C).
6. a) For $x \geq 0$,prove that f has an inverse function g whose domain of definition should be determined .
7. Let (G) be the representative curve of g and (D') its asymptote. Draw (G) and (D') in the same system as that of (C) .
8. Suppose that the area of the region bounded by (C) , (x'Ox),(y'Oy) is A .
   Calculate , in terms of A , the area of the region bounded by (G) ,its asymptote and the y-axis .

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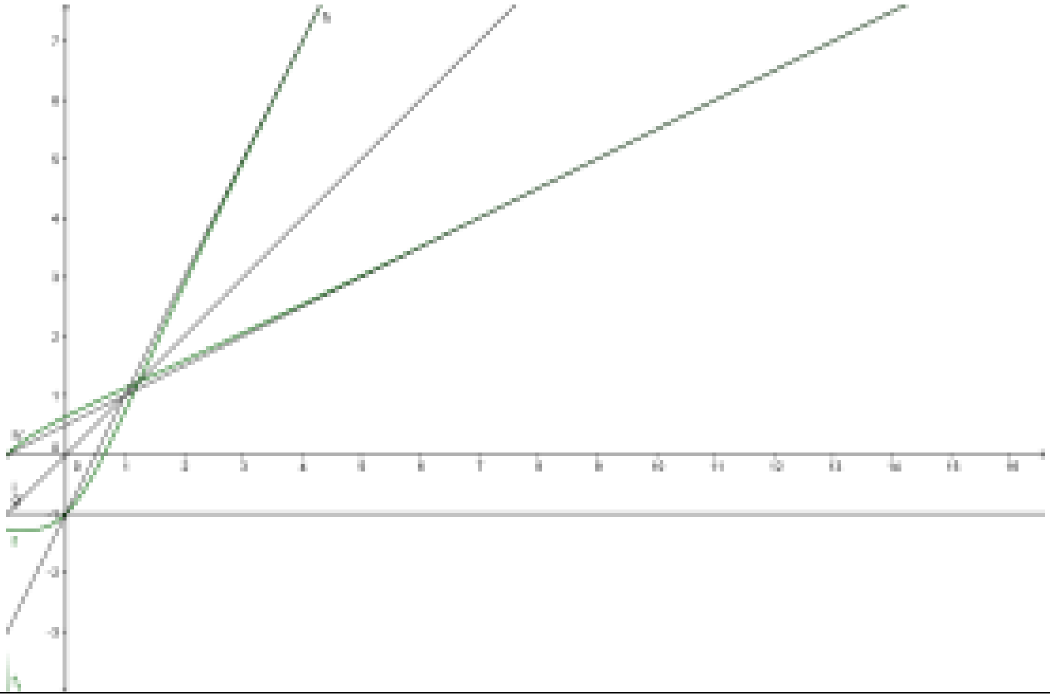
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

QI		**Notes**
1.a	E is a point on (d) for $t=-3$	**0,5**
1.b	A(3 ; 1 ; −3)	**0,5**
2.a	$\overrightarrow{EF}(-2,-2,-2) \Rightarrow (EF)\perp(p)$ Let H(1,1,-1) be the midpoint of [EF] and verify that H is on (P).	**1**
2.b	$(AH):\begin{cases} x=-2m+3 \\ y=1 \\ z=2m-3 \end{cases}$ the perpendicular bisector of [EF]	**0.5**
3.a	(Q): x+y+z+2=0	**0.5**
3.b	K(4,0,-6) $=(d)\cap(Q)$ and A is the midpoint of [EK].	**1**

QII		**Notes**
1	$P\left({}^{R}\!/_{U_1}\right)=\frac{C_6^2}{C_{10}^2}=\frac{1}{3} \quad P(R\cap U_1)=P\left({}^{R}\!/_{U_1}\right)\times P(U_1)=\frac{1}{9}$	**0,5**
2	$P(R)=P(R\cap U_1)+P(R\cap U_2)=\frac{1}{9}+\frac{5}{10}\times\frac{5}{10}\times\frac{2}{3}=\frac{5}{18}$	**1**
3	$p\left({}^{U_1}\!/_{R}\right)=\frac{P(R\cap U_1)}{P(R)}=\frac{2}{5}$	**0,5**
4	$P(X=1)=\left(\frac{6\times 4}{C_{10}^2}\right)\times\frac{1}{3}+2\left(\frac{5}{10}\times\frac{5}{10}\times\frac{2}{3}\right)=\frac{23}{45}$	**1**
5	$\begin{array}{\|c\|c\|c\|c\|}\hline X=x_i & 0 & 1 & 2 \\ \hline p(X=x_i) & \frac{19}{90} & \frac{23}{45} & \frac{5}{18} \\ \hline\end{array}$ p(X=0)=1-P(X=1)-P(X=2)	**1**

QIII		**Notes**
1	ABC is a right isosceles triangle.	**1**
2	$z'=e^{\frac{-\pi}{2}i}$	**0,5**
3.a	$z_E=-2+5i$	**0,5**
3.b	$\frac{z_A-z_E}{z_B-z_E}=2$ then A,E and B are collinear .	**0,5**
4.a	$\|z'\|=\frac{\|i\|\|z-z_A\|}{\|z-z_B\|}$, then $OM'=\frac{AM}{BM}$	**0,5**
4.b	OM′=1, then M′ is on the circle with center O and radius 1	**1**

<table>
<tr><th>QIV</th><th></th><th>Notes</th></tr>
<tr><td>1</td><td>$\lim\limits_{x\to-\infty} f(x) = -1$ then y=–1 is a horizontal asymptote .</td><td>0.5</td></tr>
<tr><td>2.a</td><td>$\lim\limits_{x\to+\infty} (f(x) - 2x + 1) = 0$ then y=2x – 1 is on O.Asymptote .</td><td>1</td></tr>
<tr><td>2.b</td><td>si x<0 then (C) is above (D)
si x>0 then (C) is below (D)
si x=0 (C) intersects (D)</td><td>1</td></tr>
<tr><td>3</td><td>$f'(x) = \dfrac{e^x\left(2e^x - 1\right)}{e^{2x} - e^x + 1}$</td><td>0,5</td></tr>
<tr><td>3</td><td><table>
<tr><td>x</td><td>$-\infty$</td><td>–ln2</td><td>$+\infty$</td></tr>
<tr><td>f′(x)</td><td>–</td><td>0</td><td>+</td></tr>
<tr><td>f(x)</td><td>↘</td><td></td><td>↗</td></tr>
</table></td><td>0,5</td></tr>
<tr><td>4</td><td>$f'(x) = 2\ then\ A(ln2; ln3 - 1)$</td><td>1</td></tr>
<tr><td>5</td><td></td><td>1</td></tr>
<tr><td>6.a</td><td>For x ≥0 , f defined ,continuous and strictly increasing then f has an inverse function g and $D_g = [-1;+\infty[$</td><td>0,5</td></tr>
<tr><td>6.b</td><td>On the figure.</td><td>1</td></tr>
<tr><td>7</td><td>Because of the symetry with respect to y=x then Area = A – (area of the region bounded by (D’) and the coordinates axes).
Then Area = A – area of the triangle bounded by the coordinates axes = A- 0.25.</td><td>1</td></tr>
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